# Introduction to Compartmental Modeling

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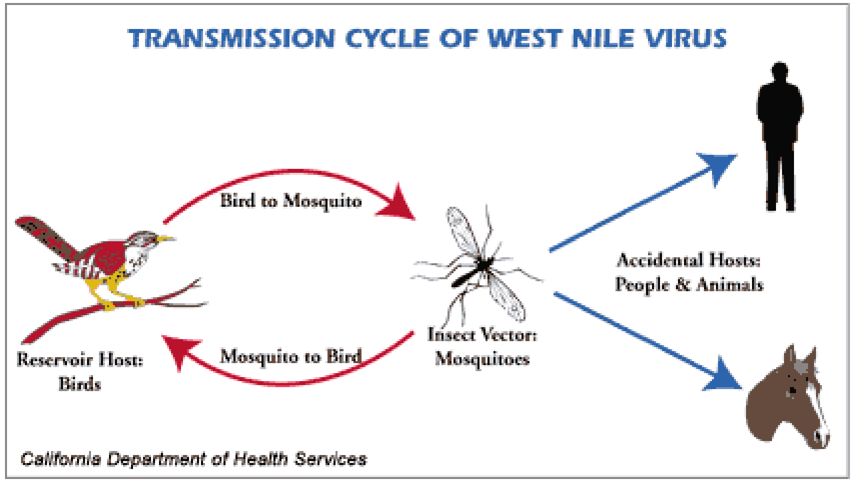
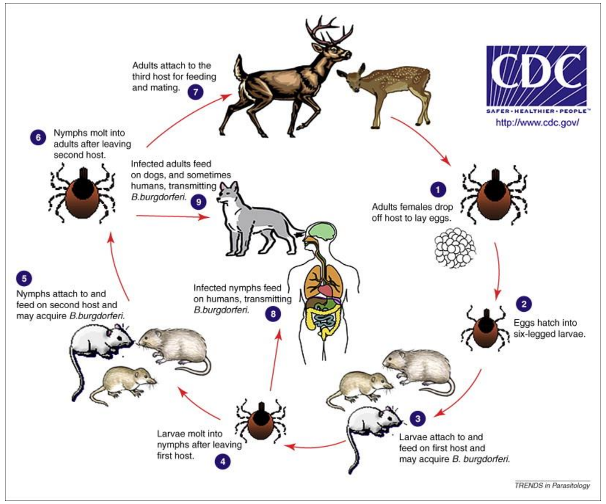
# Objectives

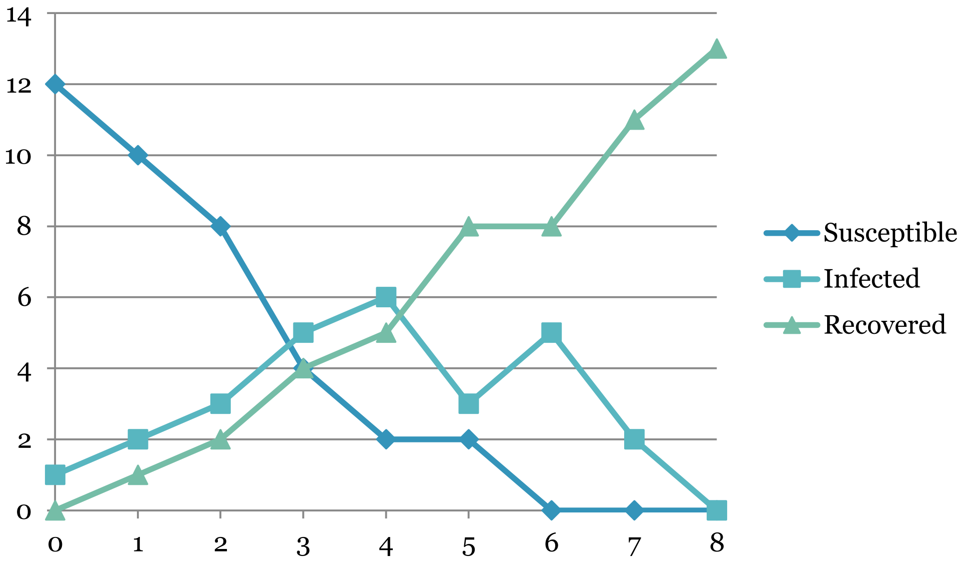
- Students should be able to draw a conceptual compartmental model for any system of interest
- Students should be able to interpret a conceptual compartmental model and translate it into a system of differential equations
- Students should be able to interpret a system of differential equations and work backwards to construct the conceptual compartmental model represented by the equations

# Modeling Infectious Diseases

- Infectious disease research has ethical limitations that many other disciplines do not
- Often rely on modeling to explore disease systems



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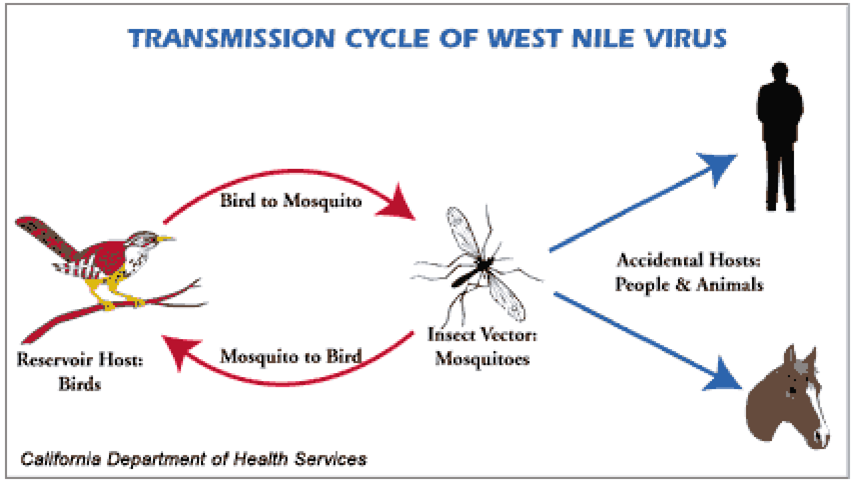


