Is Timing Everything? The Effects of Measurement Timing on the Performance of Nonlinear Longitudinal Models

by

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ABSTRACT

IS TIMING EVERYTHING? THE EFFECTS OF MEASUREMENT TIMING ON THE PERFORMANCE OF NONLINEAR LONGITUDINAL MODELS

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Despite the value that longitudinal research offers for understanding psychological processes, studies in organizational research rarely use longitudinal designs. One reason for the paucity of longitudinal designs may be the challenges they present for researchers. Three challenges of particular importance are that researchers have to determine 1) how many measurements to take, 2) how to space measurements, and 3) how to design studies when participants provide data with different response schedules (time unstructuredness). In systematically reviewing the simulation literature, I found that few studies comprehensively investigated the effects of measurement number, measurement spacing, and time structuredness (in addition to sample size) on model performance. As a consequence, researchers have little guidance when trying to conduct longitudinal research. To address these gaps in the literature, I conducted a series of simulation experiments. I found poor model performance across all measurement number/sample size pairings. That is, bias and precision were never concurrently optimized under any combination of manipulated variables. Bias was often low, however, with moderate measurement numbers and sample sizes. Although precision was frequently poor, the greatest improvements in precision resulted from using either seven measurements with $N \geq 200$ or nine measurements with $N \leq 100$. With time-unstructured data, model performance systematically

decreased across all measurement number/sample size pairings when the model incorrectly assumed an identical response pattern across all participants (i.e., time-structured data). Fortunately, when models were equipped to handle heterogeneous response patterns using definition variables, the poor model performance observed across all measurement number/sample size pairings no longer appeared. Altogether, the results of the current simulation experiments provide guidelines for researchers interested in modelling nonlinear change.

DEDICATION

I dedicate this document to Don Cameron. Without you, none of this would have been possible.

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Appendix A: Ergodicity and the Need to Conduct Longitudinal Research

To understand why cross-sectional results are unlikely to agree with longitudinal results for any given analysis, a discussion of data structures is apropos. Consider an example where a researcher obtains data from 50 people measured over 100 time points such that each row contains a p person's data over the 100 time points and each column contains data from 50 people at a t time point. For didactic purposes, all data are assumed to be sampled from a normal distribution. To understand whether findings in any given cross-sectional data set yield the same findings in any given longitudinal data set, the researcher randomly samples one cross-sectional and one longitudinal data set and computes the mean and variance in each set. To conduct a cross-sectional analysis, the researcher randomly samples the data across the 50 people at a given time point and computes a mean of the scores at the sampled time point ($\bar{X}_t$) using Equation A.1 shown below:

$$\bar{X}_t = \frac{1}{P}\sum_{p=1}^{P} x_p, \tag{A.1}$$

where the scores of all P people are summed (x_p) and then divided by the number of people (P). To compute the variance of the scores at the sampled time point (S_t^2), the researcher uses Equation A.2 shown below:

$$S_t^2 = \frac{1}{P}\sum_{p=1}^{P}(x_p - \bar{X}_t)^2, \tag{A.2}$$

where the sum of the squared differences between each person's score (x_p) and the average value at the given t time point ($\bar{X}_t$) is computed and then divided by the number of people (P). To conduct a longitudinal analysis, the researcher randomly samples one person's data across the 100 time points and also computes a mean and variance of the scores. To compute the mean across the t time points of the longitudinal data set ($\bar{X}_p$), the researcher uses Equation A.3 shown below:

$$\bar{X}_p = \frac{1}{T}\sum_{t=1}^{T} x_t, \tag{A.3}$$

where the scores at each t time point are summed (x_t) and then divided by the number of time points (T). The researcher also computes a variance of the sampled person's scores across all time points (S_p^2) using Equation A.4 shown below:

$$S_p^2 = \frac{1}{T}\sum_{t=1}^{T}(x_t - \bar{X}_p)^2, \tag{A.4}$$

where the sum of squared differences between the score at each time point (x_t) and the average value of the p person's scores ($\bar{X}_p$) is computed and then divided by the number of time points (T).

If the researcher wants to treat the mean and variance from the cross-sectional and longitudinal data sets as interchangeable, then two conditions outlined by ergodic theory must be satisfied (Molenaar, 2004; Molenaar & Campbell, 2009).[1] First, a given cross-sectional mean and variance can only closely estimate the mean and variance of any

[1]Note that ergodic theory is an entire mathematical discipline (for an introduction, see Petersen, 1983). In the current context, the most important ergodic theorems are those proven by Birkhoff (1931, for a review, see Choe, 2005, Chapter 3)

given person's data (i.e., a longitudinal data set) to the extent that each person's data originate from a normal distribution with the same mean and variance. If each person's data originate from a different normal distribution, then computing the mean and variance at a given time point would, at best, describe the values of one person. When each person's data are generated from the same normal distribution, the condition of *homogeneity* is met. Importantly, satisfying the condition of homogeneity does not guarantee that the mean and variance obtained from another cross-sectional data set will closely estimate the mean and variance of any given person (i.e., any given longitudinal data set). The mean and variance values computed from any given cross-sectional data set can only closely estimate the values of any given person to the extent that the cross-sectional mean and variance remain constant over time. If the mean and variance of observations remain constant over time, then the second condition of *stationarity* is satisfied. Therefore, the researcher can only treat means and variances from cross-sectional and longitudinal data sets as interchangeable if each person's data are generated from the same normal distribution (homogeneity) and if the mean and variance remain constant over time (stationarity). When the conditions of homogeneity and stationarity are satisfied, a process is said to be *ergodic*: Analyses of cross-sectional data sets will return the same values as analyses on longitudinal data sets.

Given that psychological studies almost never collect data from only one person, one potential reservation may be that the conditions required for ergodicity only hold when a longitudinal data set contains the data of one person. That is, if the researcher uses the full data set containing the data of 100 people sampled over 100 time points and computes 100 cross-sectional means and variances (Equation A.1 and Equation A.2, respectively) and 100 longitudinal means and variances (Equation A.3 and Equation A.4, respectively), wouldn't the average of the cross-sectional means and variances be the same as the average of the longitudinal means and variances? Although averaging the cross-sectional means returns the same value as averaging the longitudinal means, the average longitudinal variance remains different from the average cross-sectional variance (for several empirical examples, see Fisher et al., 2018). Therefore, the conditions of ergodicity apply even with larger longitudinal and cross-sectional sample sizes.

The guaranteed differences in cross-sectional and longitudinal variance values that result from non-ergodic processes have far-reaching implications. Almost every analysis employed in organizational research—whether it be correlation, regression, factor analysis, mediation, etc.—analyzes variability, and so, when a process is non-ergodic, cross-sectional variability will differ from longitudinal variability, and the results obtained from applying any given analysis on each of the variabilities will differ as a consequence. Because variability is central to so many analyses, the non-equivalence of longitudinal and cross-sectional variances that results from a non-ergodic process explains why discussions of ergodicity often point out that "for non-ergodic processes, an analysis of the structure of IEV [interindividual variability] will yield results that differ from results obtained in an analogous analysis of IAV [intraindividual variability]"(Molenaar, 2004, p. 202).[2]

[2]It is important to note that a violation of one or both ergodic conditions (homogeneity and stationarity) does not mean that an analysis of cross-sectional variability yields results that have no relation to the results gained from applying the analysis on longitudinal variability (i.e., the causes of cross-sectional variability are independent from the causes of longitudinal variability). An analysis of cross-sectional variability can still give insight into temporal dynamics if the causes of non-ergodicity can be identified (Voelkle et al., 2014; for similar discussion, see Spector, 2019). Thus, conceptualizing ergodicity on a continuum with non-erdogicity and ergodicity on opposite ends provides a more accurate perspective for understanding ergodicity (Adolf & Fried, 2019; Medaglia et al., 2019).

With an understanding of the conditions required for ergodicity, a brief review of organizational phenomena finds that these conditions are regularly violated. Focusing only on homogeneity (each person's data are generated from the same distribution), several instances in organizational research violate this condition. As examples of homogeneity violations, employees show different patterns of absenteeism over five years (Magee et al., 2016), leadership development over the course of a seminar (Day & Sin, 2011), career stress over the course of 10 years (Igic et al., 2017), and job performance in response to organizational restructuring (Miraglia et al., 2015). With respect to stationarity (constant values for statistical parameters across people over time), several examples can be generated by realizing how calendar events affect psychological processes and behaviours throughout the year. As examples of stationarity violations, consider how salespeople, on average, undoubtedly sell more products during holidays, how employees, on average, take more sick days during the winter months, and how accountants, on average, experience more stress during tax season. With violations of ergodic conditions commonly occurring in organizational psychology, it becomes fitting to echo the commonly held sentiment that few, if any, psychological processes are ergodic (Curran & Bauer, 2011; Fisher et al., 2018; Hamaker, 2012; Molenaar, 2004, 2008; Molenaar & Campbell, 2009; Wang & Maxwell, 2015). As a result, longitudinal research is necessary for understanding psychological processes.

Appendix B: Code Used to Run Monte Carlo Simulations for all Experiments

The code used to compute the simulations of each experiment are shown in Code Block B.1. Note that the cell size is 1000 (i.e., `num_iterations = 1000`).

Code Block B.1

Code Use to Run Monte Carlo Simulations for Each Simulation Experiment

```r
devtools::install_github(repo = 'sciarraseb/nonlinSims', force=T)

library(easypackages)
packages <- c('devtools', 'nonlinSims', 'parallel', 'tidyverse', "OpenMx",
"data.table", 'progress', 'tictoc')
libraries(packages)

time_period <- 360

#Population values for parameters
#fixed effects
sd_scale <- 1
common_effect_size <- 0.32
theta_fixed <- 3
alpha_fixed <- theta_fixed + common_effect_size
beta_fixed <- 180
gamma_fixed <- 20

#random effects
sd_theta <- 0.05
sd_alpha <- 0.05
sd_beta <- 10
sd_gamma <- 4
sd_error <- 0.05

#List containing population parameter values
pop_params_4l <- generate_four_param_pop_curve(
  theta_fixed =  theta_fixed, alpha_fixed = alpha_fixed,
   beta_fixed = beta_fixed, gamma_fixed = gamma_fixed,
   sd_theta = sd_theta, sd_alpha = sd_alpha,
   sd_beta = sd_beta, sd_gamma = sd_gamma, sd_error = sd_error



num_iterations <- 1e3 #n=1000 (cell size)
seed <- 27 #ensures replicability

# Experiment 1 (number measurements,  spacing, midpoint) --------
factor_list_exp_1 <- list('num_measurements' = seq(from = 5, to = 11, by = 2),
                          'time_structuredness' = c('time_structured'),
                          'spacing' = c('equal', 'time_inc', 'time_dec', 'mid_ext'),
                          'midpoint' = c(80, 180, 280),
                          'sample_size' = c(225))

tic()
exp_1_data <- run_exp_simulation(factor_list = factor_list_exp_1, num_iterations =
num_iterations, pop_params = pop_params_4l,
                                 num_cores = detectCores()-1, seed = seed)
toc()

#Average computation time is 1 iteration per second. As an example, Experiment has 48
cells x 1000 iterations/cell = 48 000 iterations and seconds/3600s/hour ~ 13.33 hours
(simulations computed with 15 cores)
write_csv(x = exp_1_data, file = '~/Desktop/exp_1_data.csv')

# Experiment 2 (number measurements, spacing,  sample size) ---
factor_list_exp_2 <- list('num_measurements' = seq(from = 5, to = 11, by = 2),
                          'time_structuredness' = c('time_structured'),
                          'spacing' = c('equal', 'time_inc', 'time_dec', 'mid_ext'),
                          'midpoint' = 180,
                          'sample_size' = c(30, 50, 100, 200, 500, 1000))

tic()
exp_2_data <- run_exp_simulation(factor_list = factor_list_exp_2, num_iterations =
num_iterations, pop_params = pop_params_4l,
```

```
                                    num_cores = detectCores(), seed = seed)
toc()

write_csv(x = exp_2_data, file = 'Desktop/exp_2_data.csv')

# Experiment 3 (number measurements, sample size, time structuredness) -----
factor_list_exp_3 <- list('num_measurements' = seq(from = 5, to = 11, by = 2),
                          'time_structuredness' = c('time_structured', 'fast_response',
                          'slow_response'),
                          'spacing' = c('equal'),
                          'midpoint' = 180,
                          'sample_size' = c(30, 50, 100, 200, 500, 1000))
tic()
exp_3_data <- run_exp_simulation(factor_list = factor_list_exp_3, num_iterations =
num_iterations, pop_params = pop_params_41,
                                    num_cores = detectCores(), seed = seed)
toc()

write_csv(x = exp_3_data, file = '~/Desktop/exp_3_data.csv')



# Experiment 3 (definition variables with slow response rate ) ------
factor_list_exp_def <- list('num_measurements' = seq(from = 5, to = 11, by = 2),
                            'time_structuredness' = c('slow_response'),
                            'spacing' = c('equal'),
                            'midpoint' = 180,
                            'sample_size' = c(30, 50, 100, 200, 500, 1000))
tic()
exp_3_def_data <- run_exp_simulation(factor_list = factor_list_exp_def, num_iterations =
num_iterations, pop_params = pop_params_41,
                                        num_cores = detectCores() - 1, seed = seed,
                                        definition = T)
toc()
#240734.993 sec elapsed (7 cores used; simulation time increased by roughly a
magnitude of 8).
write_csv(x = exp_3_def_data, file = 'exp_3_def.csv')
```

Appendix C: Procedure for Generating Measurement Schedules Measurement Schedules

Given that no procedure existed (to my knowledge) for creating measurement schedules, I devised a method for generating measurement schedules for the four spacing conditions (equal, time-interval increasing, time-interval decreasing, and middle-and-extreme spacing). The code I used to automate the generation of these schedules can be found within the code for the `compute_measurement_schedules()` function for the `nonlinSims` package (see https://github.com/sciarraseb/nonlinSims). For each measurement spacing conditions across all measurement number levels, a two-step procedure was employed to generate measurement schedules in Experiments 1 and 2. At a broad level, the first step computes values for setup variables and the second step computes the interval lengths.

C.1 Procedure for Constructing Measurement Schedules With Equal Spacing

Figure C.1 shows how the two-step procedure is implemented to construct a measurement schedule with equal spacing and five measurements. In the first step, the number of intervals (NI) is computed by subtracting one from the number of measurements (NM). With five measurements ($NM = 5$), there are four intervals ($NI = 4$). In the second step, interval lengths are calculated by dividing the length of the measurement period (MP) by the number of intervals (NI), yielding an interval length of 90 days ($\frac{MP}{NI} = \frac{360}{4} = 90$) for each interval and the following measurement days:

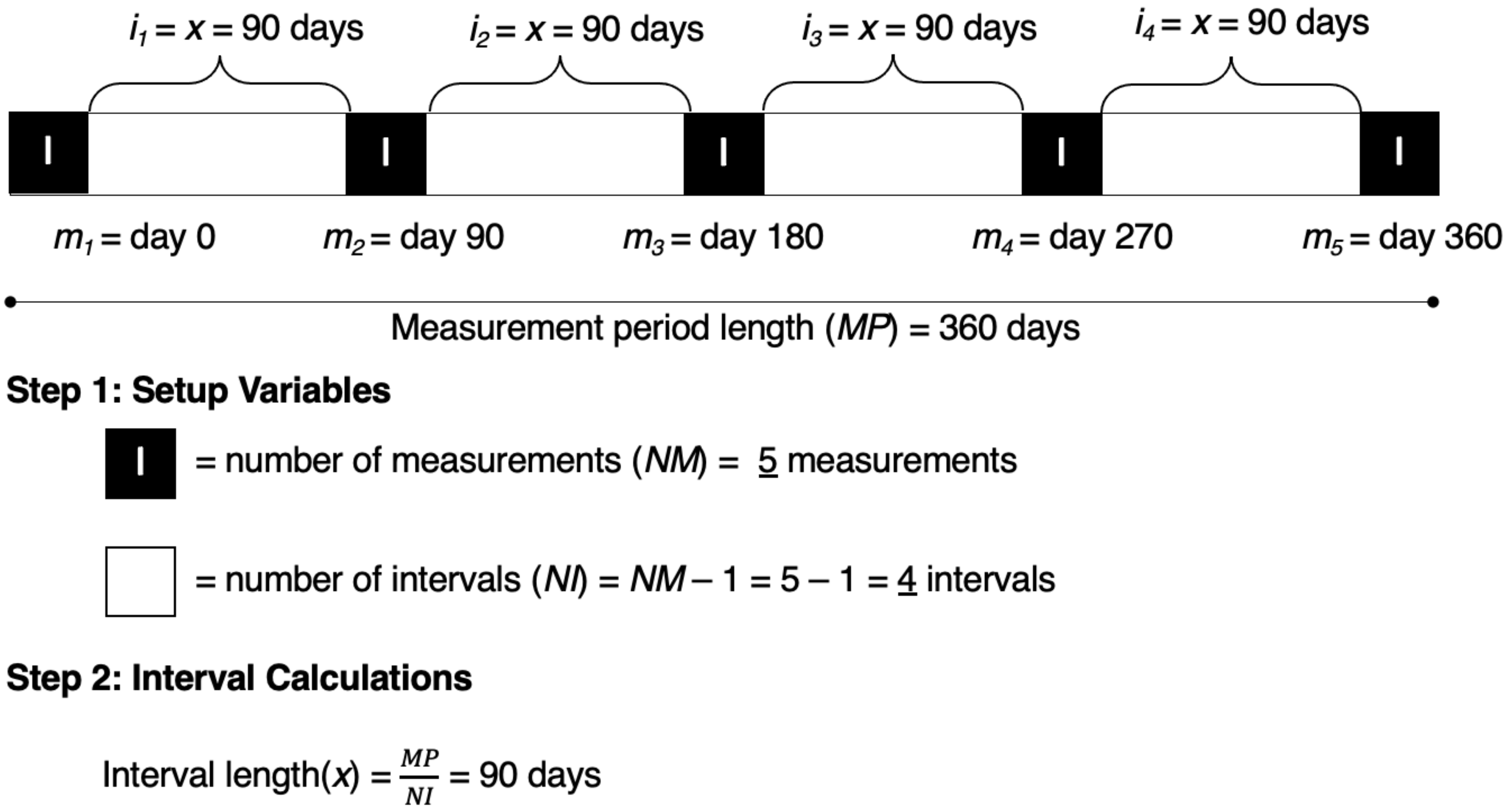
- $m_1 =$ day 0
- $m_2 =$ day 90
- $m_3 =$ day 180
- $m_4 =$ day 270
- $m_5 =$ day 360.

