2 Functions of 1 variable

$\boxed{2.1}$ $0°c = 32°F$ $y = mx + b$

$100°c = 212°F$

$$\begin{cases} 32x + b = 0 \\ 212x + b = 100 \end{cases} \Leftrightarrow \begin{cases} b = -32x \\ 212x - 32x = 100 \end{cases} \Leftrightarrow \begin{cases} b = -32x \\ 180x = 100 \end{cases} \Leftrightarrow \begin{cases} b = -32 \cdot \frac{5}{9} \\ x = 5/9 \end{cases} \Leftrightarrow$$

$$\begin{cases} b = \frac{-160}{9} \\ \cancel{x} = \frac{5}{9} \end{cases}$$ We got the equation $y = \frac{5}{9}x + \frac{160}{9}$

Condition $x = y$ $x = \frac{5}{9}x - \frac{160}{9} \Rightarrow x - \frac{5}{9}x = -\frac{160}{9} \Rightarrow$

$$x\left(1 - \frac{5}{9}\right) = -\frac{160}{9} \Leftrightarrow x \cdot \frac{4}{9} = -\frac{160}{9} \Leftrightarrow x = \frac{-\frac{160}{\cancel{9}}}{\frac{4}{\cancel{9}}} = \underline{\underline{-40}}$$

$\boxed{2.2}$ $f(x) = 3x - 12$ $f(y) = 0$ $y = ?$

$f(y) = 3y - 12 = 0 \Leftrightarrow 3x = 12 \Leftrightarrow y = 4$

$\boxed{2.3}$ $9^{x^2 - 6x + 2} = 81 \Leftrightarrow 9^{x^2 - 6x + 2} = 9^2 / \log_9 \Leftrightarrow x^2 - 6x + \cancel{2} = \cancel{2} \Rightarrow x(x-6) = 0$

$\Leftrightarrow x_1 = 0$ $x_2 = 6$

$\boxed{2.4}$ 3% in a year $3 \cdot 100 = 300\%$, so in 100 years

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$\boxed{2.5}$ $\log_\pi\left(\frac{1}{\pi^5}\right) = x \Leftrightarrow \pi^x = \frac{1}{\pi^5} \Leftrightarrow \pi^x = \pi^{-5} \Leftrightarrow x = -5$

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3. Calculus

$\boxed{3.1}$ $\sum_{n=0}^{\infty}\left(\frac{1}{5^n} + 0{,}3^n\right) = \sum_{n=0}^{\infty}\left(\frac{1}{5^n} + \left(\frac{3}{10}\right)^n\right)$

The sum of $\Sigma\left(\frac{1}{5}\right)^n$ is: $\frac{1}{1 - \frac{1}{5}} = \frac{5}{4}$

— " — of $\left(\frac{3}{10}\right)^n$: $\frac{1}{1 - \frac{3}{10}} = \frac{10}{7}$

but because both $\Sigma \frac{1}{5^n}$ converges and $\left(\frac{3}{10}\right)^n$ converges,

Ratio test: $\lim_{n \to \infty} \frac{\left(\frac{1}{5}\right)^{n+1}}{\left(\frac{1}{5}\right)^n} = \frac{\cancel{\left(\frac{1}{5}\right)^n} \cdot \left(\frac{1}{5}\right)^1}{\cancel{\left(\frac{1}{5}\right)^n}} = \frac{1}{5} < 1$, and conv

$\lim_{n \to \infty} \frac{\cancel{\left(\frac{3}{10}\right)^n} \cdot \frac{3}{10}}{\cancel{\left(\frac{3}{10}\right)^n}} = \frac{3}{10} < 1$, converges

so, $\frac{5}{4} + \frac{10}{7} = \frac{35 + 40}{28}$

$= \frac{75}{28}$

Math Problems

1 Elementary algebra

Problem 1.1

$$\frac{x^{32}}{x^9 x^2} \cdot \frac{x^7}{x^2} = \frac{x^{39}}{x^{13}} = x^{26}$$