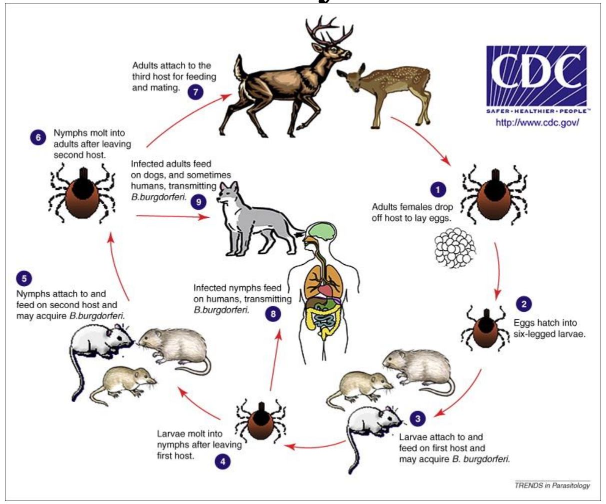


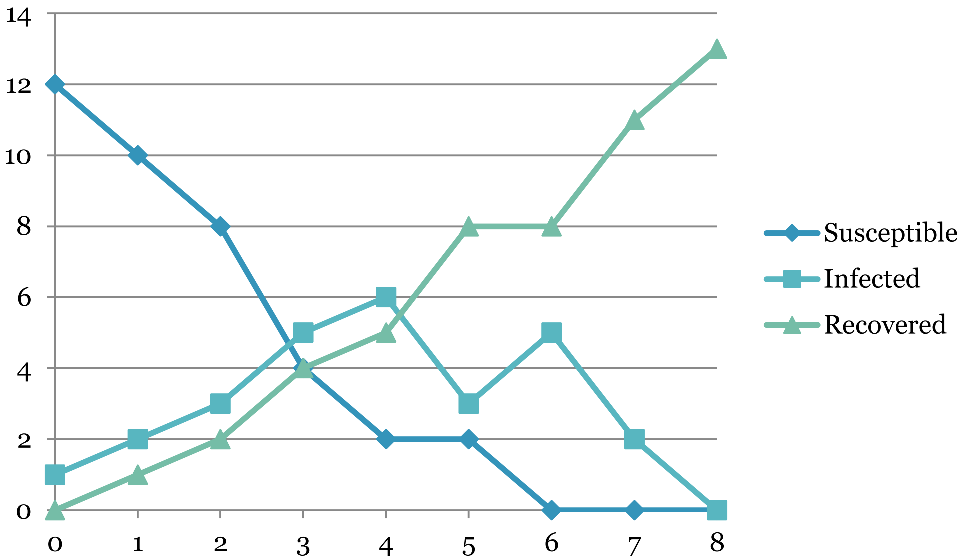


Modeling = Representing what we think is happening with MATH

DON'T BE AFRAID OF MATH!

# How do we make compartmental models?

- Identify PROCESS (e.g., disease transmission) of interest
- Identify COMPARTMENTS (e.g., populations, groups of individuals)
- Identify TRANSITIONS (e.g., how individuals move between compartments)
- Convert conceptual diagram to SYSTEM OF EQUATIONS

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|---|---|---|
| Susceptible | Infected | Recovered |

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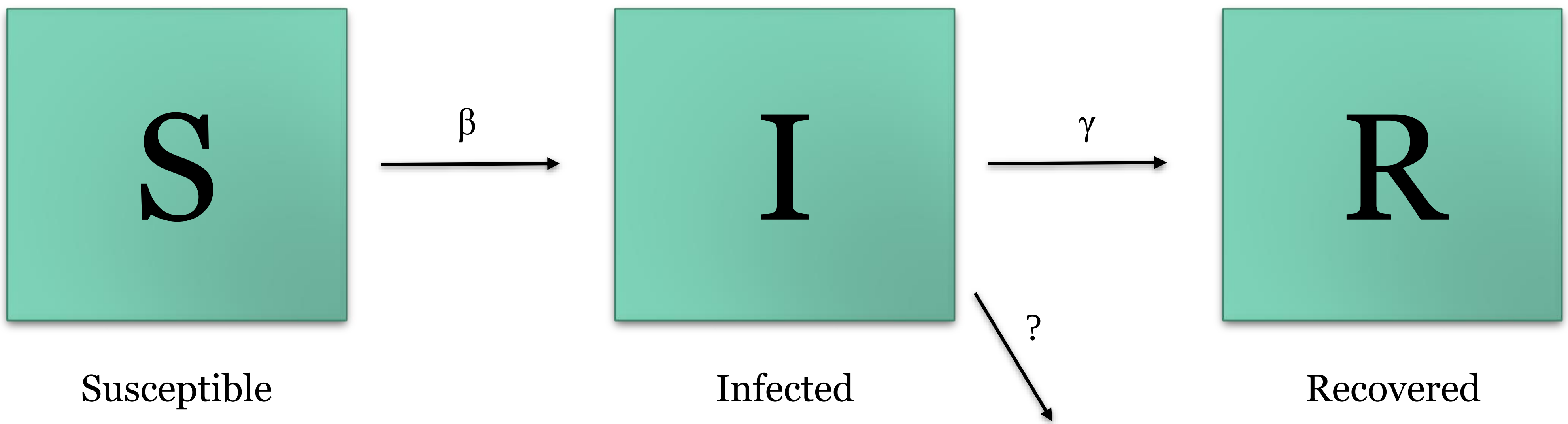
- *Do all individuals move from I to R? What else could happen to them?*