C.2 Procedure for Constructing Measurement Schedules With Time-Interval Increasing Spacing

Figure C.2 shows how the two-step procedure is implemented to construct a measurement schedule with time-interval increasing spacing and five measurements. In the first step, the number of intervals (NI) is computed by subtracting one from the number of measurements (NM). With five measurements ($NM = 5$), there are four intervals

Figure C.1

Procedure for Computing Measurement Schedules With Equal Spacing



Note. In Step 1, setup variables are calculated. With five measurements ($NM = 5$), there are four intervals ($NI = 4$). In Step 2, interval lengths are calculated by dividing the length of the measurement period (MP) by the number of intervals (NI), yielding an interval length of 90 days ($\frac{MP}{NI} = \frac{360}{4} = 90$) for each interval.

($NI = 4$). Because interval lengths increase over time, I decided that intervals would increase by an integer multiple of a constant length (c) after each measurement day (m_i) according to the function shown below in Equation C.1:

$$\text{Constant-length increment} = \sum_{x=0}^{NI-1} xc, \tag{C.1}$$

where x represents the integer multiple that increases by 1 after each measurement day. Importantly, to calculate the constant-length increment (c) by which interval lengths increase over time, it is important to realize that two terms contribute to the length of any interval: A shortest-interval length (s) and a constant-length value (c), as shown below in Equation C.2:

$$\text{Interval length} = s + \sum_{x=0}^{NI-1} xc. \tag{C.2}$$

Because the shortest-interval length (s) contributes to the length of each interval–in this example, four intervals—then the sum of these lengths can be subtracted from the measurement period length of 360 days ($MP = 360$). In the current example with five measurements, 240 days remain ($r = 240$) after subtracting the days needed for the shortest-interval lengths (see Equation C.3).

$$\text{Remaining days } (r) = MP - (NI)s = 360 - (30)4 = 240 \text{ days} \tag{C.3}$$

Having computed the number of remaining days, the constant-length value (c) can then be obtained by dividing the number of remaining days by the number of constant-value interval lengths (c_i), as shown below in Equation C.4:

$$\text{Constant-value interval length}(c) = \frac{r}{\sum_{i=0}^{NI-2} i} = \frac{240}{3+2+1} = 40 \text{ days} \tag{C.4}$$

Therefore, having computed the value for c, the following interval lengths are obtained:

- $i_1 = s + 0(c) = 30 + 0(30) = 30$ days
- $i_2 = s + 1(c) = 30 + 1(40) = 70$ days
- $i_3 = s + 0(c) = 30 + 2(40) = 110$ days
- $i_4 = s + 0(c) = 30 + 3(40) = 150$ days

and the following measurement days are obtained:

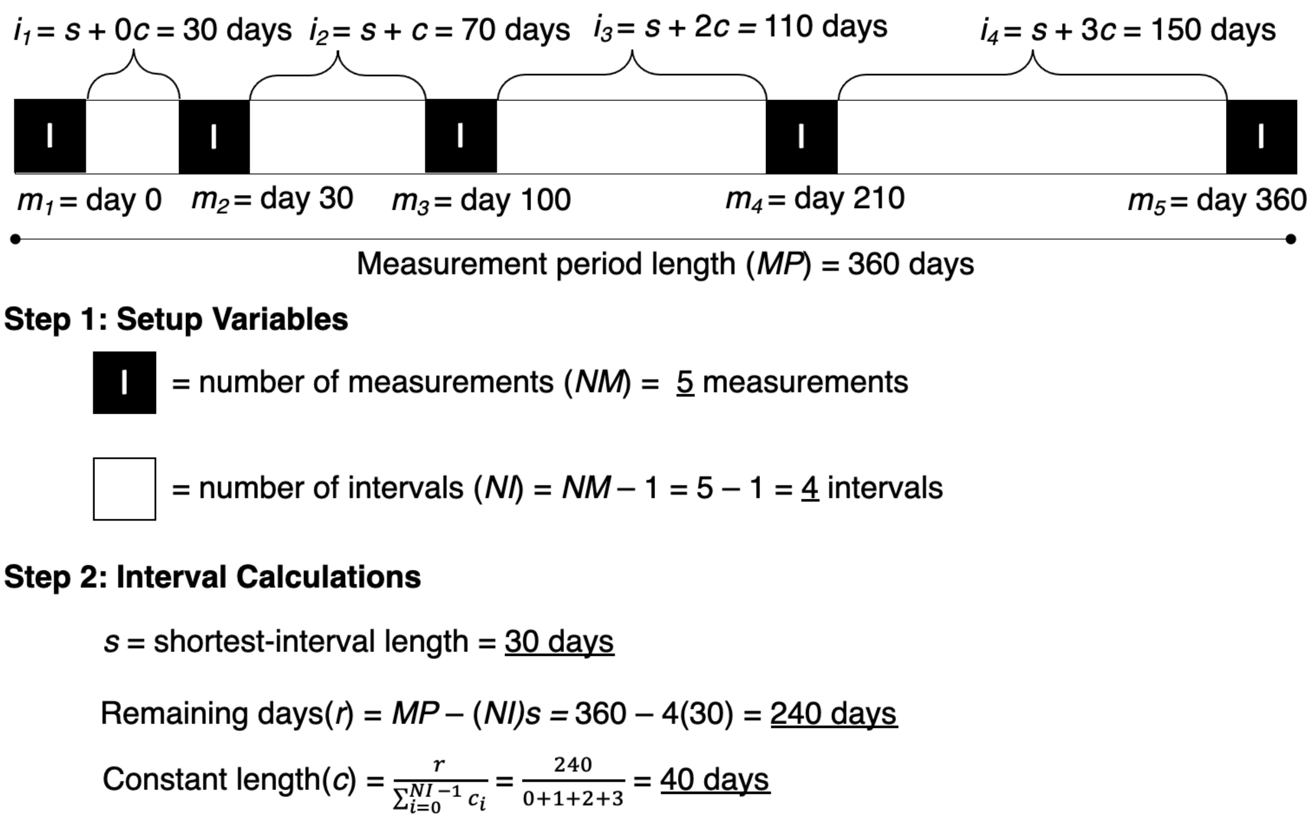
- $m_1 =$ day 0
- $m_2 =$ day 30
- $m_3 =$ day 100
- $m_4 =$ day 210
- $m_5 =$ day 360.

C.3 Procedure for Constructing Measurement Schedules With Time-Interval Decreasing Spacing

Figure C.3 shows how the two-step procedure is implemented to construct a measurement schedule with time-interval decreasing spacing and five measurements. Because the procedure for calculating time-decreasing intervals simply requires that the order of time-interval increasing intervals are reversed, the procedure is, thus, essentially identical to the procedure shown in the previous section. Therefore, with five measurements, time-interval decreasing spacing produces the following intervals:

Figure C.2

Procedure for Computing Measurement Schedules With Time-Interval Increasing Spacing



Note. In Step 1, setup variables are calculated. With five measurements ($NM = 5$), there are four intervals ($NI = 4$). In Step 2, two components contribute to each interval length: A shortest-interval length (s) and a constant-length value (c), as shown in Equation C.2. Because the shortest-interval length (s) contributes to each interval, the sum of these lengths can be subtracted from the measurement period length of 360 days ($MP = 360$). In the current example with five measurements, 240 days remain ($r = 240$) after subtracting the days needed for the shortest-interval lengths (see Equation C.3). To calculate the constant-length value (c), the remaining days (r) are divided by the number of constant-value interval lengths (c_i), as shown in Equation C.4.

- $i_1 = s + 0(c) = 30 + 3(40) = 150$ days
- $i_2 = s + 0(c) = 30 + 2(40) = 110$ days
- $i_3 = s + 1(c) = 30 + 1(40) = 70$ days
- $i_4 = s + 0(c) = 30 + 0(30) = 30$ days

and the following measurement days are obtained:

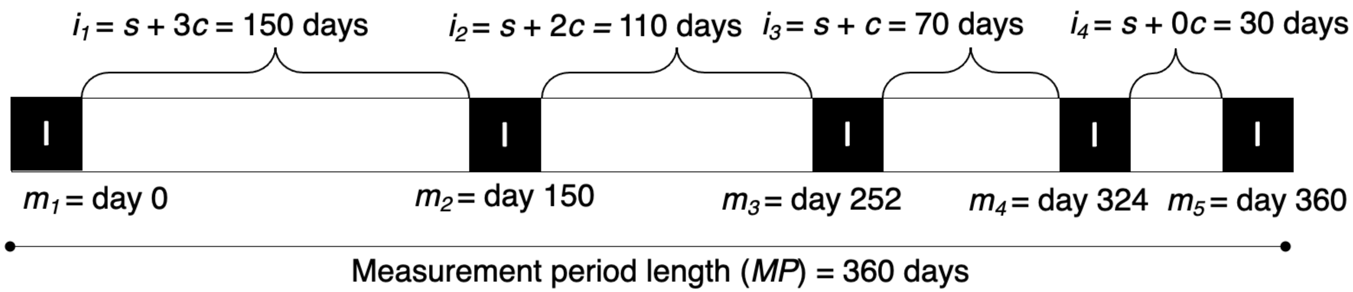
- $m_1 =$ day 0
- $m_2 =$ day 150
- $m_3 =$ day 260
- $m_4 =$ day 330
- $m_5 =$ day 360.

C.4 Procedure for Constructing Measurement Schedules With Middle-and-Extreme Spacing

Figure C.4 shows how the two-step procedure is implemented to construct a measurement schedule with middle-and-extreme spacing and five measurements. In the first step, the number of intervals (NI) is computed by subtracting one from the number of measurements (NM). With five measurements ($NM = 5$), there are four intervals ($NI = 4$). Importantly, because middle-and-extreme spacing places measurements near the extremities and the middle of the measurement window, the number of measurements in both these sections must also be calculated. The number of extreme measurements is first calculated by dividing the number of measurements by 3 and taking the floor (i.e., rounded-down value [$\lfloor x \rfloor$]) of this value and multiplying it by 2, as shown below in Equation C.5:

Figure C.3

Procedure for Computing Measurement Schedules With Time-Interval Decreasing Spacing



Step 1: Setup Variables

= number of measurements (*NM*) = <u>5</u> measurements

= number of intervals (*NI*) = *NM* – 1 = 5 – 1 = <u>4</u> intervals

Step 2: Interval Calculations

s = shortest-interval length = <u>30 days</u>

Remaining days(*r*) = *MP* – (*NI*)*s* = 360 – 4(30) = <u>240 days</u>

Constant length(*c*) = $\frac{r}{\sum_{i=0}^{NI-1} c_i} = \frac{240}{0+1+2+3}$ = <u>40 days</u>

Note. In Step 1, setup variables are calculated. With five measurements ($NM = 5$), there are four intervals ($NI = 4$). In Step 2, two components contribute to each interval length: A shortest-interval length (s) and a constant-length value (c), as shown in Equation C.2. Because the shortest-interval length (s) contributes to each interval, the sum of these lengths can be subtracted from the measurement period length of 360 days ($MP = 360$). In the current example with five measurements, 240 days remain ($r = 240$) after subtracting the days needed for the shortest-interval lengths (see Equation C.3). To calculate the constant-length value (c), the remaining days (r) are divided by the number of constant-value interval lengths (c_i), as shown in Equation C.4.

$$\text{Number of extreme measurements}(ex) = 2\lfloor\frac{NM}{3}\rfloor = 2\lfloor\frac{5}{3}\rfloor = 2. \tag{C.5}$$

The number of middle measurements can then be calculated by subtracting the number of extreme measurements (ex) from the number of measurements (NM), as shown below in Equation C.7:

$$\text{Number of middle measurements}(mi) = NM - ex = 5 - 2 = 3. \tag{C.6}$$

In Step 2, interval lengths are calculated. For middle-and-extreme spacing, there are two types of interval lengths: 1) Intervals separating either two middle or two extreme measurements and 2) intervals separating one middle and one extreme measurement. Intervals separating two middle or two extreme measurements (w_i) are set to the shortest-interval length (s), which I set to be 30 days ($w_i = s = 30$). Intervals separating one middle and one extreme measurement (b_i) are set to the sum of two components: 1) A shortest-interval length (s) and a 2) constant-value interval length (c), as shown below in Equation C.7:

$$b_i = s + c. \tag{C.7}$$

To obtain the constant-value interval length (c), the sum of shortest-value interval lengths (s) is subtracted from the measurement period of 360 days ($MP = 360$). In the current example with five measurements, 240 days remain ($r = 240$) after subtracting the days needed for the shortest-interval lengths (see Equation C.8).

$$\text{Remaining days } (r) = MP - (NI)s = 360 - (30)4 = 240 \text{ days} \tag{C.8}$$

Having computed the number of remaining days, the constant-length value (c) can then be obtained by dividing the number of remaining days by the number of intervals separating middle and extreme measurements, which will always be 2, as shown below in Equation C.9:

$$\text{Constant-value interval length}(c) = \frac{r}{2} = \frac{240}{2} = 120 \text{ days} \tag{C.9}$$

Therefore, having computed the value for c, the following interval lengths are obtained:

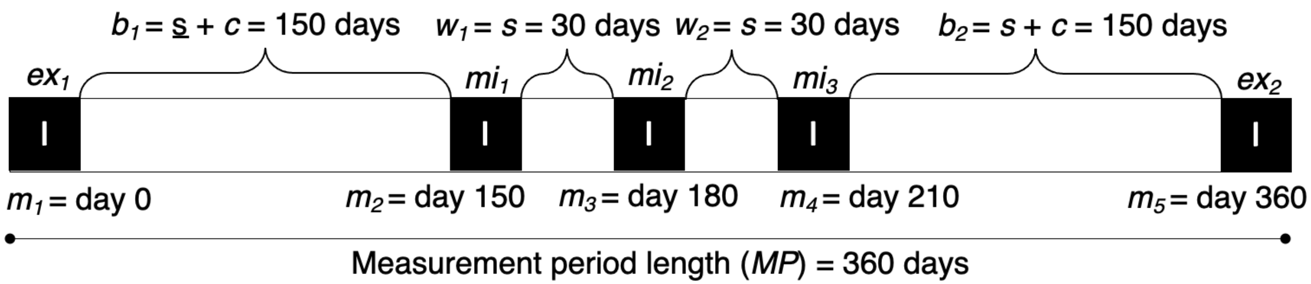
- $b_1 = s + c = 30 + 120 = 150$ days
- $w_1 = s = 30 = 30$ days
- $w_2 = s = 30 = 30$ days
- $b_2 = s + c = 30 + 120 = 150$ days

and the following measurement days are obtained:

- $m_1 =$ day 0
- $m_2 =$ day 150
- $m_3 =$ day 180
- $m_4 =$ day 21
- $m_5 =$ day 360.

Figure C.4

Procedure for Computing Measurement Schedules With Middle-and-Extreme Spacing



Step 1: Setup Variables

= number of measurements (NM) = <u>5</u> measurements

= number of intervals (NI) = $NM - 1 = 5 - 1 =$ <u>4</u> intervals

Number of extreme measurements (ex) = $2(\lfloor \frac{NM}{3} \rfloor)$ = <u>2</u> extreme measurements

Number of middle measurements (mi) = $NM - ex$ = <u>3</u> middle measurements

Step 2: Interval Calculations

s = shortest-interval length = <u>30 days</u>

Constant length(c) = $\frac{r}{2} = \frac{240}{2}$ = <u>120 days</u>

Remaining days(r) = $MP - (NI)s = 360 - 4(30)$ = <u>240 days</u>

Note. In Step 1, setup variables are calculated. With five measurements ($NM = 5$), there are four intervals ($NI = 4$). Importantly, because middle-and-extreme spacing places measurements near the extremities and the middle of the measurement window, the number of measurements in both these sections must also be calculated. The number of extreme measurements is first calculated by dividing the number of measurements by 3 and taking the taking the floor (i.e., rounded-down value [$\lfloor x \rfloor$]) of this value and multiplying it by 2 (see Equation C.5). The number of middle measurements can then be calculated by subtracting the number of extreme measurements (ex) from the number of measurements (NM; see Equation C.7). In Step 2, interval lengths are calculated. For middle-and-extreme spacing, there are two types of interval lengths: 1) Intervals separating either two middle or two extreme measurements and 2) intervals separating one middle and one extreme measurement. Intervals separating two middle or two extreme measurements are set to the shortest-interval length (s), which I set to be 30 days ($w_i = s = 30$). Intervals separating one middle and one extreme measurement are set to the sum of two components: 1) A shortest-interval length (s) and a 2) constant-value interval length (c; see Equation C.9). To obtain the constant-value interval length (c), the sum of shortest-value interval lengths (s) is subtracted from the measurement period of 360 days ($MP = 360$). In the current example with five measurements, 240 days remain ($r = 240$) after subtracting the days needed for the shortest-interval lengths (see Equation C.8). Having computed the number of remaining days, the constant-length value (c) can then be obtained by dividing the number of remaining days by the number of intervals separating middle and extreme measurements, which will always be 2 (see Equation C.9).