1.2

$$8^2 \cdot 4^x \cdot 2^x = 8^4$$

$$2^6 \cdot 2^{2x} \cdot 2^x = 2^{12}$$

$$2^{(6+3x)} = 2^{12} \,/\, \log_2$$

$$6+3x=12$$

$$3x=6 \Rightarrow x=2$$

1.3 if $\frac{x}{y}=3$ $\quad x^{-4}y^4$

$$\frac{y}{x}=\frac{1}{3} \qquad \frac{y^4}{x^4}=\frac{1^4}{3^4}$$

1.4

$$\frac{\sqrt{4^{15}}}{\sqrt{16^7}} = \frac{4^{\frac{15}{2}}}{16^{\frac{7}{2}}} = \frac{4^{\frac{15}{2}}}{4^7} = \frac{\cancel{4^7} \cdot 4^{\frac{1}{2}}}{\cancel{4^7}} = 2$$

1.5

a) $x+(y+z)=(y+x)+z$ True

b) $y(x+z)=xy+zy \rightarrow$ True

c) $x^{y+z}=x^z+x^y$ False $\quad x^{y+z}=x^y \cdot x^z$

d) $\frac{x^z}{x^y}=x^{y-z}$ False $\quad \frac{x^z}{x^y}=x^{(z-y)}$

1.6 $\ln(x) \geq e \,/\, {}^{\wedge}e$

$$e^{\ln(x)} \geq e^e$$

$$x \geq e^e$$

$\boxed{3.8}$ $f(x,y) = x^3 - y^2$

$f(2,3) = 2^3 - 3^2 = 8 - 9 = -1$

$\boxed{3.9}$ $f(x,y) = \ln(x - 3y)$

$x - 3y > 0$

$\boxed{x > 3y}$

$\boxed{3.10}$

$$\frac{\partial}{\partial x}\left(x^5y^7 + \frac{x^2}{y^3}\right) = 5x^4 \cdot y^7 + \frac{1}{y^3}2x$$

$\boxed{3.11}$ $f(x,y) = \sqrt{xy} - x - y$

$$\frac{\partial f(x,y)}{\partial x} = \frac{y}{2\sqrt{xy}} - 1 = 0 \qquad \frac{1}{2\sqrt{xy}} = t \Rightarrow \begin{cases} t \cdot y - 1 = 0 \\ t \cdot x - 1 = 0 \end{cases} \Leftrightarrow \begin{cases} y = \frac{1}{t} \\ x = \frac{1}{t} \end{cases} \Rightarrow x = y$$

$$\frac{\partial f(x,y)}{\partial y} = \frac{x}{2\sqrt{xy}} - 1 = 0$$

$$\frac{x}{2\sqrt{x^2}} - 1 = 0 \Rightarrow \frac{\cancel{x}}{2\cancel{x}} - 1 = 0 \Rightarrow \frac{1}{2} = 1 \perp$$ which means it doesn't exist

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$\boxed{3.12}$ $x^2y^2 \to \max$ s.t. $2x + y = 9 \Rightarrow 9 - 2x - y = 0$

$$L = x^2y^2 + \lambda(9 - 2x - y)$$

$$\frac{\partial L}{\partial x} = 2xy^2 - 2\lambda = 0 \Rightarrow \cancel{-2\lambda = -2xy^2 \Rightarrow \lambda = 2x^2y}$$

$$\frac{\partial L}{\partial y} = 2x^2y - \lambda = 0 \Rightarrow$$

$$\frac{\partial L}{\partial \lambda} = 9 - 2x - y = 0$$

$$\begin{cases} -2\lambda = -2xy^2 \Rightarrow \lambda = xy^2 \Rightarrow \\ -\lambda = -2x^2y \Rightarrow \lambda = 2x^2y \Rightarrow \\ 9 - 2x - 2x = 0 \Rightarrow 9 - 4x = 0 \Rightarrow -4x = -9 \Rightarrow \boxed{x = \frac{9}{4}} \quad y = \boxed{\frac{18}{4}} \end{cases} \quad xy^2 = 2x^2y \Rightarrow \underline{y = 2x}$$

$\boxed{3.2}$ $\lim_{x\to 5} \frac{x^2-25}{x-5} = \lim_{x\to 5} \frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}} = \lim_{x\to 5}(x+5) = \underline{\underline{10}}$