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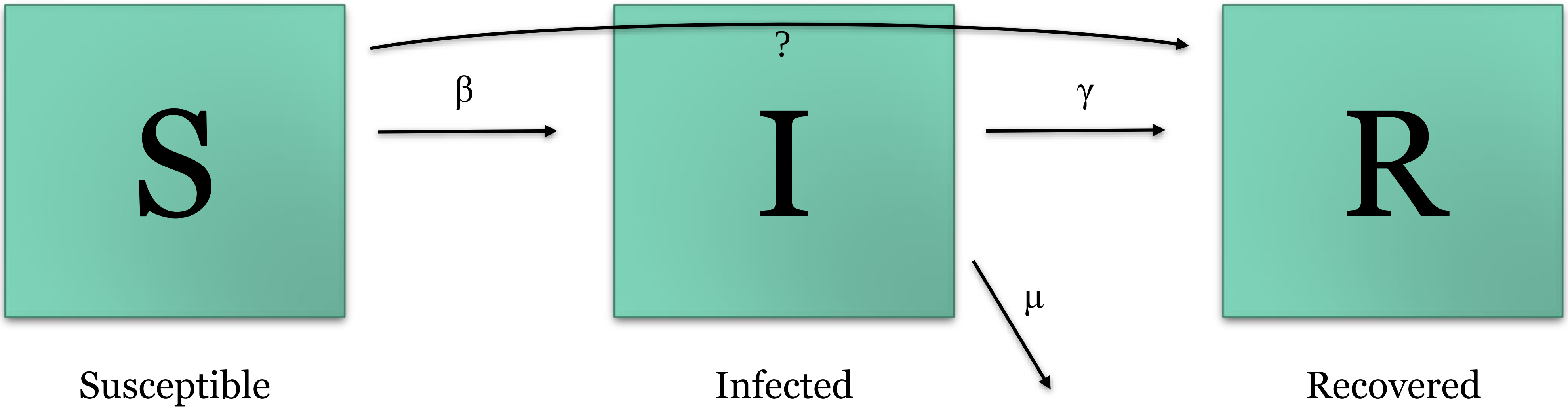
- Do all individuals move from I to R? What else could happen to them? (PATHOGEN-INDUCED MORTALITY/VIRULENCE=μ)
- *Can individuals move from S to R? Why or why not?*



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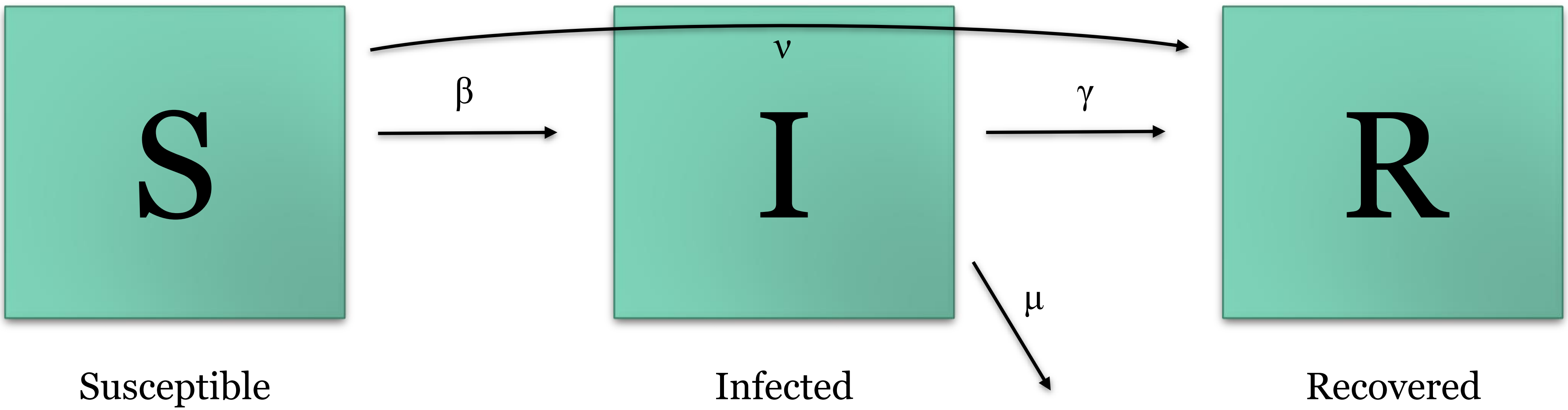
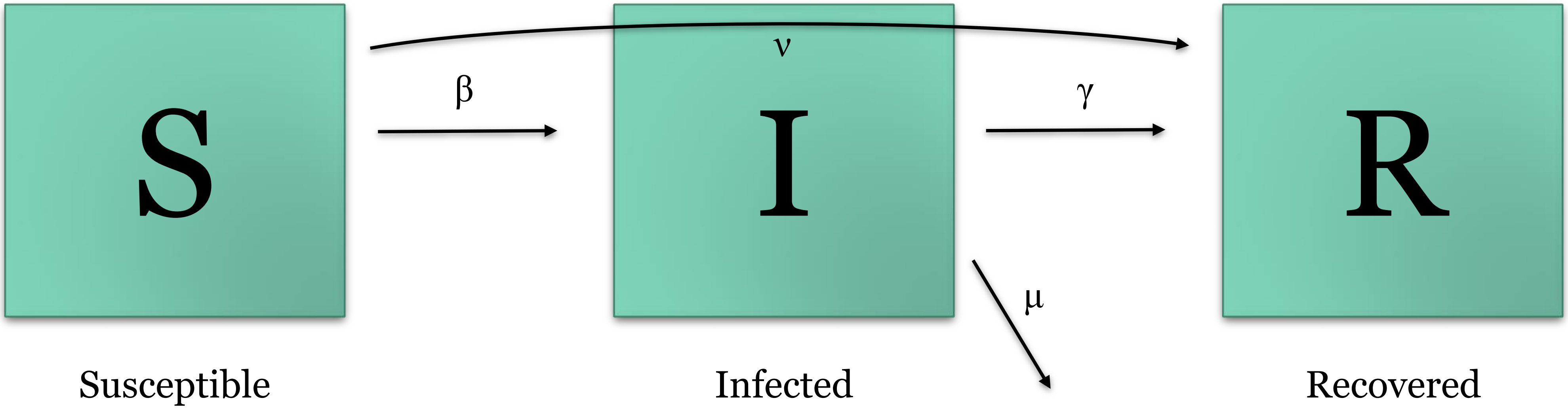
- Do all individuals move from I to R? What else could happen to them? (PATHOGEN-INDUCED MORTALITY/VIRULENCE=μ)
- Can individuals move from S to R? Why or why not? (VACCINATION=ν)