Appendix D: Using Nonlinear Function in the Structural Equation Modelling Framework

D.1 Nonlinear Latent Growth Curve Model Used to Analyze Each Generated Data Set

The sections that follow will first review the framework used to build latent growth curve models and then explain how nonlinear functions can be modified to fit into this framework.

D.1.1 Brief Review of the Latent Growth Curve Model

The latent growth curve model proposed by Meredith and Tisak (1990) is briefly reviewed here (for a review, see K. Preacher et al., 2008). Consider an example where data are collected at five time points ($T = 5$) from p people ($\mathbf{y_p} = [y_1, y_2, y_3, y_4, y_5]$). A simple model to fit is one where change over time is defined by a straight line and each person's pattern of change is some variation of this straight line. In modelling parlance, an intercept-slope model is fit where both the intercept and slope are random effects whose values are allowed to vary for each person.

To fit a random-effect intercept-slope model, a general linear pattern can first be specified in the Λ matrix shown below in Equation D.1:

$$\Lambda = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}. \tag{D.1}$$

In each column of Λ, the effect a parameter is specified over the five time points; that is, Λ is a matrix with two columns (one for the intercept and one for the slope parameter) and five rows (one for each time point).[3] The first column of Λ specifies the intercept parameter. Because the effect of the intercept parameter is constant over time, a column of 1s is used to represent its effect. The second column of Λ specifies the slope parameter. Because a linear pattern of growth is assumed, the second column contains a series of monotonically increasing integer numbers across the time points and begins with 0. [4]

To specify the intercept and slope parameters as random effects that vary across people, a weight can be applied to each column of Λ and each weight can vary across people. That is, a p person's pattern of change can be reproduced with a unique set of weights in $\boldsymbol{\iota}_\mathbf{p}$ that determines the extent to which each basis column of Λ contributes to the person's observed change over time. By allowing the weights for the intercept and slope parameters to vary across people, variability can be estimated in these parameters. Discrepancies between the values predicted by $\Lambda\boldsymbol{\iota}_\mathbf{p}$ and a person's observed scores across all five time points are stored in an error vector $\mathcal{E}_\mathbf{p}$. Thus, a person's observed data ($\mathbf{y}_\mathbf{p}$) is reproduced using the function shown below in Equation D.2:

$$y_p = \Lambda\boldsymbol{\iota}_\mathbf{p} + \mathcal{E}_\mathbf{p}. \tag{D.2}$$

Note that Equation D.2 defines the general structural equation modelling framework.

---

[3]The columns of Λ are often called basis curves (Blozis, 2004) or basis functions (Meredith & Tisak, 1990; Browne, 1993) because each column specifies a particular component of change.

[4]The set of numbers specified for the slope starts at zero because there is presumably no effect of any variable at the first time point.

D.1.2 Fitting a Nonlinear Function in the Structural Equation Modelling Framework

Unfortunately, the logistic function of Equation **??**—where each parameter is estimated as a fixed- and random-effect—cannot be directly used in a latent growth curve model because it violates the linear nature of the structural equation modelling framework (Equation D.2). Structural equation models only permit linear combinations—specifically, the products of matrix-vector and/or matrix-matrix multiplication—and so directly fitting a nonlinear function such as the logistic function in Equation **??** is not possible.

One solution to fitting the logistic function within the structural equation modelling framework is to implement the structured latent curve modelling approach (Browne, 1993; Browne & du Toit, 1991; for an excellent review, see K. J. Preacher & Hancock, 2015). Briefly, the structured latent curve modelling approach constructs a Taylor series approximation of a nonlinear function so that the nonlinear function can be fit into the structural equation modelling framework (Equation D.2). The sections that follow will present the structured latent curve modelling approach in four parts such that 1) Taylor series approximations will first be reviewed, 2) a Taylor series approximation will then be constructed for the logistic function, 3) the logistic Taylor series approximation will be modified and fit into the structural equation modelling framework, and 4) the process of parameter estimation will be reviewed.

D.1.2.1 Taylor Series': Approximations of Linear Functions

A Taylor series uses derivative information of a nonlinear function to construct a linear function that is an approximation of the nonlinear function.[5] Equation D.3 shows the general formula for a Taylor series such that

$$P^N(f(x), a) = \sum_{n=0}^{N} \frac{f^n a}{n!}(x-a)^n, \tag{D.3}$$

where N is the highest derivative order of the function $f(a)$ that is taken beginning from a zero-value derivative order ($n = 0$), a is the point where the Taylor series is derived (i.e., the point of derivation), and x is the point where the Taylor series is evaluated (i.e., the point of evaluation). As an example of a Taylor series, consider the second-order Taylor series of $f(x) = \cos(x)$. Note that, across the continuum of x values (i.e., from $-\infty$ to ∞), $\cos(x)$ returns values between -1 and 1 in an oscillatory manner. Computing the second-order Taylor series of $f(x) = \cos(x)$ yields the following function shown in

[5]Linear functions are defined as functions where no parameter exists within its own partial derivative (at any order). For example, none of the parameters in the polynomial equation of $y = a + bt + ct^2 + dt^3$ exist within their own partial derivative: $\frac{\partial y}{\partial a} = 1$, $\frac{\partial y}{\partial b} = t$, $\frac{\partial y}{\partial c} = t^2$, and $\frac{\partial y}{\partial d} = t^3$. Conversely, the logistic function is nonlinear because β and γ exist in their own partial derivatives. For example, the derivative of the logistic function $y = \theta + \frac{\alpha - \theta}{1 + e^{\frac{\beta - t}{\gamma}}}$ with respect to β is $\frac{(\theta - \alpha)(e^{\frac{\beta - t}{\gamma}})(\frac{1}{\gamma})}{1 + (e^{\frac{\beta - t}{\gamma}})^2}$ and so is nonlinear because it contains β.

Equation D.4:

$$\begin{aligned}
P^2(\cos(x), a) &= \frac{\frac{\partial^0 \cos(a)}{\partial a^0}}{0!}(x-a)^0 + \frac{\frac{\partial^1 \cos(a)}{\partial a^1}}{1!}(x-a)^1 + \frac{\frac{\partial^2 \cos(a)}{\partial a^2}}{2!}(x-a)^2 \\
&= \frac{\cos(0)}{0!}(x-0)^0 - \frac{\sin(0)}{1!}(x-0)^1 - \frac{\cos(0)}{2!}(x-0)^2 \\
&= \frac{1}{1}1 - \frac{0}{1}x - \frac{1}{2}x^2 \\
P^2(\cos(x), 0) &= 1 - \frac{1}{2}x^2. \qquad \text{(D.4)}
\end{aligned}$$

Importantly, the second-order Taylor series of $\cos(x)$ shown in Equation D.4 is linear, whereas the function $\cos(x)$ is not linear. To show that the second-order Taylor series of $1 - \frac{1}{2}x^2$ is linear, we can reformulate it by adding placeholder parameters in front of each term (b and c), resulting in the following modified equation of Equation D.5:

$$P^2_{reform}(\cos(x), a) = b1 - c\frac{1}{2}x^2. \qquad \text{(D.5)}$$

If the partial derivative of $P^2(\cos(x), a)$ is taken with respect to b and c, no parameter exists within its own partial derivative, meaning the function is linear (see Equations D.6–D.7 below).

$$\frac{\partial P^2_{reform}(\cos(x), a)}{\partial b} = 1 \text{ and} \qquad \text{(D.6)}$$

$$\frac{\partial P^2_{reform}(\cos(x), c)}{\partial c} = -\frac{1}{2}x^2. \qquad \text{(D.7)}$$

Conversely, the fourth-order partial derivative of $\cos(x)$ contains itself (see Equation D.8), and so is a nonlinear function.

$$\frac{\partial^4 \cos(x)}{\partial x^4} = \cos(x). \tag{D.8}$$

Therefore, Taylor series' can generate linear versions of nonlinear functions by using local derivative information.

Although Taylor series' provide linear versions of nonlinear functions, it is important to emphasize that the linear versions are approximations. More specifically, the second-order Taylor series of $\cos(x)$ perfectly estimates $\cos(x)$ when the point of evaluation x is set equal to the point of derivation a, but estimates $\cos(x)$ with an increasing amount of error as the difference between x and a increases (see Example D.1). Thus, Taylor series are approximations because they are only locally accurate (i.e., near the point of derivation).

Example D.1. *Estimates of Taylor series approximation of $f(x) = \cos(x)$ as the difference between the point of evaluation* x *and the point of derivation* a *increases.*

Taylor series approximation of $\cos(x)$ (specifically, the second-order Taylor series; $P^2[\cos(x), a]$) estimates values that are exactly equal to the values returned by $\cos(x)$ when the point of evaluation (x) is set to the point of derivation (a). The example below computes the value predicted by the Taylor series approximation of $P^2[\cos(x), a]$ and by

$\cos(x)$ when $x = a = 0$.

$$
\begin{aligned}
P^2(\cos(x=0), a=0) &= \cos(x=0) \\
1 - \frac{1}{2}x^2 &= \cos(0) \\
1 - \frac{1}{2}0^2 &= 1 \\
1 - 0 &= 1 \\
1 &= 1
\end{aligned}
$$

Taylor series approximation of $\cos(x)$ (specifically, the second-order Taylor series; $P^2[\cos(x), a]$) estimates a value that is approximately equal ($\approx$) to the value returned by $f\cos(x)$ when the difference between the point of evaluation x and the point of derivation a is small. The example below computes the value predicted by the Taylor series approximation of $P^2[\cos(x), a]$ and by $\cos(x)$ when $x = 1$ and $a = 0$.

$$
\begin{aligned}
P^2(\cos(x=1), 0) &\approx \cos(x=1) \\
1 - \frac{1}{2}x^2 &\approx \cos(1) \\
1 - \frac{1}{2}1^2 &\approx 0.54 \\
1 - 0.5 &\approx 0.54 \\
0.5 &\approx 0.54
\end{aligned}
$$

Taylor series approximation of $f\cos(x)$ (specifically, the second-order Taylor series; $P^2[\cos(x), a]$) estimates a a value that is clearly not equal ($\neq$) to the value returned by $f\cos(x)$ when the difference between the point of evaluation x and the point of derivation a is large. The example below computes the value predicted by the Taylor series approximation of $P^2[\cos(x), a]$ and by $\cos(x)$ when $x = 4$ and $a = 0$.

$$
\begin{aligned}
P^2(\cos(x = 4), 0) &\neq \cos(x = 4) \\
1 - \frac{1}{2}x^2 &\neq \cos(4) \\
1 - \frac{1}{2}4^2 &\neq -0.65 \\
1 - 16 &\neq -0.65 \\
0.5 &\neq -0.65
\end{aligned}
$$

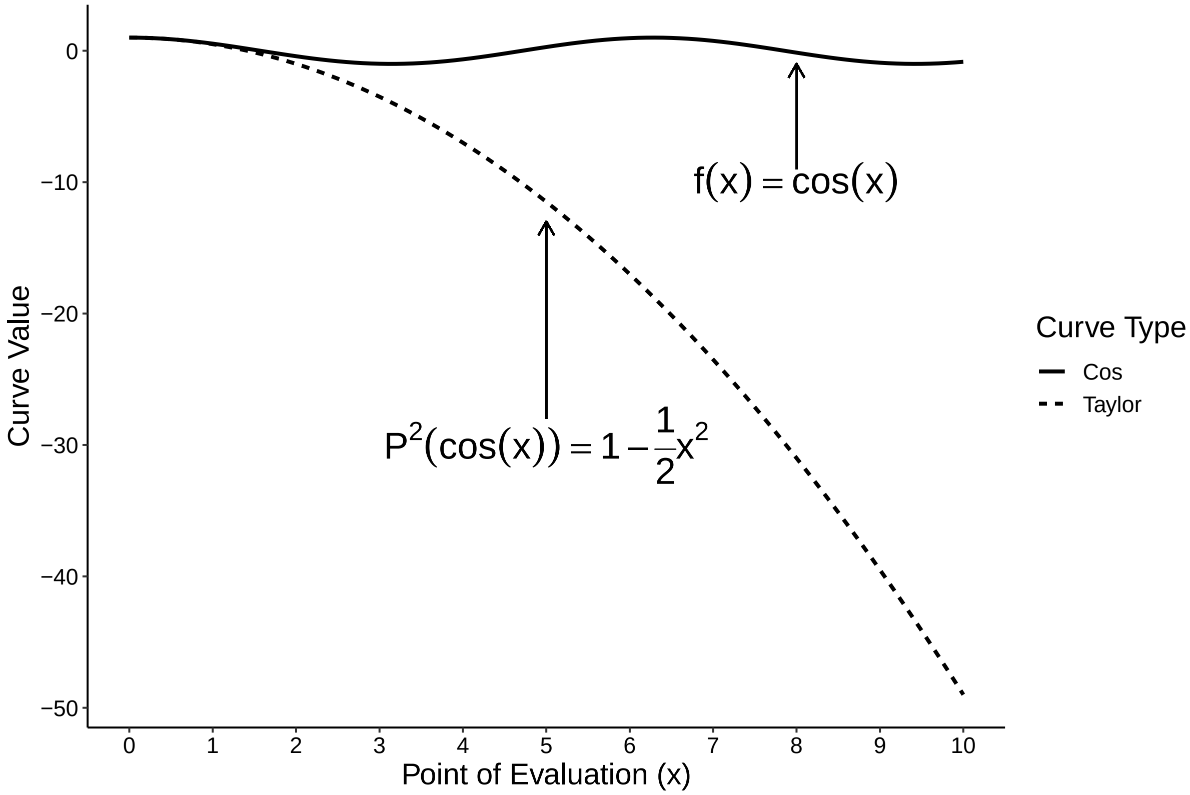
Figure D.1 provides a comprehensive visualization of the of the point conveyed in Example D.1 about the accuracy of Taylor series approximations. In Figure D.1, the values returned by the nonlinear function of $\cos(x)$ and its second-order Taylor series $P^2[\cos(x)] = 1 - \frac{1}{2}x^2$ are shown. The second order Taylor series perfectly estimates $\cos(x)$ when the point of evaluation (x) equals the point of derivation (a; $x = a = 0$), but incurs an increasingly large amount of error as the difference between the point of evaluation and the point of derivation increases. For example, at $x = 10$, $\cos(10) = -0.84$, but the Taylor series outputs a value of -49.50 ($P^2[cos(50)] = 1 - \frac{1}{2}10^2 = -49.50$).

D.1.2.2 Taylor Series of the Logistic Function

Given that the Taylor series provides a linear version of a nonlinear function, the structured latent curve modelling approach uses Taylor series' to fit nonlinear functions into the linear nature of the structural equation modelling framework (Browne, 1993; Browne & du Toit, 1991). In the current simulations, the logistic function was used to generate data (see Equation D.10), and so a Taylor series approximation was constructed

Figure D.1

Estimation Accuracy of Taylor Series Approximation of Nonlinear Function (cos(x))



Note. The second order Taylor series perfectly estimates $\cos(x)$ when the point of evaluation (x) equals the point of derivation (a; $x = a = 0$), but incurs an increasingly large amount of error as the difference between the point of evaluation and the point of derivation increases. For example, at $x = 10$, $\cos(x) = -0.84$, but the Taylor series outputs a value of -49.50 ($P^2[cos(50)] = 1 - \frac{1}{2}10^2 = -49.50$).

for the logistic function in the analysis. Note that, because the logistic function had four parameters (θ, α, β, γ), partial derivatives were computed with respect to each of the parameters. Using a derivative order set to one ($n = 1$), the following Taylor series was constructed for the logistic function (Equation D.9):

$$P^1(L(\Theta, t)) = L + \frac{\partial L}{\partial \theta}(x_\theta - a_\theta)^1 + \frac{\partial L}{\partial \alpha}(x_\alpha - a_\alpha)^1 + \frac{\partial L}{\partial \beta}(x_\beta - a_\beta)^1 + \frac{\partial L}{\partial \gamma_\gamma}(x_\gamma - a_\gamma)^1, \tag{D.9}$$

where $\mathbf{L}(\Theta, \mathbf{t})$ represents the logistic function shown below in Equation D.10:

$$\mathbf{L}(\Theta, \mathbf{t}) = \theta + \frac{\alpha - \theta}{1 + e^{\frac{\beta - t}{\gamma}}} + \epsilon, \tag{D.10}$$

with $\Theta = [\theta, \alpha, \beta, \gamma]$ and $\mathbf{L}(\Theta, \mathbf{t})$ being a vector of scores across all $\mathbf{t}$ time points. Because each parameter of the logistic function has a unique meaning (see section on data generation), they are unlikely to have the same population value, and so the derivation (a) will, therefore, differ for each parameter. To set the derivation values (a), the mean values estimated by the structured latent growth curve model for each parameter (i.e., fixed-effect values) are used, meaning that each derivation value in Equation D.9 is replaced with a model estimate as shown below:

- $a_\theta = \hat{\theta}$
- $a_\alpha = \hat{\alpha}$
- $a_\beta = \hat{\beta}$
- $a_\gamma = \hat{\gamma}$

where a caret ($\hat{}$) denotes a parameter value that is estimated by the analysis. In order to compute curves for each p person, evaluation points for each parameter (x_θ, x_α, x_β, x_γ) are set to the value computed for a given person (θ_p, α_p, β_p, γ_p). Thus, each evaluation value in Equation D.9 is replaced with a person-specfic value as shown below:

- $x_\theta = \theta_p$
- $x_\alpha = \alpha_p$
- $x_\beta = \beta_p$
- $x_\gamma = \gamma_p$.

