1 Computation proofs

1.1 Multivariate case toy example

Fact 1 *Let* $[D_w]_{j,j} = w_j \in \mathbb{C}^{k\times k}$ *and*

$$V_d = \begin{pmatrix} e^{i\pi\mu_1^{(1)}} & \dots & e^{i\pi\mu_1^{(k)}} \\ \vdots & \ddots & \vdots \\ e^{i\pi\mu_d^{(1)}} & \dots & e^{i\pi\mu_d^{(k)}} \end{pmatrix} \in \mathbb{C}^{d\times k},$$

where $\mu_n^{(i)}$ *are i.i.d.* $\sim \mathcal{U}([-1,+1])$. *Furthermore, let* $F_{n_1,n_2,n_3} = f(s)\big|_{s=e_{n_1}+e_{n_2}+e_{n_3}}$, *for all* $n_1, n_2, n_3 \in [d]$. *Then,* F *admits the tensor decomposition* $F = V_d \otimes V_d \otimes (V_d D_w)$.

Proof We wish to show that

$$f(e_1 + e_2 + e_3) = \sum_{j=1}^{k} w_j e^{i\pi(\mu_1^{(j)}+\mu_2^{(j)}+\mu_3^{(j)})}.$$

To do so, we first start by computing the matrix product $V_d D_w$. We now have

$$\begin{aligned} V_d D_w &= \begin{pmatrix} e^{i\pi\mu_1^{(1)}} & \dots & e^{i\pi\mu_1^{(k)}} \\ \vdots & \ddots & \vdots \\ e^{i\pi\mu_d^{(1)}} & \dots & e^{i\pi\mu_d^{(k)}} \end{pmatrix} \begin{pmatrix} w_1 & 0 & \dots & 0 \\ 0 & w_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & w_k \end{pmatrix} \\ &= \begin{pmatrix} w_1 e^{i\pi\mu_1^{(1)}} & w_1 e^{i\pi\mu_1^{(2)}} & \dots & w_1 e^{i\pi\mu_1^{(k)}} \\ w_1 e^{i\pi\mu_2^{(1)}} & w_1 e^{i\pi\mu_2^{(2)}} & \dots & w_1 e^{i\pi\mu_2^{(k)}} \\ \vdots & \vdots & \ddots & \vdots \\ w_1 e^{i\pi\mu_d^{(1)}} & w_1 e^{i\pi\mu_d^{(2)}} & \dots & w_1 e^{i\pi\mu_d^{(k)}} \end{pmatrix}, \end{aligned}$$

so that

$$\begin{aligned} F_{n_1,n_2,n_3} &= \sum_{j=1}^{k} [V_d]_{n_1,j} [V_d]_{n_2,j} [V_d D_w]_{n_3,j} \\ &= \sum_{j=1}^{k} e^{i\pi\mu_{n_1}^{(j)}} e^{i\pi\mu_{n_2}^{(j)}} w_j e^{i\pi\mu_{n_3}^{(j)}} \\ &= \sum_{j=1}^{k} w_j e^{i\pi(\mu_{n_1}^{(j)}+\mu_{n_2}^{(j)}+\mu_{n_3}^{(j)})} = f(e_1, e_2, e_3), \end{aligned}$$

as required. ∎

1.2 Tensor decomposition in the exact recovery case