# Convert to System of Equations

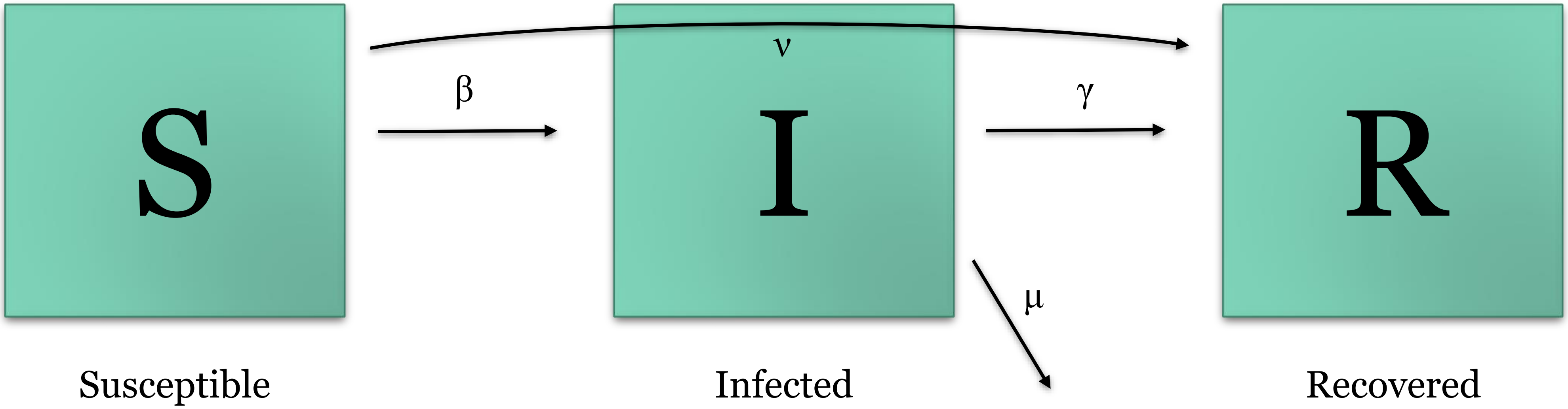
- Represent as differential equations (ODEs) = change in state variables (compartments) over time
- Flows in to a compartment are represented as additions in ODE
- Flows out of a compartment are represented as subtractions



# Convert to System of Equations

All parameters get multiplied by AT LEAST ONE state variable (compartment)