Substituting the above values for the derivation and evaluation values of x and a in the initial logistic Taylor series (Equation D.9) yields the following function (Equation D.11):

$$P^1(L(\Theta, t)) = L(\Theta, t) + \frac{\partial L}{\partial \theta}(\theta_p - \hat{\theta})^1 + \frac{\partial L}{\partial \alpha}(\alpha_p - \hat{\alpha})^1 + \frac{\partial L}{\partial \beta}(\beta_p - \hat{\beta})^1 + \frac{\partial L}{\partial \gamma}(\gamma_p - \hat{\beta})^1. \tag{D.11}$$

Two important points about Equation D.9 deserve mentioning. First, the average population logistic curve (i.e., the fixed-effect parameter values) will have a perfect logistic function shape. In estimating the average population logistic curve, the evaluation values (x) are set equal to the derivation value counterparts (a); that is, each mean value estimated for a parameter ($\hat{\theta}$, $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$) replaces the corresponding derivation-evaluation pair in Equation D.9. Second, it is possible that estimates of random-effect parameters (i.e., variability observed in a parameter's value across people) may be misleading. To compute the values for the random-effect parameters, the evaluation values (a) are set to the logistic function values needed to compute each p person's observed curve (θ_p, α_p, β_p, γ_p). Because Taylor series approximations are only locally accurate, the curves computed for individuals can accommodate shapes that do not resemble a logistic (i.e., s-shaped) pattern (see Example D.1). Thus, estimates of random-effect parameters (i.e., variability observed in a parameter's value across people) can be influenced by curves that do not have a logistic shape and, therefore, may be misleading.

D.1.2.3 Fitting the Logistic Taylor Series Into the Structual Equation Modelling Framework

With the logistic Taylor series computed in Equation D.11, it can be fit into the structural equation modelling framework by transforming it from its scalar form (Equation D.11) into its matrix form (see Equation D.16). In transforming the scalar form of the logistic Taylor series into a matrix form, three steps will be completed, with each step transforming a component of the scalar form into a matrix representation. The paragraphs that follow detail each of these three steps.

First, the partial derivative information must be transformed into their matrix form. The matrix Λ shown below contains the partial derivative information presented in the scalar Taylor series function (see Equation D.11):[6]

$$\Lambda = \begin{bmatrix} \frac{\partial L(\Theta,t_1)}{\partial \theta} & \frac{\partial L(\Theta,t_1)}{\partial \alpha} & \frac{\partial L(\Theta,t_1)}{\partial \beta} & \frac{\partial L(\Theta,t_1)}{\partial \gamma} \\ \frac{\partial L(\Theta,t_2)}{\partial \theta} & \frac{\partial L(\Theta,t_2)}{\partial \alpha} & \frac{\partial L(\Theta,t_2)}{\partial \beta} & \frac{\partial L(\Theta,t_2)}{\partial \gamma} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial L(\Theta,t_n)}{\partial \theta} & \frac{\partial L(\Theta,t_n)}{\partial \alpha} & \frac{\partial L(\Theta,t_n)}{\partial \beta} & \frac{\partial L(\Theta,t_n)}{\partial \gamma} \end{bmatrix}.$$

As in the structural equation modelling framework (see Equation D.2) where each column of Λ specifies a basis curve (i.e., loadings of a growth parameter onto all time points that specify the effect of the parameter over time), each column of Λ in the structured latent curve modelling approach similarly contains the loadings of a logistic function parameter across all the n time points, but the loading values are now determined by the partial derivative of the logistic function with respect to that parameter.

Second, the difference between the evaluation and derivation values $(x - a)$ must be transformed into their matrix form. As a reminder, the difference between the evaluation and derivation values is needed so that person-specific curves can be computed. Thus, the difference between the evaluation and derivation values can be conceptualized as person-specific deviation. The vector $\boldsymbol{\iota}_\mathbf{p}$ contains the person-specific deviations (e.g., $\hat{\theta} - \theta_p$) from

[6]This is also known as a Jacobian matrix.

each mean estimated parameter value as shown below:

$$\boldsymbol{\iota}_\mathbf{p} = \begin{bmatrix} \hat{\theta} - \theta_p \\ \\ \hat{\alpha} - \alpha_p \\ \\ \hat{\beta} - \beta_p \\ \\ \hat{\gamma}_i - \gamma_p \end{bmatrix},$$

where a caret ($\hat{}$) denotes the mean value estimated for a given parameter and a subscript p indicates a parameter value computed for a person.

With a matrix of logistic function loadings (Λ) and the vector of person-specific deviations ($\boldsymbol{\iota}_\mathbf{p}$), person-specific deviations can be computed for each basis column of Λ. Specifically, the person-specific basis column deviations can be computed by post-multiplying the matrix of loadings (Λ) by the vector of person-specific deviations ($\boldsymbol{\iota}_\mathbf{p}$), as shown below in Equation D.12:

$$\text{Basis column deviations}_p = \Lambda\boldsymbol{\iota}_\mathbf{p}. \tag{D.12}$$

Because $\Lambda\boldsymbol{\iota}_\mathbf{p}$ only provides the extent to which each person's curve deviates from the average curve ($\mathbf{L}(\Theta, \mathbf{t})$), it cannot alone be used to compute person-specfic curves. To compute person-specific curves ($\mathbf{y}_\mathbf{p}$), the average logistic curve must be added to Equation D.12, as shown below in Equation D.13:

$$\mathbf{y}_\mathbf{p} = \mathbf{L}(\Theta, \mathbf{t}) + \Lambda\boldsymbol{\iota}_\mathbf{p} + \mathcal{E}_\mathbf{p}. \tag{D.13}$$

Unfortunately, the logistic function ($\mathbf{L}(\Theta, \mathbf{t})$) in the above expression (Equation D.13) is simply the original logistic function (see Equation D.10), and so Equation D.13 above is nonlinear. Because Equation D.13 is nonlinear, it cannot be inserted in the structural equation modelling framework, which requires a linear function (see Equation D.2). Thus, the logistic function term in Equation D.13 ($\mathbf{L}(\Theta, \mathbf{t})$) must be linearized so that the logistic Taylor series can be used in the structural equation modelling framework.

Third, and last, the logistic function component ($\mathbf{L}(\Theta, \mathbf{t})$) must be linearized. By taking advantage of some clever linear algebra, the logistic function component can be rewritten as the product of the partial derivative matrix (Λ) and a mean vector (τ; Browne, 1993; Shapiro & Browne, 1987) as shown below in Equation D.14:

$$\mathbf{L}(\Theta, \mathbf{t}) = \boldsymbol{\Lambda}\tau. \tag{D.14}$$

Importantly, the values of the mean vector τ need to be determined so that a linear representation of the logistic function can be created. Example D.2 below solves for the mean vector (τ) and shows that the values obtained for the linear parameters (i.e., θ and α) constitute the mean values estimated by the analysis (i.e., the fixed-effect values) and zeroes are obtained for the nonlinear parameters (i.e., θ and α). Given that the vector τ contains mean estimated values, it is often called the mean vector (Blozis, 2004; K. J. Preacher & Hancock, 2015).

Example D.2. *Computation of mean vector* τ*.*

Given the parameter estimates of $\hat{\theta} = 3.00$, $\hat{\alpha} = 3.32$, $\hat{\beta} = 180.00$, and $\hat{\gamma} = 20.00$ and $\mathbf{t}$

$= [0,\ 1,\ 2,\ 3]$, $\tau = [3.00,\ 3.32,\ 0,\ 0]$, then

$$\mathbf{L}(\Theta, \mathbf{t}) = \mathbf{\Lambda}\tau$$

$$[3.00, 3.02, 3.30, 3.32] = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 \\ 0.95 & 0.05 & -0.00 & 0.00 \\ 0.05 & 0.95 & -0.00 & -0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 \end{bmatrix} \tau$$

$$\begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 \\ 0.95 & 0.05 & -0.00 & 0.00 \\ 0.05 & 0.95 & -0.00 & -0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 \end{bmatrix}^{-1} \begin{bmatrix} 3.00 \\ 3.02 \\ 3.30 \\ 3.32 \end{bmatrix} = \mathbf{\Lambda}\tau$$

$$\tau = [3.00, 3.32, 0, 0]$$

With $\mathbf{L}(\Theta, \mathbf{t}) = \Lambda\tau$, Equation D.13 can be rewritten in a linear equation as shown below in Equation D.15:

$$\mathbf{y_p} = \Lambda\tau + \Lambda\iota_\mathbf{p} + \mathcal{E}_\mathbf{p}. \tag{D.15}$$

Two important points should be made about Equation D.15. First, with some algebraic modification, it can be shown to have the exact same form as the general structural equation modelling framework (see Equation D.2) that expresses a person's score (y_p) as the sum of a loading matrix (Λ) post-multiplied by a vector of person-specific deviations (ι_p)

and an error vector ($\mathcal{E}_\mathbf{p}$). To show the equivalence between Equation D.15 and Equation D.2, the mean vector τ and vector of person-specific deviations $\iota_\mathbf{p}$ can be combined into a new vector $\mathbf{s_p}$ that, like the product of $\mathbf{\Lambda}\tau$ (see Equation D.14), also represents the person-specific weights applied to the basis curves in Λ such that

$$\mathbf{s_p} = \tau + \iota_\mathbf{p} = \begin{bmatrix} \hat{\theta} + \hat{\theta} - \theta_p \\ \\ \hat{\alpha} + \hat{\alpha} - \alpha_p \\ \\ 0 + \hat{\beta} - \beta_p \\ \\ 0 + \hat{\gamma} - \gamma_p \end{bmatrix},$$

which allows Equation D.15 to be reexpressed in Equation D.16 below and, thus, take on the exact same form as the general structural equation modelling framework (see Equation D.2)

$$\mathbf{y_p} = \Lambda \mathbf{s_p} + \mathcal{E}_\mathbf{p}. \tag{D.16}$$

Second, the logistic Taylor series shown in Equation D.15 reproduces the nonlinear logistic function. Because the expected value of the person-specific weights ($\mathbf{s_p}$) is the mean vector (τ; $\mathbb{E}[\mathbf{s_p}] = \tau$), the expected set of scores predicted across all people ($\mathbb{E}[\mathbf{y_p}]$) gives back the original expression for the logistic function matrix-vector product in Equation D.14 as shown below in Equation D.17:

$$\mathbb{E}[\mathbf{y_p}] = \Lambda\tau = \mathbf{L}(\Theta, \mathbf{t}). \tag{D.17}$$

Therefore, the structured latent curve modelling approach successfully reproduces the output of the nonlinear logistic function (Equation D.10) with the linear function of Equation D.16. Note that that no error term exists in Equation D.17 because the expected value of the error values is zero ($\mathbb{E}[\mathcal{E}_{\mathbf{p}}] = 0$).

D.1.2.4 Estimating Parameters in the Structured Latent Curve Modelling Approach

To estimate the parameter values, the full-information maximum likelihood shown in Equation D.18 is computed for each person (i.e., likelihood of observing a p person's data given the estimated parameter values):

$$\mathcal{L}_p = k_p \ln(2\pi) + \ln(|\mathbf{\Sigma_p}| + (\mathbf{y_p} - \mathbf{\mu_p})^\top \mathbf{\Sigma_p}^{-1}(\mathbf{y_p} - \mathbf{\mu_p}), \tag{D.18}$$

where k_p is the number of non-missing values for a given p person, $\mathbf{\Sigma_p}$ is the model-implied covariance matrix with rows and columns filtered at time points where person p has missing data, $\mathbf{y_p}$ is a vector containing the data points collected for a p person (i.e., filtered data), and $\mathbf{\mu_p}$ is the model-implied mean vector that is filtered at time points where person p has missing data. Note that, because all my simulations assumed complete data across all times points, no filtering procedures were executed (for a review of the filtering procedure, see Boker et al., 2020, Chapter 5). Thus, computing the above full-information maximum likelihood in Equation D.18 is equivalent to computing the below likelihood function in Equation D.19:

$$\mathcal{L}_p = k_p \ln(2\pi) + \ln(|\mathbf{\Sigma}| + (\mathbf{y_p} - \mathbf{\mu})^\top \mathbf{\Sigma}^{-1}(\mathbf{y_p} - \mathbf{\mu}), \tag{D.19}$$

where $\mathbf{\Sigma}$ is the model-implied covariance matrix, $\mathbf{y_p}$ contains the data collected from a p person, and μ is the model-implied mean vector. The model-implied covariance matrix $\mathbf{\Sigma}$ is computed using Equation D.20 below:

$$\mathbf{\Sigma} = \Lambda\Psi\Lambda + \Omega_{\mathcal{E}}, \tag{D.20}$$

where Ψ is the random-effect covariance matrix and $\Omega_{\mathcal{E}}$ contains the error variances at each time point. The mean vector μ is computed using Equation D.21 shown below:

$$\mu = \Lambda\tau. \tag{D.21}$$

Parameter estimation is conducted by finding values for the model-implied covariance matrix $\mathbf{\Sigma}$ and the model-implied mean vector μ that maximizes the sum of log-likelihoods across all P people (see Equation D.22 below):

$$\mathcal{L} = \underset{\mathbf{\Sigma},\mu}{\arg\max} \sum_{p=1}^{P} \mathcal{L}_p. \tag{D.22}$$

In OpenMx, the above problem is solved using the sequential least squares quadratic program (for a review, see Kraft, 1994).

Appendix E: OpenMx Code for Structured Latent Growth Curve Model Used in Simulation Experiments

The code that I used to model logistic pattern of change (see data generation) is shown in Code Block E.1. Note that, the code is largely excerpted from the `run_exp_simulations()` and `create_logistic_model_ns()` functions from the nonlinSims package, and so readers interested in obtaining more information should consult the source code of this package. One important point to mention is that the model specified in Code Block E.1 assumes time-structured data.