$\boxed{3.3}$

$f(x) = x^3-4$

$f'(x) = 3x^2$

The slope at the point is $3\cdot(-2)^2 = 3\cdot 4 = \underline{\underline{12}}$

$\boxed{3.4}$ $f(x) = \frac{x^5+3}{x^2-1} = \frac{(x^5+3)'\cdot(x^2-1) - (x^5+3)\cdot(x^2-1)'}{(x^2-1)^2} = \frac{5x^4(x^2-1) - (x^5+3)\cdot 2x}{(x^2-1)^2}$

$\boxed{3.5}$ $f(x) = x^9+3$,

$\frac{\partial f(x)}{\partial x} = 9x^8 \qquad \frac{\partial f^2(x)}{\partial x^2} = 9\cdot 8\cdot x^7 = 72\cdot x^7$

$\boxed{3.6}$ $f(x) = \frac{1}{x}$, continious at 0

$\lim_{x\to 0^-} \frac{1}{x} = -\infty$, and $\lim_{x\to 0^+} \frac{1}{x} = +\infty$, which proves that is not continous at 0

$\boxed{3.7}$

$f(x) = 4x^3 - 12x$

$f'(x) = 4\cdot 3\cdot x^2 - 12 = 12x^2 - 12 = 0 \qquad 12(x^2-1) = 0 \Leftrightarrow$

$x^2-1=0 \qquad (x-1)(x+1) = 0 \quad \begin{array}{l} x=1 \to \min \\ x=-1 \to \max \end{array}$

$f''(x) = 12\cdot 2\cdot x = 24x \qquad x=0 \to$ inflection point

5 Probability theory

$\boxed{5.1}$ $\Omega = \{(a,b) \mid a,b \in \{1,2,3,4,5,6\}\}$

$\boxed{5.2}$ A: person is a drug user $\quad P(A) = 0{,}1$

$\bar{A}$: — " — not a drug user $\quad P(\bar{A}) = 0{,}99$

B: test positive

$\bar{B}$: test negative

$P(B \setminus A) = 0{,}98$

$P(\bar{B} \setminus \bar{A}) = 0{,}997$

$$P(A \setminus B) = \frac{P(B\setminus A)\cdot P(A)}{P(B\setminus A)\cdot P(A) + P(B\setminus \bar{A})\cdot P(\bar{A})} = \frac{0{,}98\cdot 0{,}1}{0{,}98\cdot 0{,}1 + 0{,}02\cdot 0{,}99} = \frac{0{,}098}{0{,}1178}$$

$\approx \underline{\underline{0{,}83}}$

$\boxed{5.3}$ A: # of times ended with 5

$\Omega = \{0, 1, 2 \ldots 20\}$

but this is a classic case of binomiae distribution, so the expected value is $E(X) = n\cdot p = 20 \cdot \frac{1}{6} = 3{,}33(3)$

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4 Linear Algebra

$\boxed{4.1}$ $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \\ 7 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 & 1 \\ 9 & 1 & 5 \end{bmatrix}$

$B \cdot A = \begin{bmatrix} 1 & 0 & 1 \\ 9 & 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 5 \\ 2 & 1 \\ 7 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 11 \\ 55 & 76 \end{bmatrix}$

$\boxed{4.2}$ $A = \begin{bmatrix} 5 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} \cdot B = \begin{bmatrix} 8 & 4 & 0 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 46 & 23 & 6 \\ 2 & 1 & 2 \\ 12 & 6 & 4 \end{bmatrix}$

$\boxed{4.3}$ $\begin{bmatrix} e & 93 & 4.7 \\ 2 & 6.1 & 4.22 \\ 4 & \pi & 0 \end{bmatrix} = \begin{bmatrix} e & 2 & 4 \\ 93 & 6.1 & \pi \\ 4.7 & 4.22 & 0 \end{bmatrix}$

$\boxed{4.4}$

$\begin{bmatrix} 2 & 6 \\ 2 & 8 \end{bmatrix} = 16 - 12 = 4$