$$\frac{dS}{dt} = \qquad \frac{dI}{dt} = \qquad \frac{dR}{dt} =$$



# Convert to System of Equations

$$\frac{dS}{dt} = -\beta SI \ - \ \nu S$$

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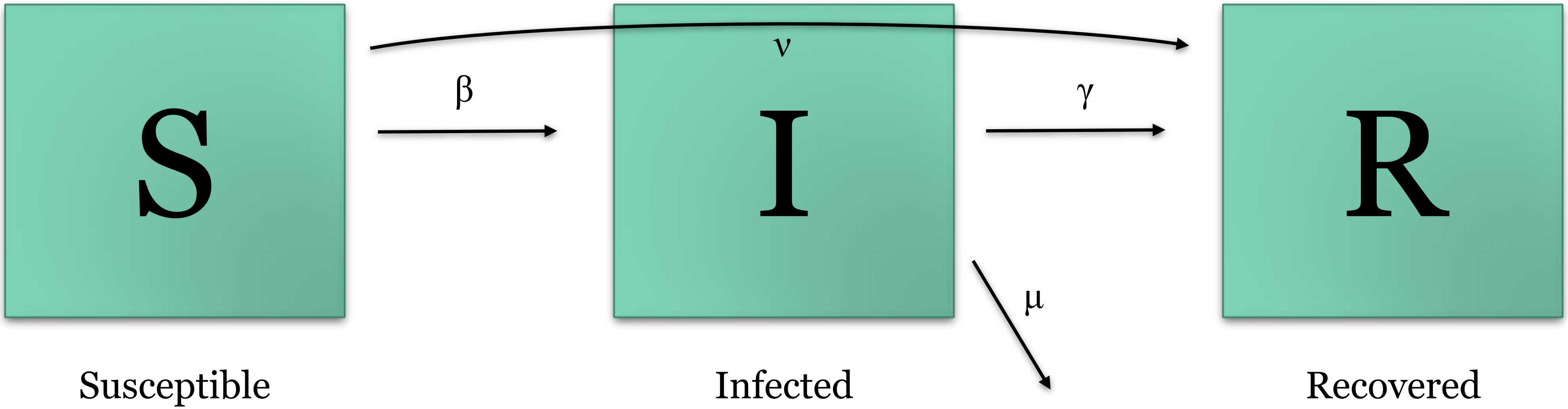

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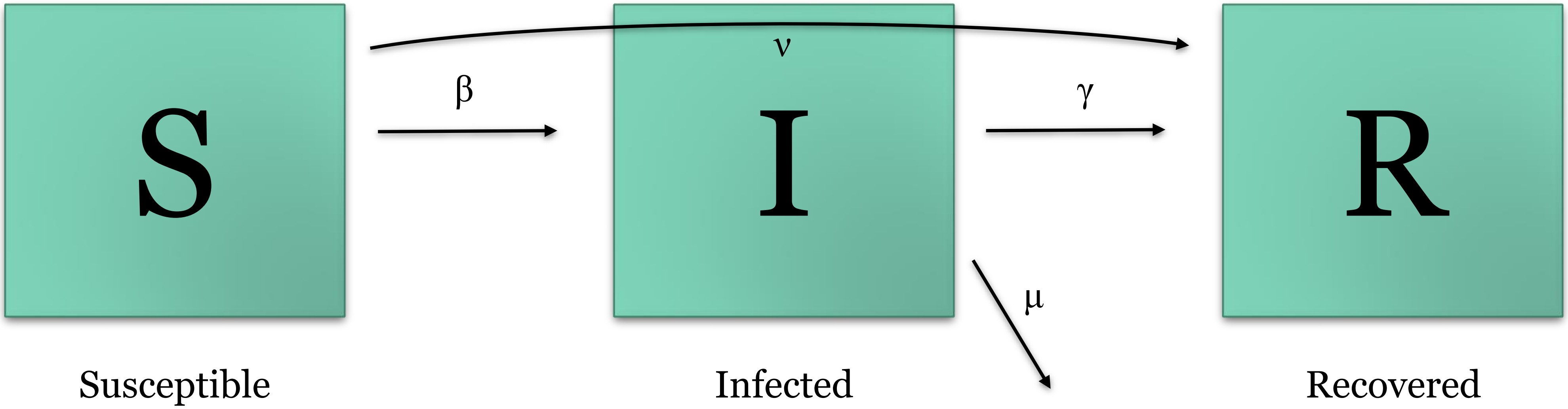
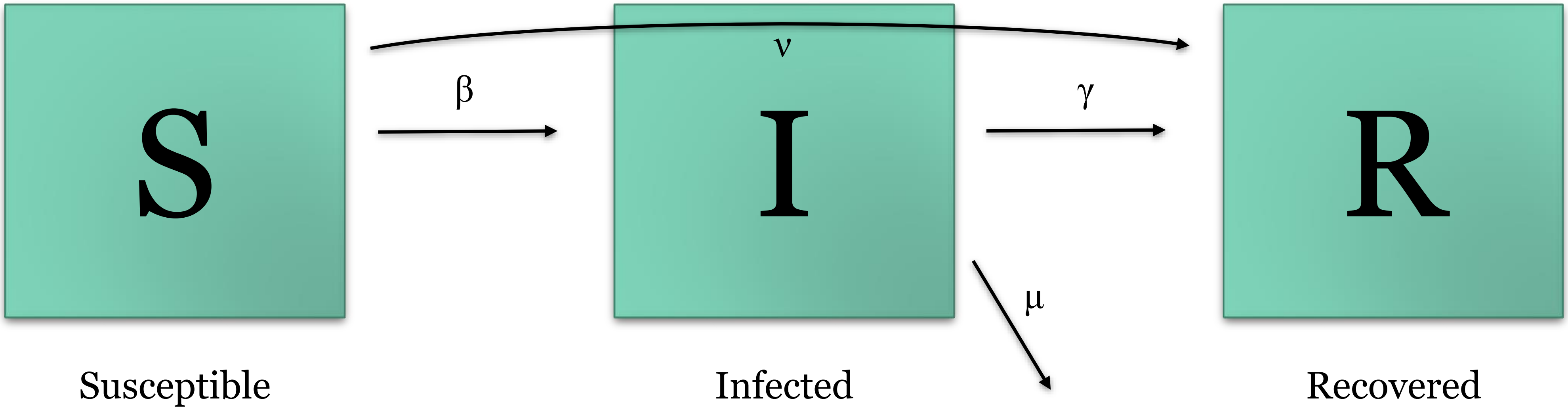