Code Block E.1

OpenMx Code for Structured Latent Growth Curve Model That Assumes Time-Structured Data

```
#Days on which measurements are assumed to be taken (note that model assumes
time-structured data; that is, at each time point, participants provide data at the
exact same moment). The measurement days obtained by finding the unique values in the
`measurement_day` column of the generated data set.
measurement_days <- unique(data$measurement_day)

#Manifest variable names (i.e., names of columns containing data at each time point,
manifest_vars <- nonlinSims:::extract_manifest_var_names(data_wide = data_wide)

#Now convert data to wide format (needed for OpenMx)
data_wide <- data[ , c(1:3, 5)] %>%
    pivot_wider(names_from = measurement_day, values_from = c(obs_score,
    actual_measurement_day))

#Remove . from column names so that OpenMx does not run into error (this occurs
because, with some spacing schedules, measurement days are not integer values.)
names(data_wide) <- str_replace(string = names(data_wide), pattern = '\\.', replacement
= '_')

#Latent variable names (theta = baseline, alpha = maximal elevation, beta =
days-to-halfway elevation, gamma = triquarter-haflway elevation)
latent_vars <- c('theta', 'alpha', 'beta', 'gamma')

latent_growth_curve_model <- mxModel(
  model = model_name,
  type = 'RAM', independent = T,
  mxData(observed = data_wide, type = 'raw'),

  manifestVars = manifest_vars,
  latentVars = latent_vars,

  #Residual variances; by using one label, they are assumed to all be equal
  (homogeneity of variance). That is, there is no complex error structure.
  mxPath(from = manifest_vars,
         arrows=2, free=TRUE,  labels='epsilon', values = 1, lbound = 0),

  #Latent variable covariances and variances (note that only the variances are
  estimated. )
  mxPath(from = latent_vars,
         connect='unique.pairs', arrows=2,
         free = c(TRUE,FALSE, FALSE, FALSE,
                  TRUE, FALSE, FALSE,
                  TRUE, FALSE,
                  TRUE),
         values=c(1, NA, NA, NA,
                  1, NA, NA,
                  1, NA,
                  1),
         labels=c('theta_rand', 'NA(cov_theta_alpha)', 'NA(cov_theta_beta)',
                  'NA(cov_theta_gamma)',
                  'alpha_rand','NA(cov_alpha_beta)', 'NA(cov_alpha_gamma)',
                  'beta_rand', 'NA(cov_beta_gamma)',
```

```
                    'gamma_rand'),
          lbound = c(1e-3, NA, NA, NA,
                     1e-3, NA, NA,
                     1, NA,
                     1),
          ubound = c(2, NA, NA, NA,
                     2, NA, NA,
                     90^2, NA,
                     45^2)),

  # Latent variable means (linear parameters). Note that the parameters of beta and
  gamma do not have estimated means because they are nonlinear parameters (i.e., the
  logistic function's first-order partial derivative with respect to each of those two
  parameters contains those two parameters. )
  mxPath(from = 'one', to = c('theta', 'alpha'), free = c(TRUE, TRUE), arrows = 1,
         labels = c('theta_fixed', 'alpha_fixed'), lbound = 0, ubound = 7,
         values = c(1, 1)),

  #Functional constraints (needed to estimate mean values of fixed-effect parameters)
  mxMatrix(type = 'Full', nrow = length(manifest_vars), ncol = 1, free = TRUE,
           labels = 'theta_fixed', name = 't', values = 1, lbound = 0,  ubound = 7),
  mxMatrix(type = 'Full', nrow = length(manifest_vars), ncol = 1, free = TRUE,
           labels = 'alpha_fixed', name = 'a', values = 1, lbound = 0,  ubound = 7),
  mxMatrix(type = 'Full', nrow = length(manifest_vars), ncol = 1, free = TRUE,
           labels = 'beta_fixed', name = 'b', values = 1, lbound = 1, ubound = 360),
  mxMatrix(type = 'Full', nrow = length(manifest_vars), ncol = 1, free = TRUE,
           labels = 'gamma_fixed', name = 'g', values = 1, lbound = 1, ubound = 360),

  mxMatrix(type = 'Full', nrow = length(manifest_vars), ncol = 1, free = FALSE,
           values = measurement_days, name = 'time'),

  #Algebra specifying first-order partial derivatives;
  mxAlgebra(expression = 1 - 1/(1 + exp((b - time)/g)), name="Tl"),
  mxAlgebra(expression = 1/(1 + exp((b - time)/g)), name = 'Al'),

  mxAlgebra(expression = -((a - t) * (exp((b - time)/g) * (1/g))/(1 + exp((b -
  time)/g))^2), name = 'Bl'),
  mxAlgebra(expression =  (a - t) * (exp((b - time)/g) * ((b - time)/g^2))/(1 + exp((b
  -time)/g))^2, name = 'Gl'),

  #Factor loadings; all fixed and, importantly, constrained to change according to
  their partial derivatives (i.e., nonlinear functions)
  mxPath(from = 'theta', to = manifest_vars, arrows=1, free=FALSE,
         labels = sprintf(fmt = 'Tl[%d,1]', 1:length(manifest_vars))),
  mxPath(from = 'alpha', to = manifest_vars, arrows=1, free=FALSE,
         labels = sprintf(fmt = 'Al[%d,1]', 1:length(manifest_vars))),
  mxPath(from='beta', to = manifest_vars, arrows=1,  free=FALSE,
         labels =  sprintf(fmt = 'Bl[%d,1]', 1:length(manifest_vars))),
  mxPath(from='gamma', to = manifest_vars, arrows=1,  free=FALSE,
         labels =  sprintf(fmt = 'Gl[%d,1]', 1:length(manifest_vars))),

  #Fit function used to estimate free parameter values.
  mxFitFunctionML(vector = FALSE)
)

#Use starting value function from OpenMx to generate good starting values (uses
weighted least squares)
latent_growth_model <- mxAutoStart(model = latent_growth_model)

#Fit model using mxTryHard(). Increases probability of convergence by attempting model
convergence by randomly shifting starting values.
model_results <- mxTryHard(latent_growth_model)
```

Appendix F: Complete Versions of Bias/Precision Plots (Day- and Likert-Unit Parameters)

F.1 Experiment 1

F.1.1 Equal Spacing

Figure F.1

Bias/Precision Plots for Day- and Likert-Unit Parameters With Equal Spacing in Experiment 1

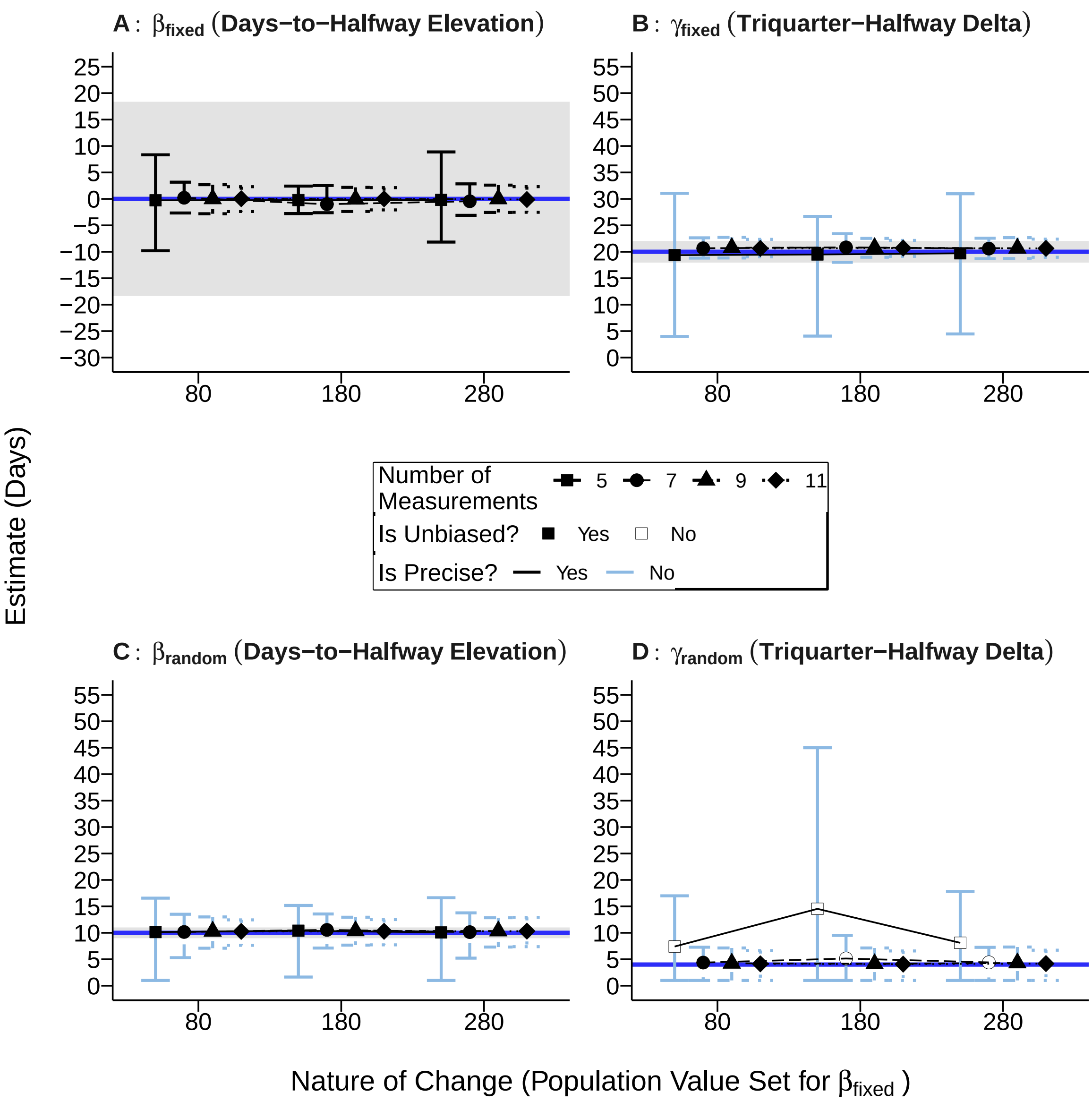
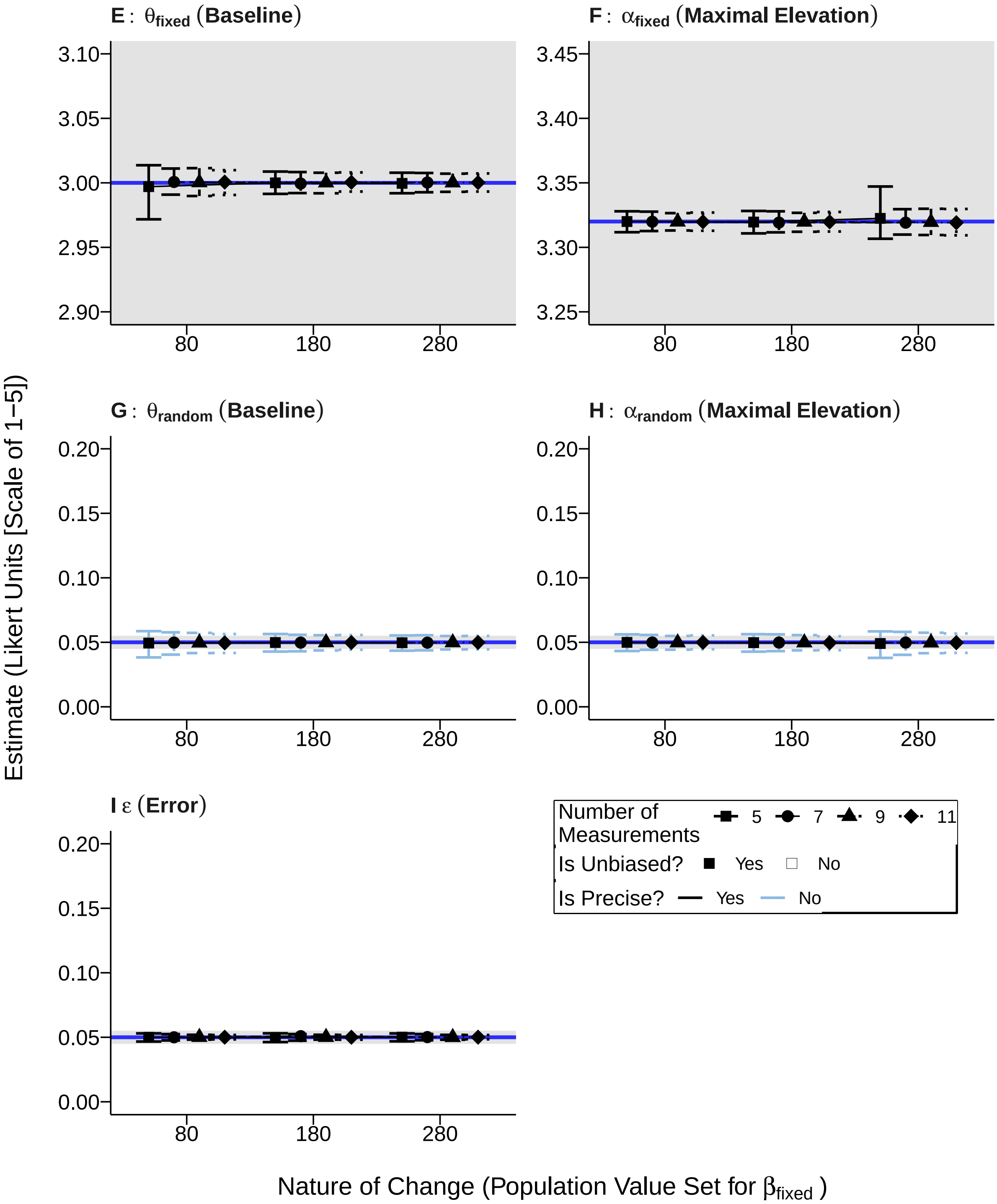


Figure F.1

Bias/Precision Plots for Day- and Likert-Unit Parameters With Equal Spacing in Experiment 1 (continued)



Note. Panels A–B: Bias/precision plots for the fixed- and random-effect days-to-halfway elevation parameters, respectively (β_{fixed} and β_{random}). Panels C–D: Bias/precision plots for the fixed- and random-effect triquarter-halfway elevation parameters, respectively (γ_{fixed} and γ_{random}). Panels E–F: Bias/precision plots for the fixed- and random-effect baseline parameters, respectively (θ_{fixed} and θ_{random}).

Panels G–H: Bias/precision plots for the fixed- and random-effect maximal elevation parameters, respectively (α_{fixed} and α_{random}). Blue horizontal lines in each panel represent the population value for each parameter. Population values for each day-unit parameter are as follows: $\beta_{fixed} \in$ 80, 180, 280, β_{random} = 10.00, γ_{fixed} = 20.00, γ_{random} = 4.00, θ_{fixed} = 3.00, θ_{random} = 0.05, α_{fixed} = 3.32, α_{random} = 0.05, ϵ = 0.05. Gray bands indicate the $\pm 10\%$ margin of error for each parameter and unfilled dots indicate cells with average parameter estimates outside of the margin or biased estimates. Error bars represent the middle 95% of estimated values, with light blue error bars indicating imprecise estimation. I considered dots that fell outside the gray bands as biased and error bar lengths with at least one whisker length exceeding the 10% cutoff (i.e., or longer than the portion of the gray band underlying the whisker) as imprecise. Note that random-effect parameter units are in standard deviation units. Importantly, across all nature-of-change values (i.e., population values used for β_{fixed}), the acceptable amount of bias and precision was based on a population value of 180. See Table H.1 for specific values estimated for each parameter.

F.1.2 Time-Interval Increasing Spacing

Figure F.2

Bias/Precision Plots for Day- and Likert-Unit Parameters With Time-Interval Increasing Spacing in Experiment 1

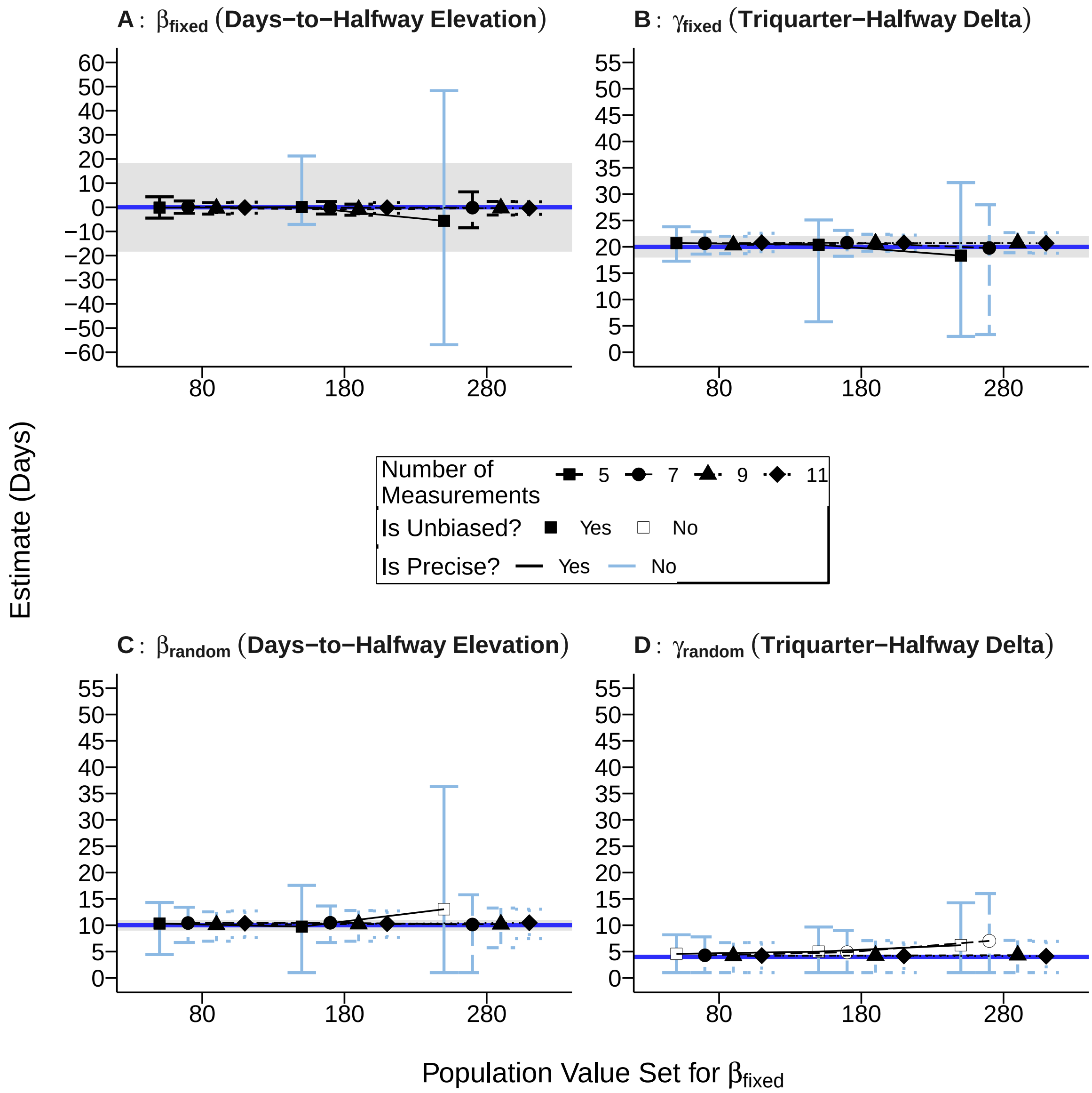
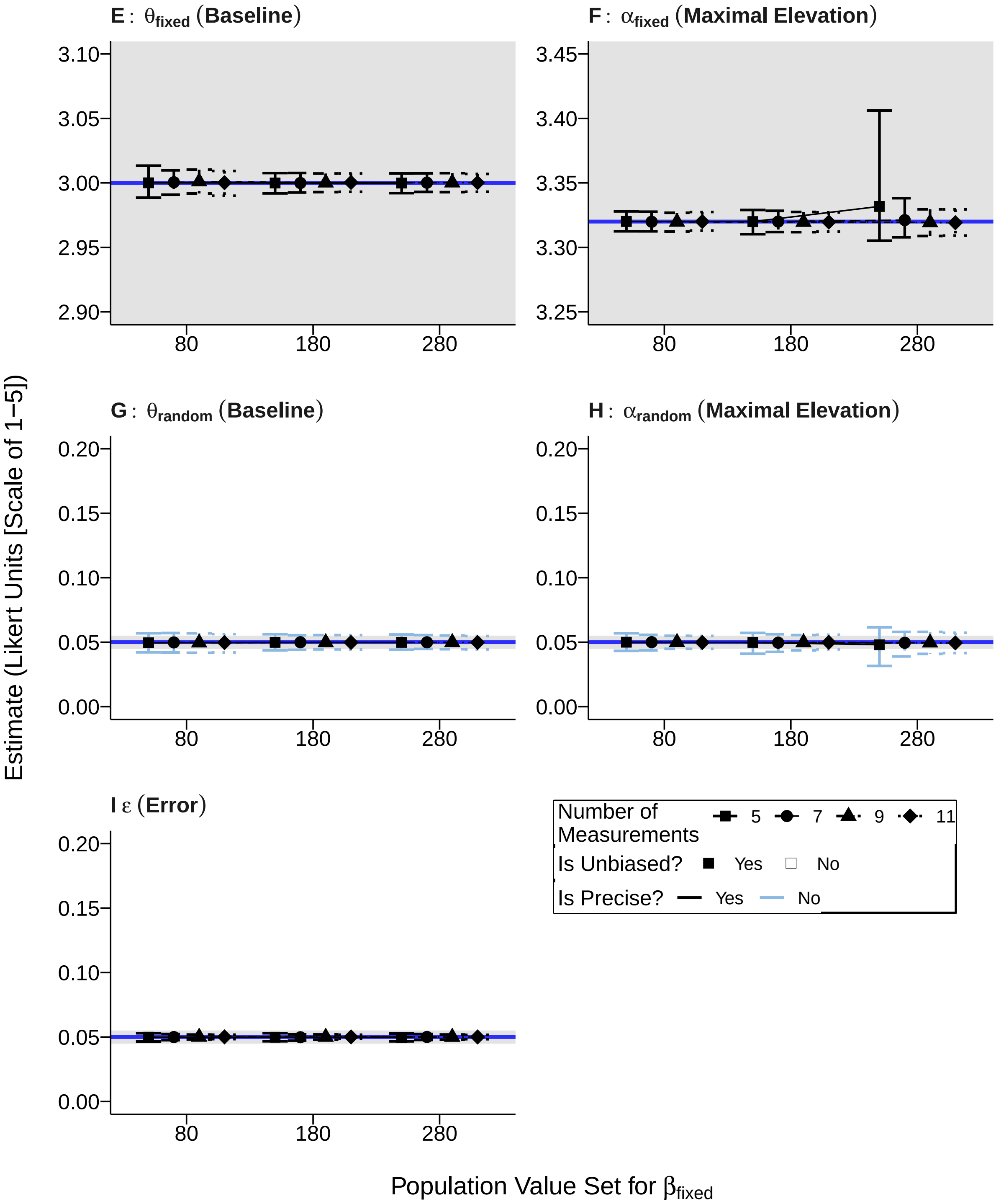