# Convert to System of Equations

$$\frac{dS}{dt} = -\beta SI \; - \; \nu S \qquad \frac{dI}{dt} = \beta SI \; - \; \gamma I \; - \; \mu I \qquad \frac{dR}{dt} =$$

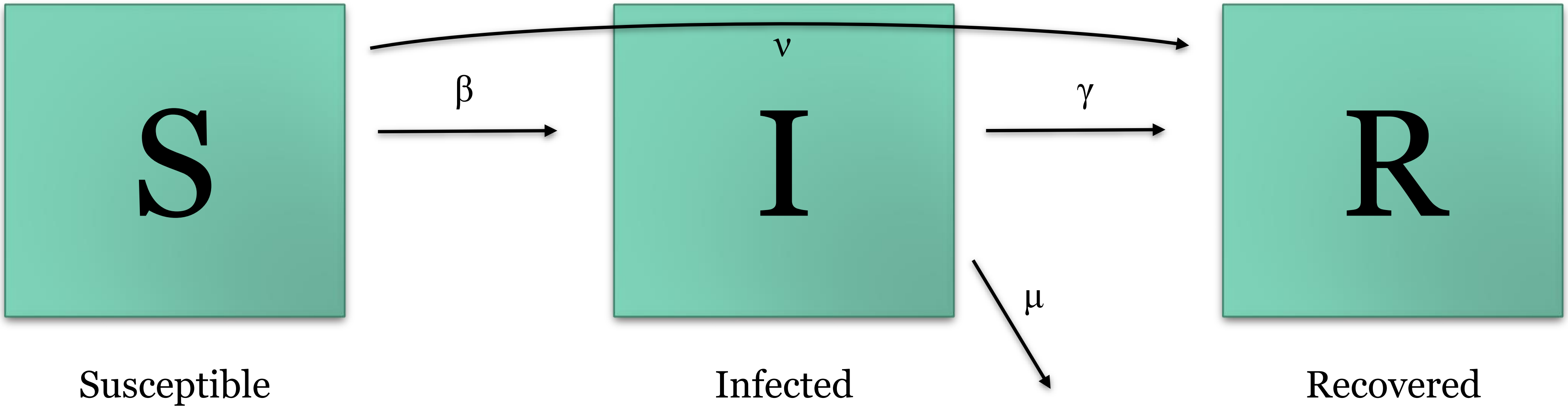


# Convert to System of Equations

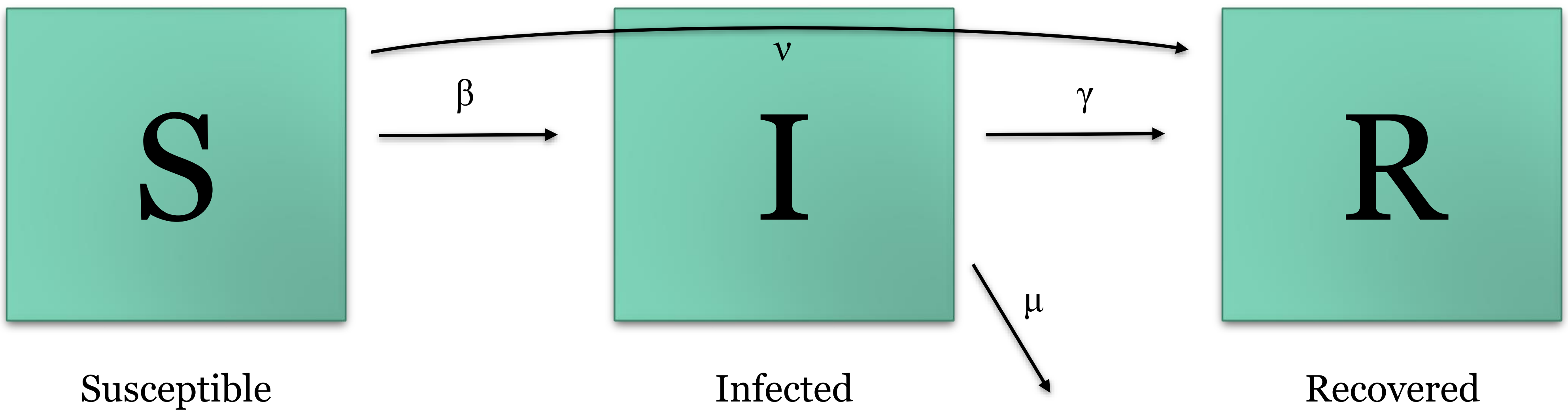
$$\frac{dS}{dt} = -\beta SI - \nu S$$

$$\frac{dI}{dt} = \beta SI - \gamma I - \mu I$$

$$\frac{dR}{dt} = \gamma I + \nu S$$

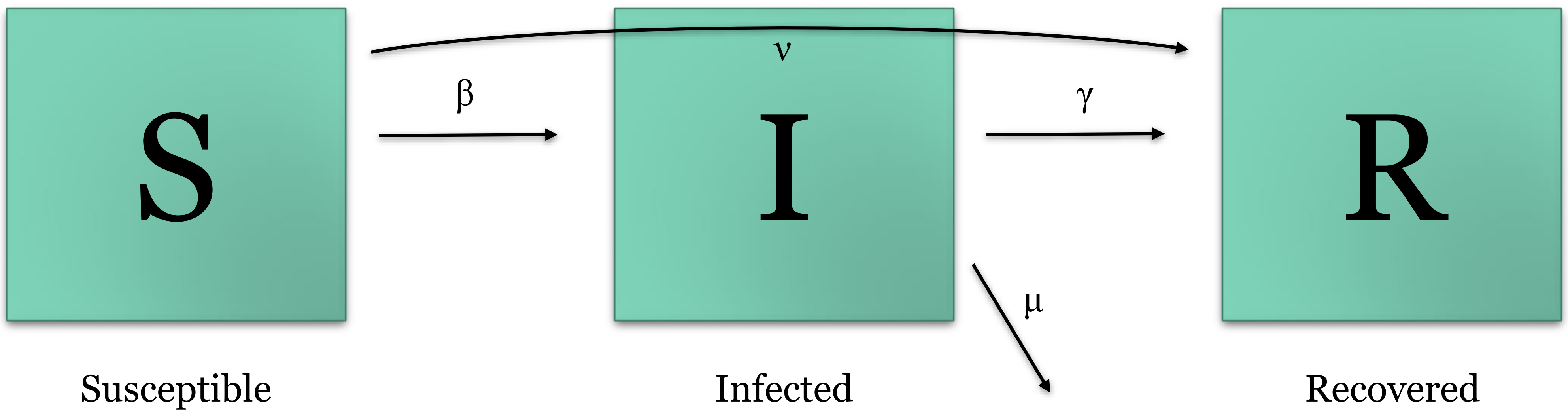


# Practical applications: How would we use this model:?



# Practical applications: How would we use this model:?

- Estimate the expected number of cases of flu under different vaccination regimes (e.g., no vaccination, 50% coverage, 90% coverage)