Figure F.2

Bias/Precision Plots for Day- and Likert-Unit Parameters With Time-Interval Increasing Spacing in Experiment 1 (continued)



Note. Panels A–B: Bias/precision plots for the fixed- and random-effect days-to-halfway elevation parameters, respectively (β_{fixed} and β_{random}). Panels C–D: Bias/precision plots for the fixed- and random-effect triquarter-halfway elevation parameters, respectively (γ_{fixed} and γ_{random}). Panels E–F: Bias/precision plots for the fixed- and random-effect baseline parameters, respectively (θ_{fixed} and θ_{random}).

Panels G–H: Bias/precision plots for the fixed- and random-effect maximal elevation parameters, respectively (α_{fixed} and α_{random}). Blue horizontal lines in each panel represent the population value for each parameter. Population values for each day-unit parameter are as follows: $\beta_{fixed} \in$ 80, 180, 280, β_{random} = 10.00, γ_{fixed} = 20.00, γ_{random} = 4.00, θ_{fixed} = 3.00, θ_{random} = 0.05, α_{fixed} = 3.32, α_{random} = 0.05, ϵ = 0.05. Gray bands indicate the $\pm 10\%$ margin of error for each parameter and unfilled dots indicate cells with average parameter estimates outside of the margin or biased estimates. Error bars represent the middle 95% of estimated values, with light blue error bars indicating imprecise estimation. I considered dots that fell outside the gray bands as biased and error bar lengths with at least one whisker length exceeding the 10% cutoff (i.e., or longer than the portion of the gray band underlying the whisker) as imprecise. Note that random-effect parameter units are in standard deviation units. Importantly, across all nature-of-change values (i.e., population values used for β_{fixed}), the acceptable amount of bias and precision was based on a population value of 180. See Table H.1 for specific values estimated for each parameter.

F.1.3 Time-Interval Decreasing Spacing

Figure F.3

Bias/Precision Plots for Day- and Likert-Unit Parameters With Time-Interval Decreasing Spacing in Experiment 1

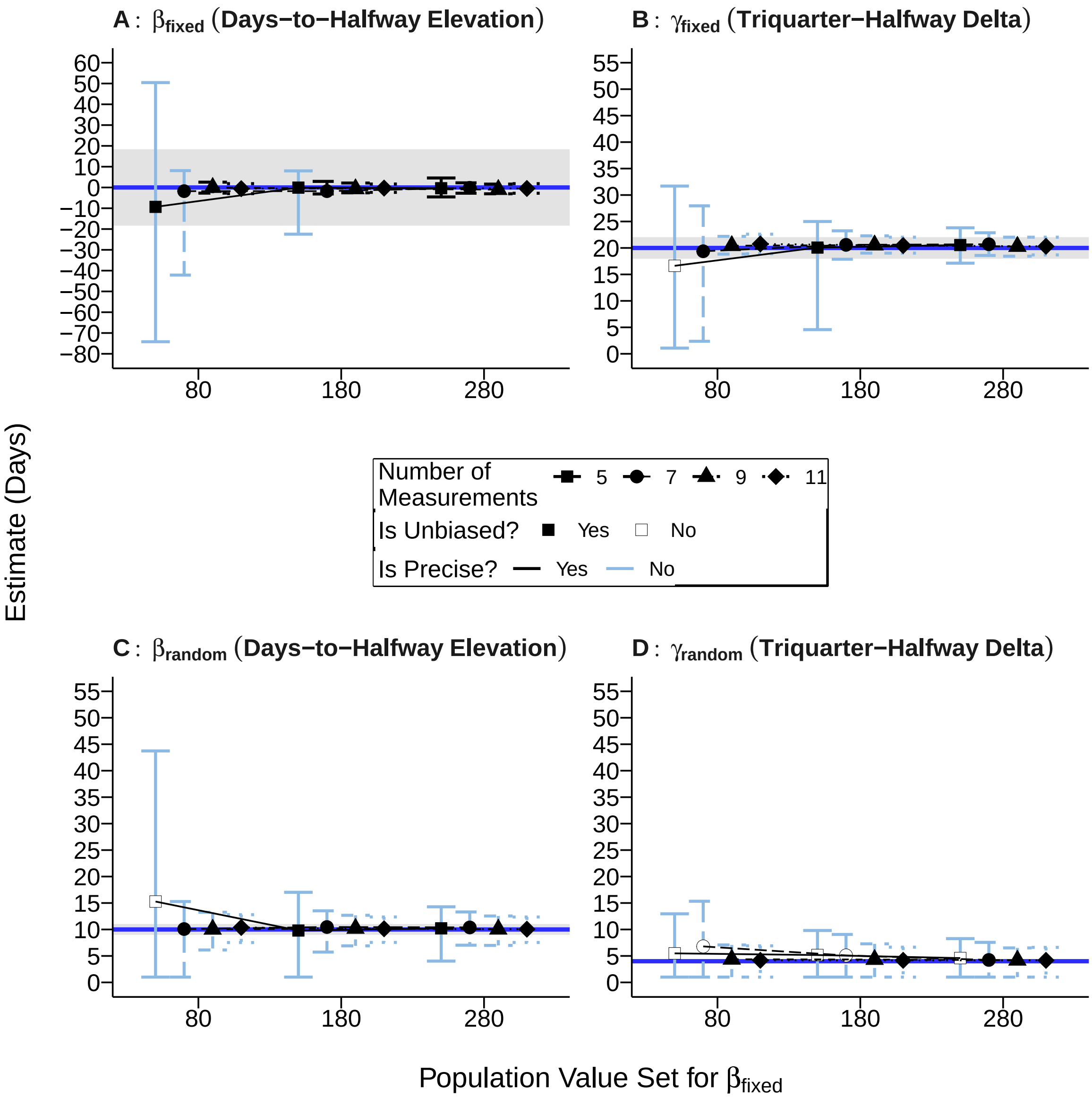
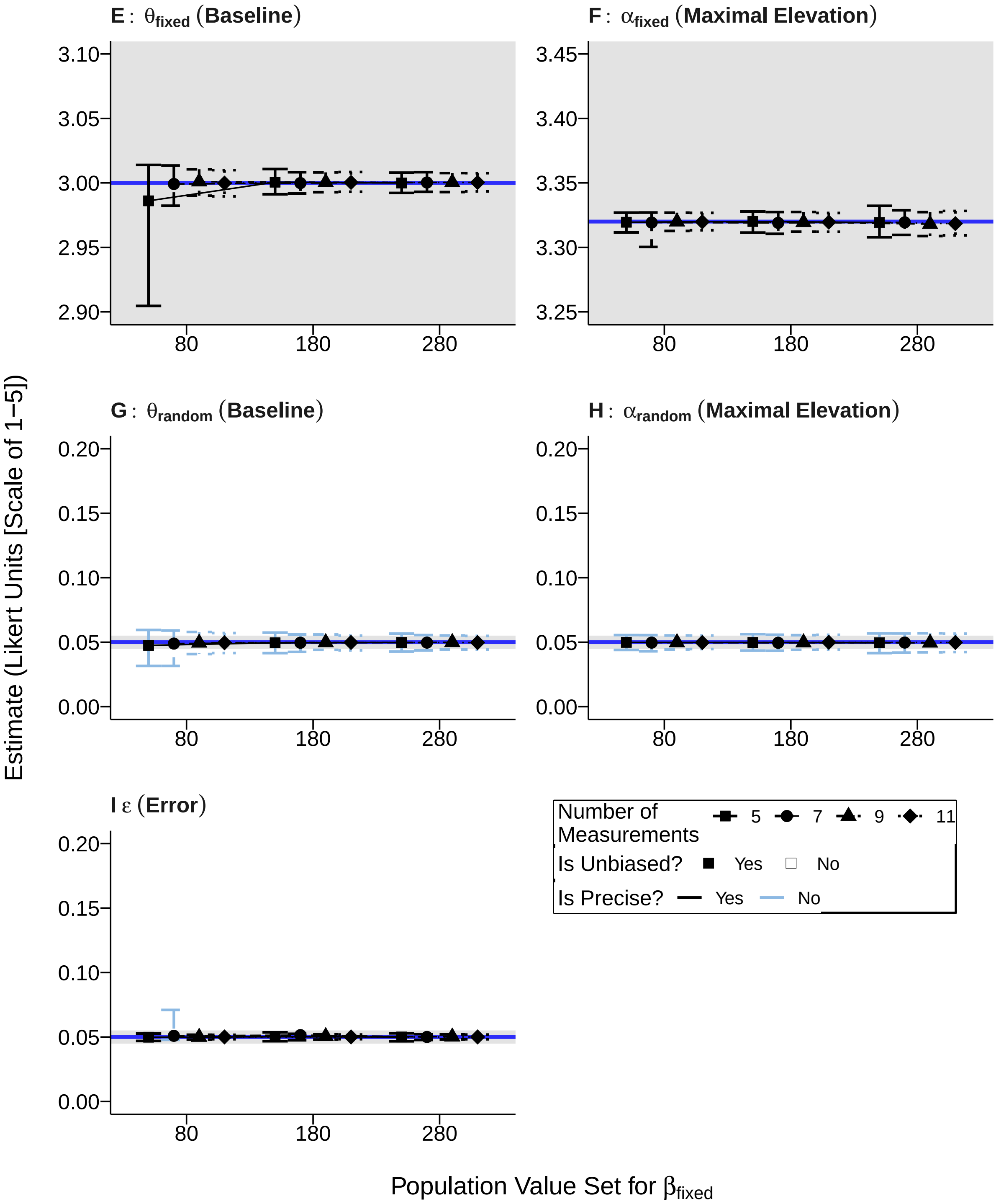


Figure F.3

Bias/Precision Plots for Day- and Likert-Unit Parameters With Time-Interval Decreasing Spacing in Experiment 1 (continued)



Note. Panels A–B: Bias/precision plots for the fixed- and random-effect days-to-halfway elevation parameters, respectively (β_{fixed} and β_{random}). Panels C–D: Bias/precision plots for the fixed- and random-effect triquarter-halfway elevation parameters, respectively (γ_{fixed} and γ_{random}). Panels E–F: Bias/precision plots for the fixed- and random-effect baseline parameters, respectively (θ_{fixed} and θ_{random}).

Panels G–H: Bias/precision plots for the fixed- and random-effect maximal elevation parameters, respectively (α_{fixed} and α_{random}). Blue horizontal lines in each panel represent the population value for each parameter. Population values for each day-unit parameter are as follows: $\beta_{fixed} \in$ 80, 180, 280, β_{random} = 10.00, γ_{fixed} = 20.00, γ_{random} = 4.00, θ_{fixed} = 3.00, θ_{random} = 0.05, α_{fixed} = 3.32, α_{random} = 0.05, ϵ = 0.05. Gray bands indicate the $\pm 10\%$ margin of error for each parameter and unfilled dots indicate cells with average parameter estimates outside of the margin or biased estimates. Error bars represent the middle 95% of estimated values, with light blue error bars indicating imprecise estimation. I considered dots that fell outside the gray bands as biased and error bar lengths with at least one whisker length exceeding the 10% cutoff (i.e., or longer than the portion of the gray band underlying the whisker) as imprecise. Note that random-effect parameter units are in standard deviation units. Importantly, across all nature-of-change values (i.e., population values used for β_{fixed}), the acceptable amount of bias and precision was based on a population value of 180. See Table H.1 for specific values estimated for each parameter.

F.1.4 Middle-and-Extreme Spacing

Figure F.4

Bias/Precision Plots for Day- and Likert-Unit Parameters With Middle-and-Extreme Spacing in Experiment 1