- Estimate the transmission rate during a previous flu epidemic



# Working backwards: from equations to conceptual model

A basic system of equations describing HIV transmission

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dE}{dt} = \beta SI - \alpha E$$

$$\frac{dI}{dt} = \alpha E - \mu I$$

How does this model differ from our previous model for flu? Why?

What does the "E" compartment represent?

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**Step 1: Identify compartments**

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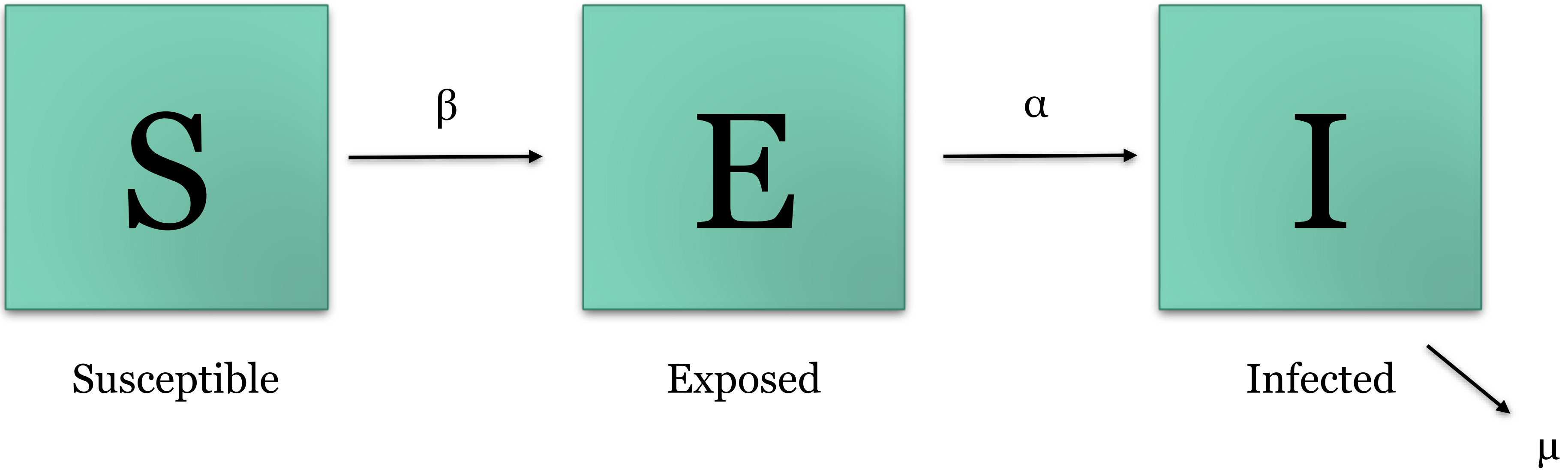
**Step 2: Identify transitions**