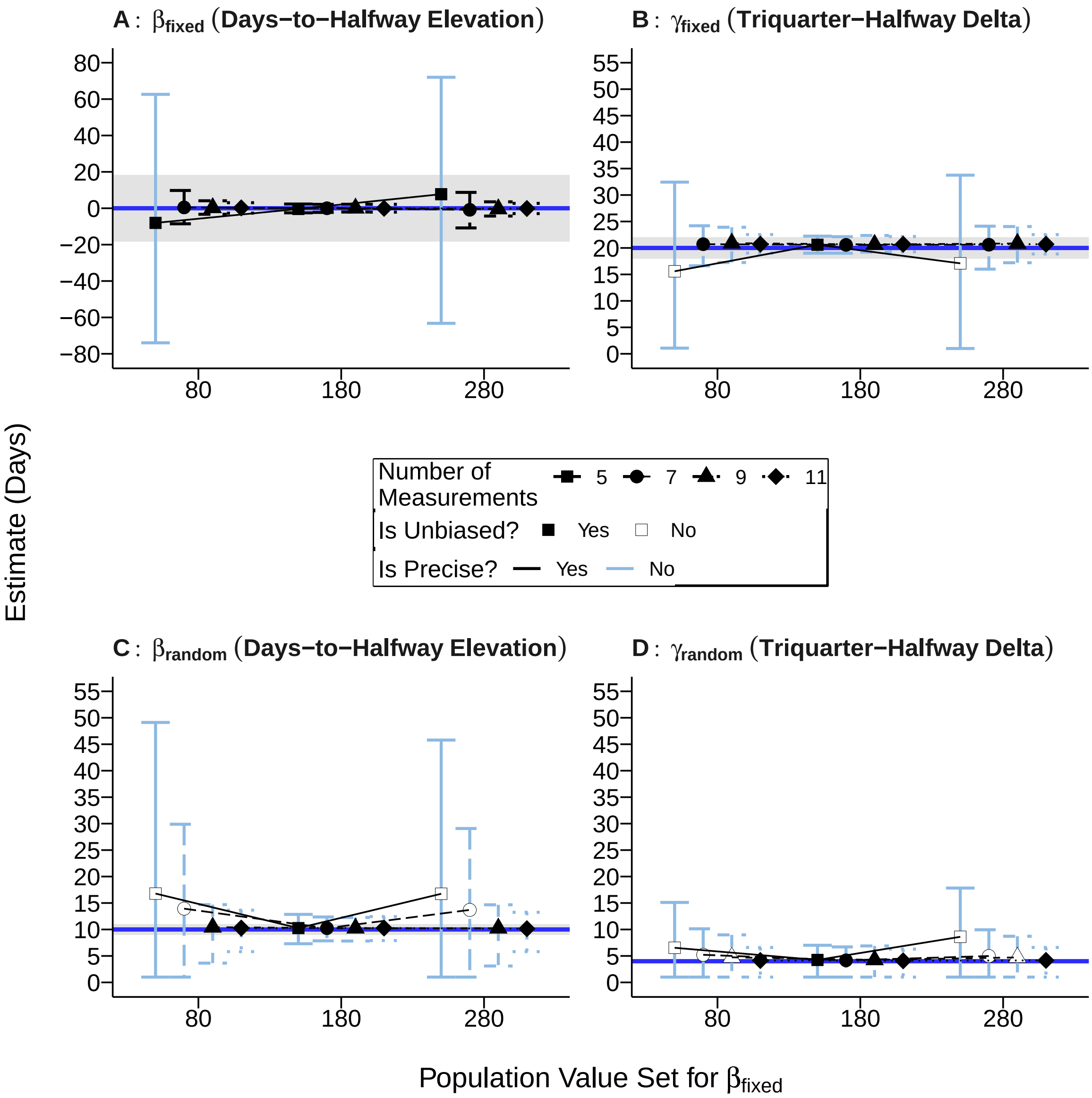
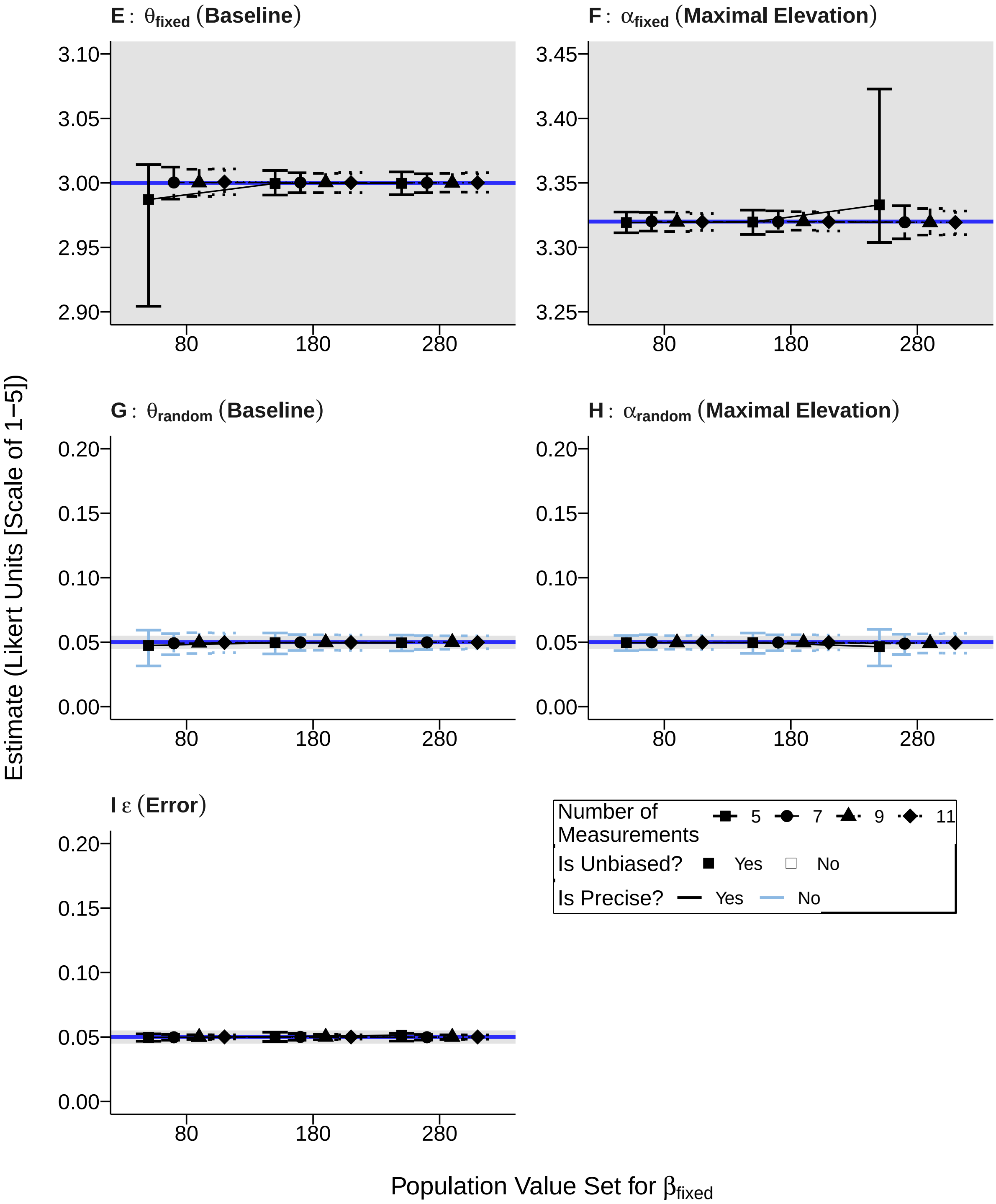


Figure F.4

Bias/Precision Plots for Day- and Likert-Unit Parameters With Middle-and-Extreme Spacing in Experiment 1 (continued)



Note. Panels A–B: Bias/precision plots for the fixed- and random-effect days-to-halfway elevation parameters, respectively (β_{fixed} and β_{random}). Panels C–D: Bias/precision plots for the fixed- and random-effect triquarter-halfway elevation parameters, respectively (γ_{fixed} and γ_{random}). Panels E–F: Bias/precision plots for the fixed- and random-effect baseline parameters, respectively (θ_{fixed} and θ_{random}).

Panels G–H: Bias/precision plots for the fixed- and random-effect maximal elevation parameters, respectively (α_{fixed} and α_{random}). Blue horizontal lines in each panel represent the population value for each parameter. Population values for each day-unit parameter are as follows: $\beta_{fixed} \in$ 80, 180, 280, β_{random} = 10.00, γ_{fixed} = 20.00, γ_{random} = 4.00, θ_{fixed} = 3.00, θ_{random} = 0.05, α_{fixed} = 3.32, α_{random} = 0.05, ϵ = 0.05. Gray bands indicate the $\pm 10\%$ margin of error for each parameter and unfilled dots indicate cells with average parameter estimates outside of the margin or biased estimates. Error bars represent the middle 95% of estimated values, with light blue error bars indicating imprecise estimation. I considered dots that fell outside the gray bands as biased and error bar lengths with at least one whisker length exceeding the 10% cutoff (i.e., or longer than the portion of the gray band underlying the whisker) as imprecise. Note that random-effect parameter units are in standard deviation units. Importantly, across all nature-of-change values (i.e., population values used for β_{fixed}), the acceptable amount of bias and precision was based on a population value of 180. See Table H.1 for specific values estimated for each parameter.

F.2 Experiment 2

F.2.5 Equal Spacing

Figure F.5

Bias/Precision Plots for Day- and Likert-Unit Parameters With Equal Spacing in Experiment 2

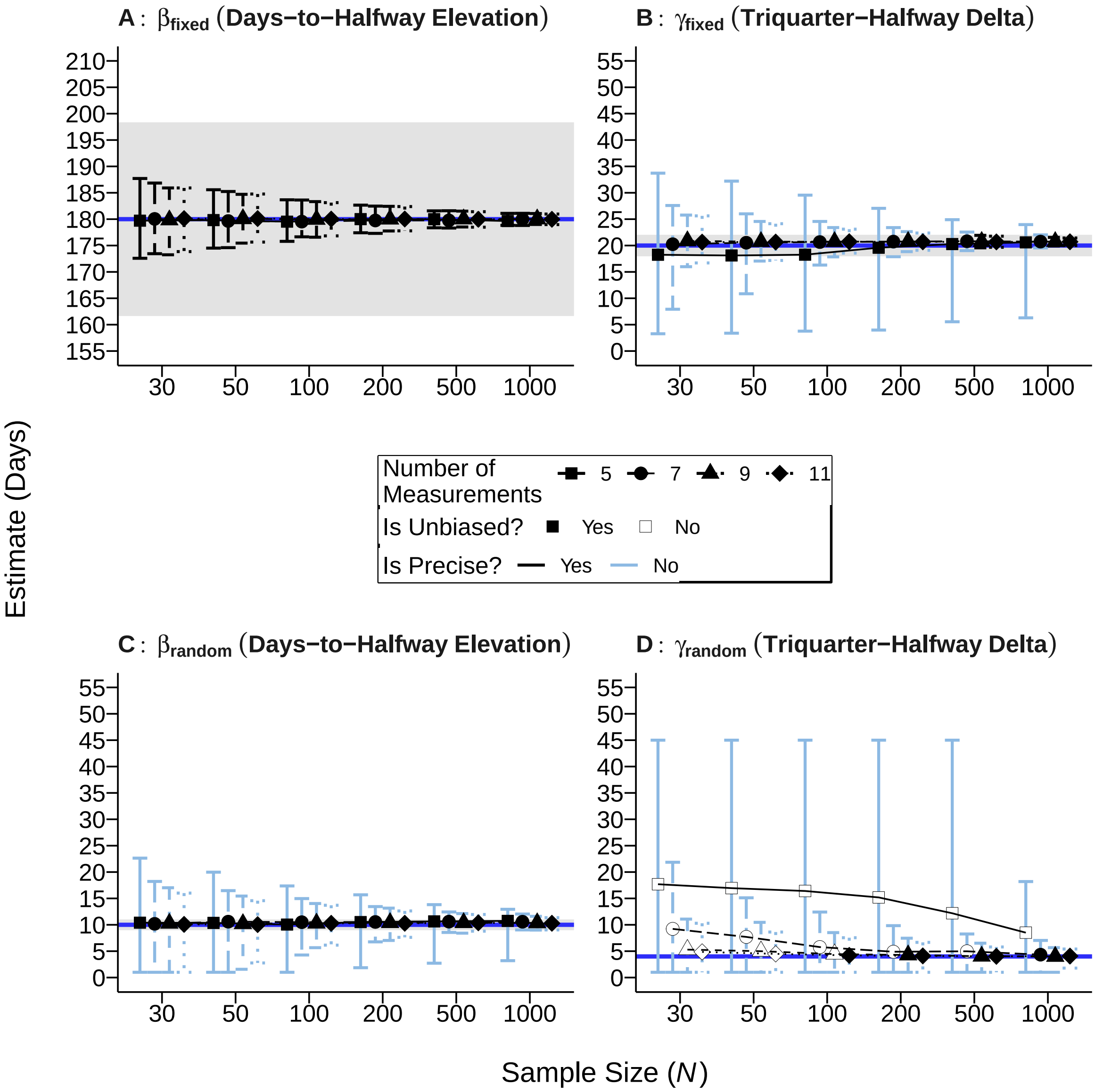
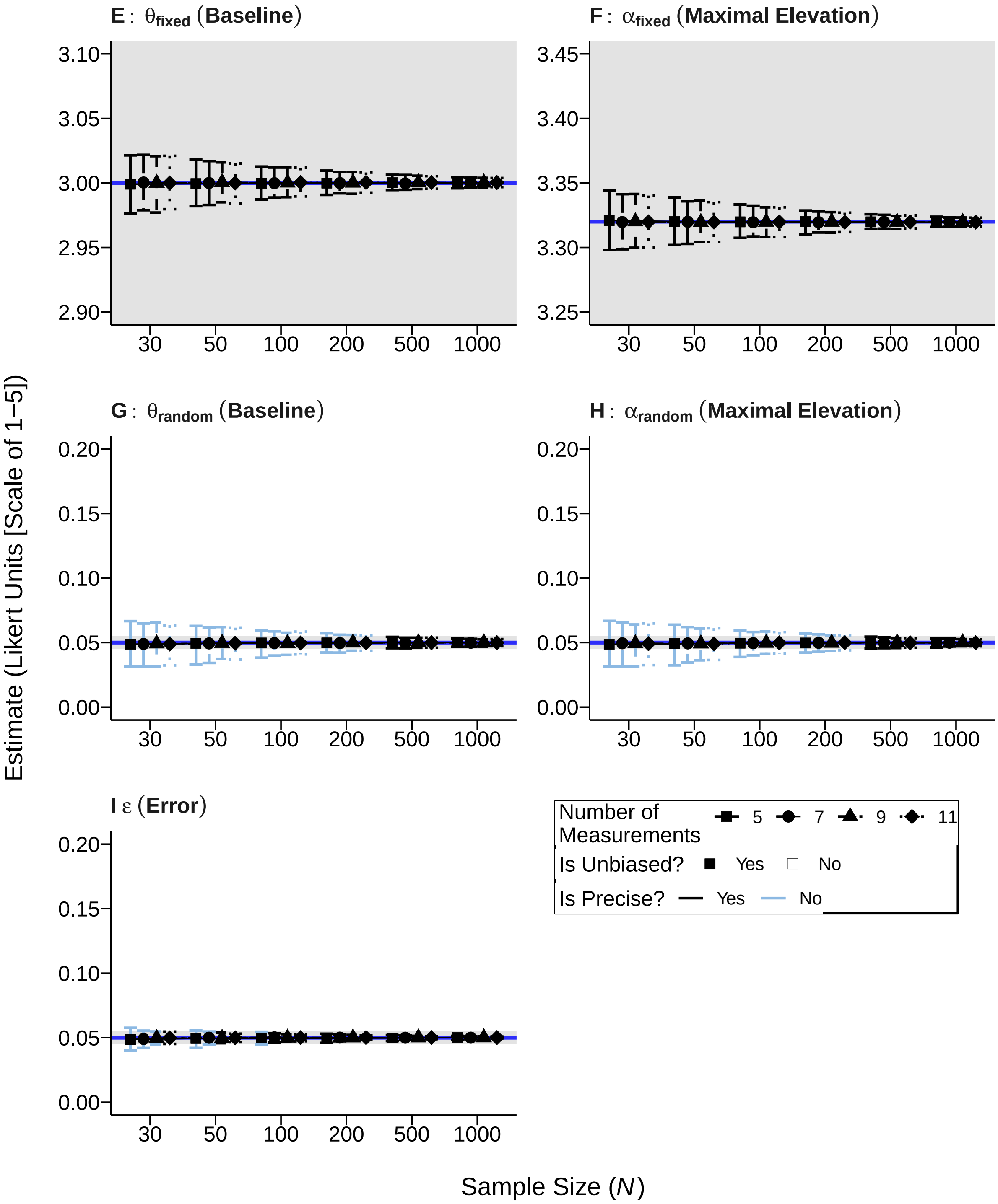


Figure F.5

Bias/Precision Plots for Day- and Likert-Unit Parameters With Equal Spacing in Experiment 2 (continued)



Note. Panels A–B: Bias/precision plots for the fixed- and random-effect days-to-halfway elevation parameters, respectively (β_{fixed} and β_{random}). Panels C–D: Bias/precision plots for the fixed- and random-effect triquarter-halfway elevation parameters, respectively (γ_{fixed} and γ_{random}). Panels E–F: Bias/precision plots for the fixed- and random-effect baseline parameters, respectively (θ_{fixed} and θ_{random}).

Panels G–H: Bias/precision plots for the fixed- and random-effect maximal elevation parameters, respectively (α_{fixed} and α_{random}). Blue horizontal lines in each panel represent the population value for each parameter. Population values for each day-unit parameter are as follows: $\beta_{fixed} \in$ 80, 180, 280, β_{random} = 10.00, γ_{fixed} = 20.00, γ_{random} = 4.00, θ_{fixed} = 3.00, θ_{random} = 0.05, α_{fixed} = 3.32, α_{random} = 0.05, ϵ = 0.05. Gray bands indicate the $\pm 10\%$ margin of error for each parameter and unfilled dots indicate cells with average parameter estimates outside of the margin or biased estimates. Error bars represent the middle 95% of estimated values, with light blue error bars indicating imprecise estimation. I considered dots that fell outside the gray bands as biased and error bar lengths with at least one whisker length exceeding the 10% cutoff (i.e., or longer than the portion of the gray band underlying the whisker) as imprecise. Note that random-effect parameter units are in standard deviation units. See Table H.2 for specific values estimated for each parameter.