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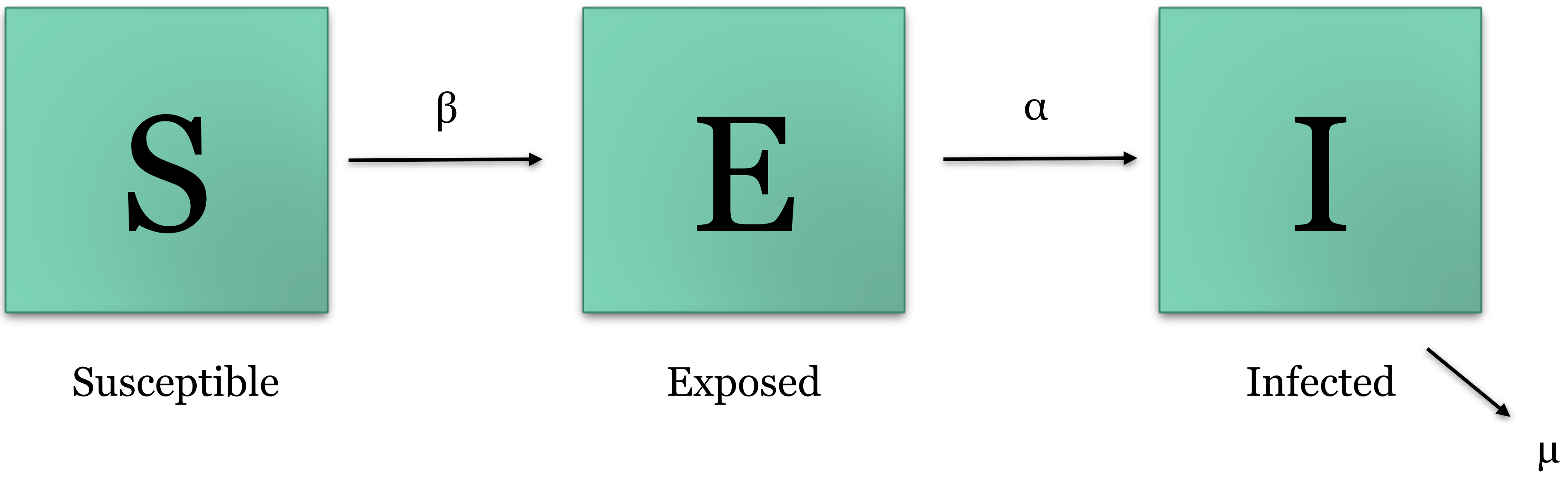
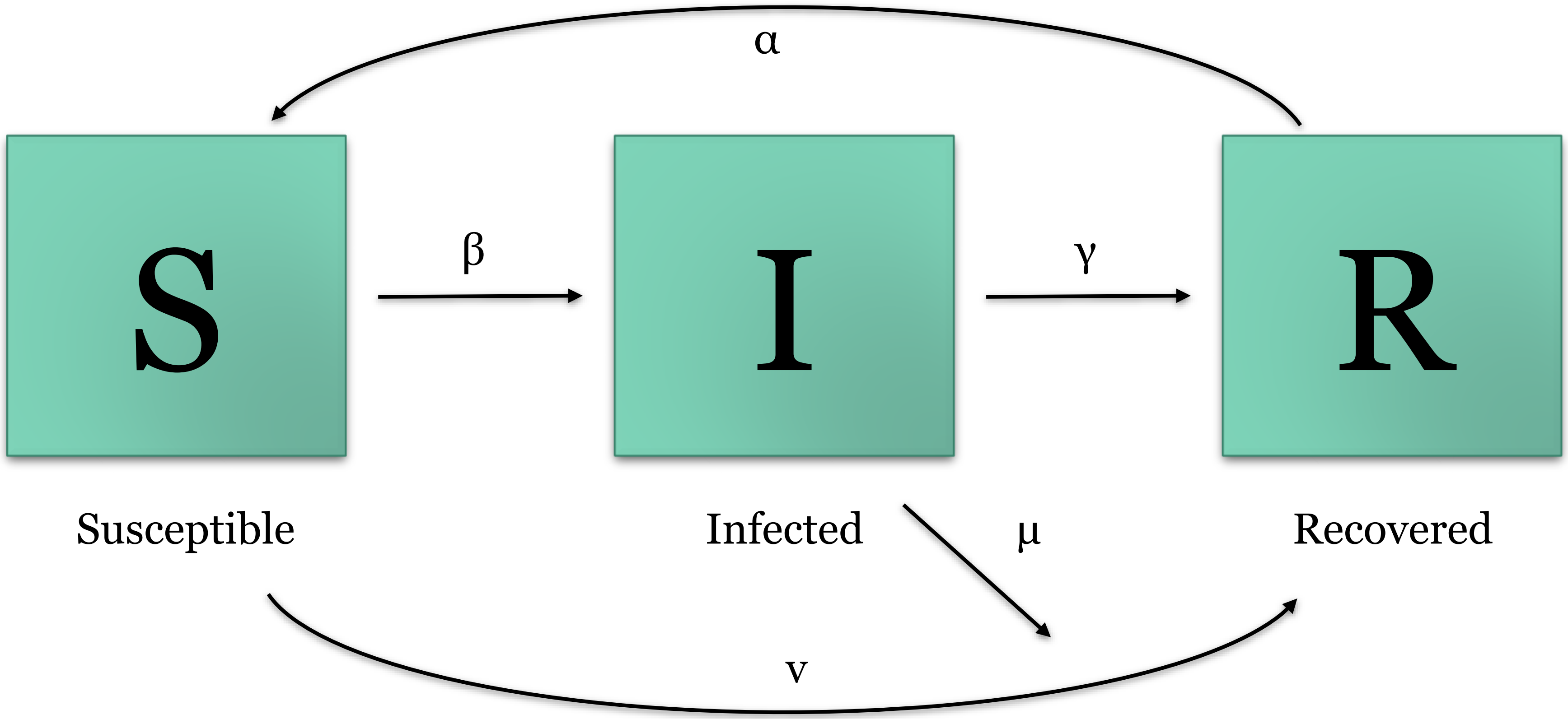
Step 1: Identify compartments
Step 2: Identify transitions
**Step 3: Interpret the model in biological terms**



# Practice: from conceptual model to system of equations

A conceptual model for measles or mumps transmission



# Answer: measles/mumps model

# Practice: from system of equations to conceptual model

A model describing transmission dynamics of Ebola

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dE}{dt} = \beta SI - \alpha E$$

$$\frac{dI}{dt} = \alpha E - vI - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

# Answer: Ebola model