F.2.6 Time-Interval Increasing Spacing

Figure F.6

Bias/Precision Plots for Day- and Likert-Unit Parameters With Time-Interval Increasing Spacing in Experiment 2

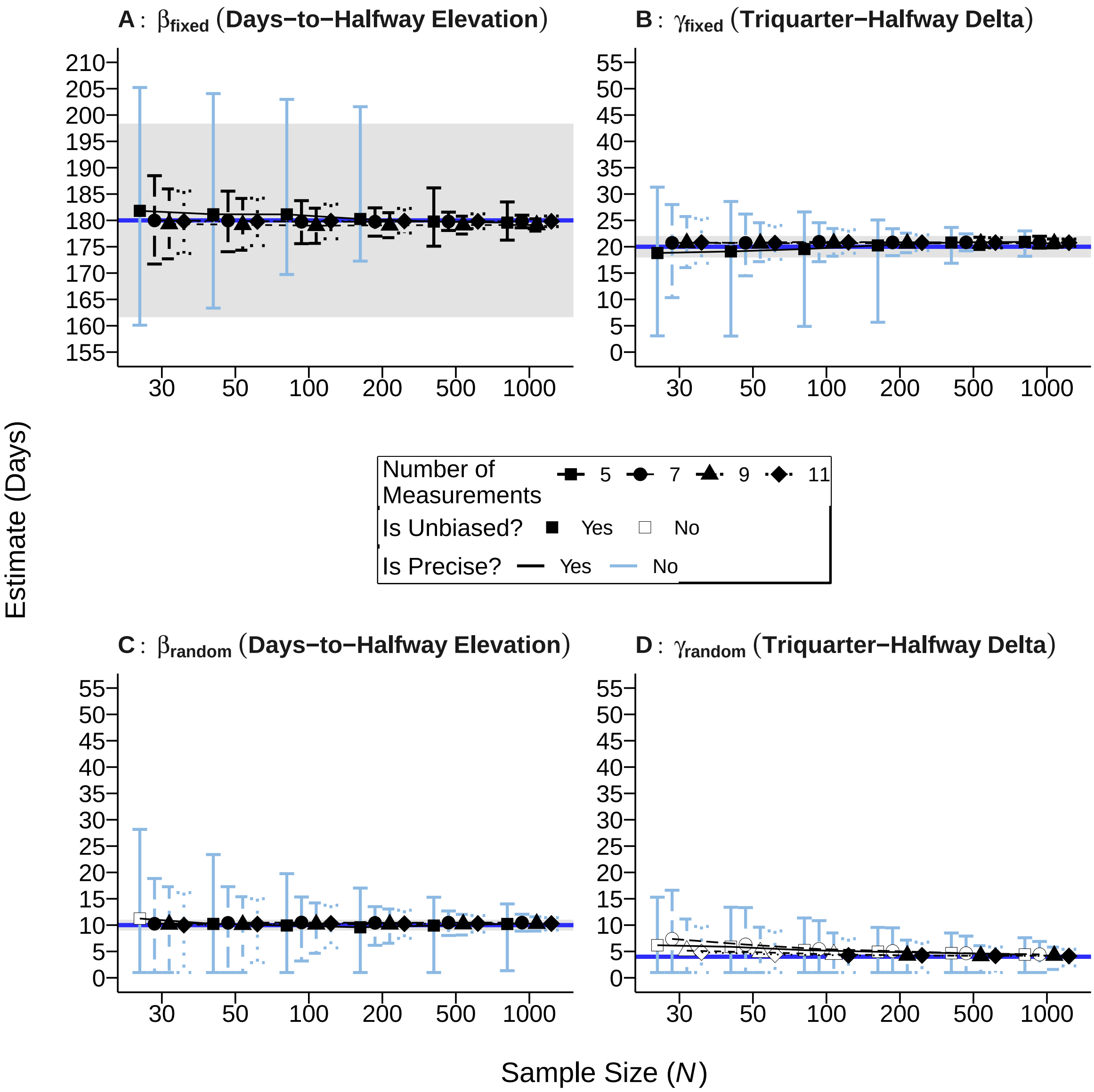
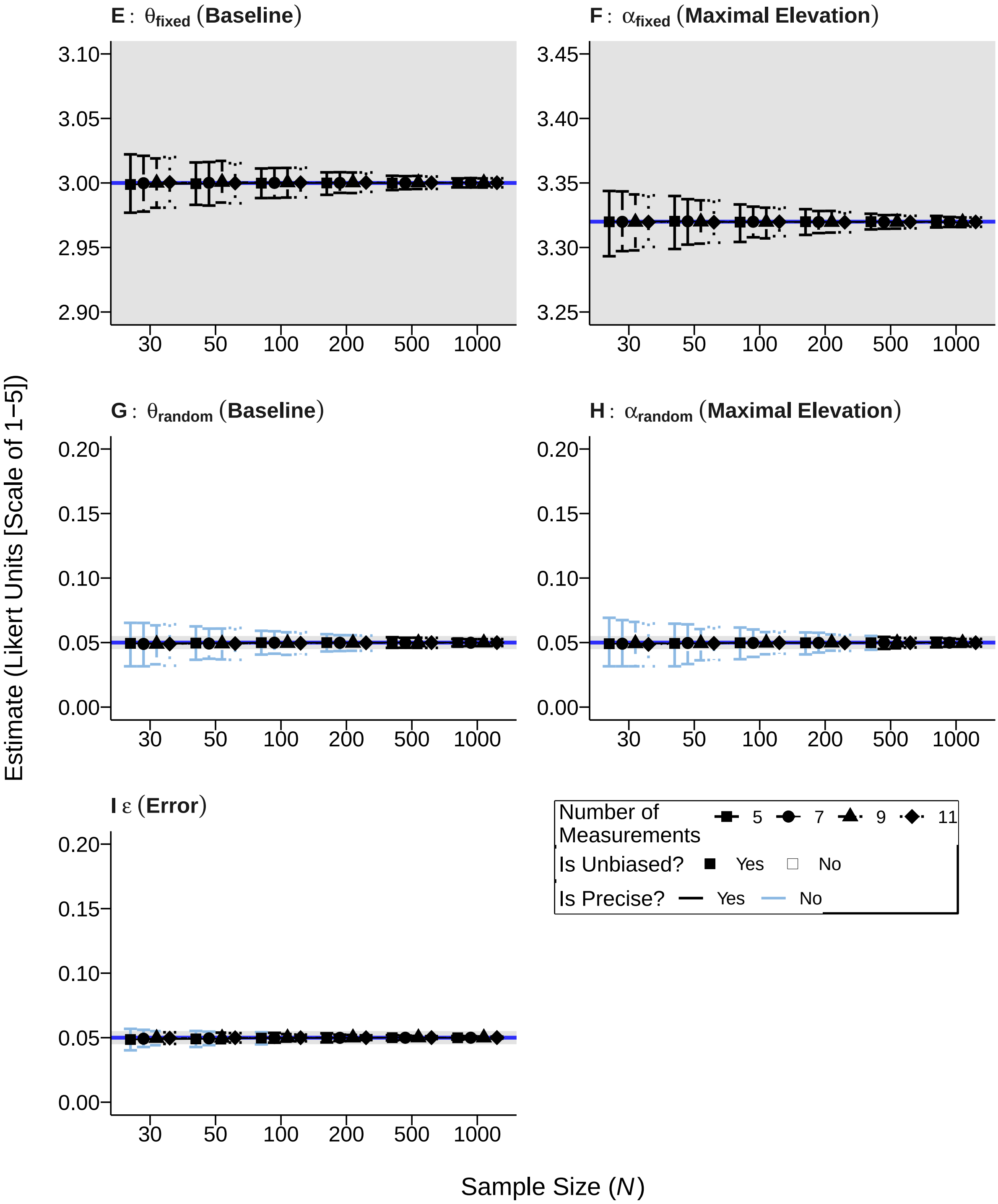


Figure F.6

Bias/Precision Plots for Day- and Likert-Unit Parameters With Time-Interval Increasing Spacing in Experiment 2 (continued)



Note. Panels A–B: Bias/precision plots for the fixed- and random-effect days-to-halfway elevation parameters, respectively (β_{fixed} and β_{random}). Panels C–D: Bias/precision plots for the fixed- and random-effect triquarter-halfway elevation parameters, respectively (γ_{fixed} and γ_{random}). Panels E–F: Bias/precision plots for the fixed- and random-effect baseline parameters, respectively (θ_{fixed} and θ_{random}).

Panels G–H: Bias/precision plots for the fixed- and random-effect maximal elevation parameters, respectively (α_{fixed} and α_{random}). Blue horizontal lines in each panel represent the population value for each parameter. Population values for each day-unit parameter are as follows: $\beta_{fixed} \in$ 80, 180, 280, β_{random} = 10.00, γ_{fixed} = 20.00, γ_{random} = 4.00, θ_{fixed} = 3.00, θ_{random} = 0.05, α_{fixed} = 3.32, α_{random} = 0.05, ϵ = 0.05. Gray bands indicate the $\pm 10\%$ margin of error for each parameter and unfilled dots indicate cells with average parameter estimates outside of the margin or biased estimates. Error bars represent the middle 95% of estimated values, with light blue error bars indicating imprecise estimation. I considered dots that fell outside the gray bands as biased and error bar lengths with at least one whisker length exceeding the 10% cutoff (i.e., or longer than the portion of the gray band underlying the whisker) as imprecise. Note that random-effect parameter units are in standard deviation units. See Table H.2 for specific values estimated for each parameter.

F.2.7 Time-Interval Decreasing Spacing

Figure F.7

Bias/Precision Plots for Day- and Likert-Unit Parameters With Time-Interval Decreasing Spacing in Experiment 2

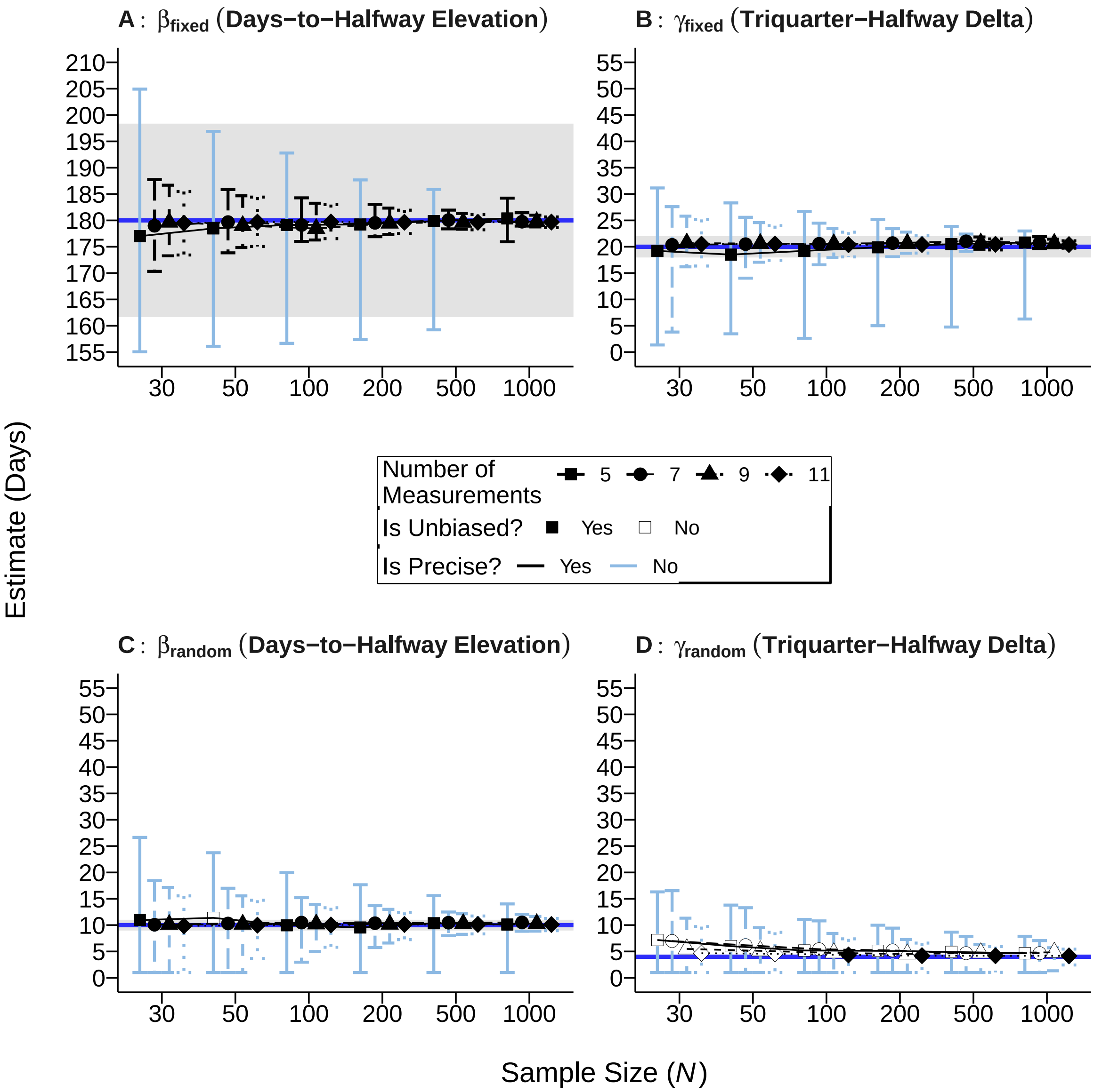
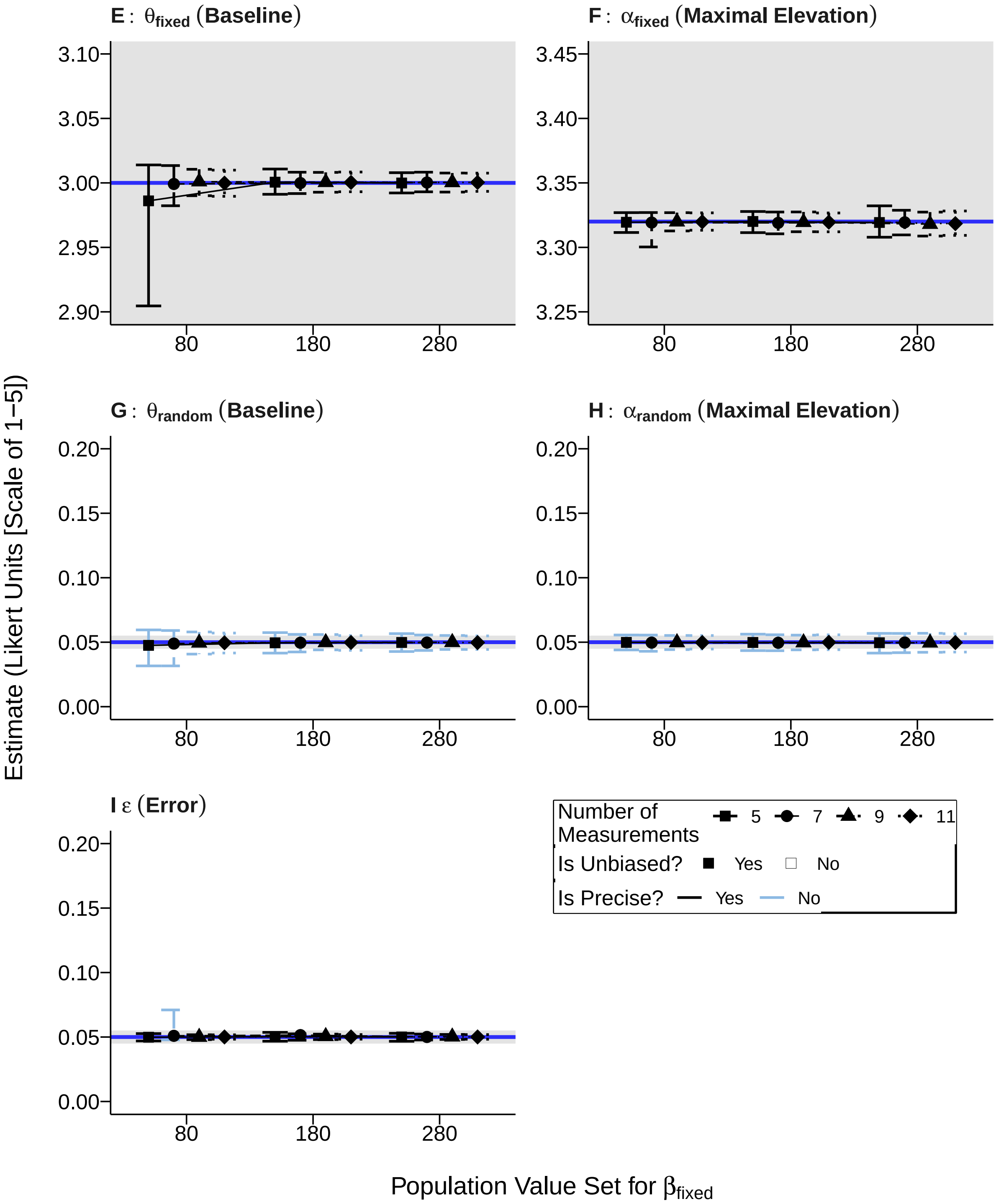


Figure F.7

Bias/Precision Plots for Day- and Likert-Unit Parameters With Time-Interval Decreasing Spacing in Experiment 2 (continued)



Note. Panels A–B: Bias/precision plots for the fixed- and random-effect days-to-halfway elevation parameters, respectively (β_{fixed} and β_{random}). Panels C–D: Bias/precision plots for the fixed- and random-effect triquarter-halfway elevation parameters, respectively (γ_{fixed} and γ_{random}). Panels E–F: Bias/precision plots for the fixed- and random-effect baseline parameters, respectively (θ_{fixed} and θ_{random}).

Panels G–H: Bias/precision plots for the fixed- and random-effect maximal elevation parameters, respectively (α_{fixed} and α_{random}). Blue horizontal lines in each panel represent the population value for each parameter. Population values for each day-unit parameter are as follows: $\beta_{fixed} \in$ 80, 180, 280, β_{random} = 10.00, γ_{fixed} = 20.00, γ_{random} = 4.00, θ_{fixed} = 3.00, θ_{random} = 0.05, α_{fixed} = 3.32, α_{random} = 0.05, ϵ = 0.05. Gray bands indicate the $\pm 10\%$ margin of error for each parameter and unfilled dots indicate cells with average parameter estimates outside of the margin or biased estimates. Error bars represent the middle 95% of estimated values, with light blue error bars indicating imprecise estimation. I considered dots that fell outside the gray bands as biased and error bar lengths with at least one whisker length exceeding the 10% cutoff (i.e., or longer than the portion of the gray band underlying the whisker) as imprecise. Note that random-effect parameter units are in standard deviation units. See Table H.2 for specific values estimated for each parameter.

F.2.8 Middle-and-Extreme Spacing

Figure F.8

Bias/Precision Plots for Day- and Likert-Unit Parameters With Middle-and-Extreme Spacing in Experiment 2

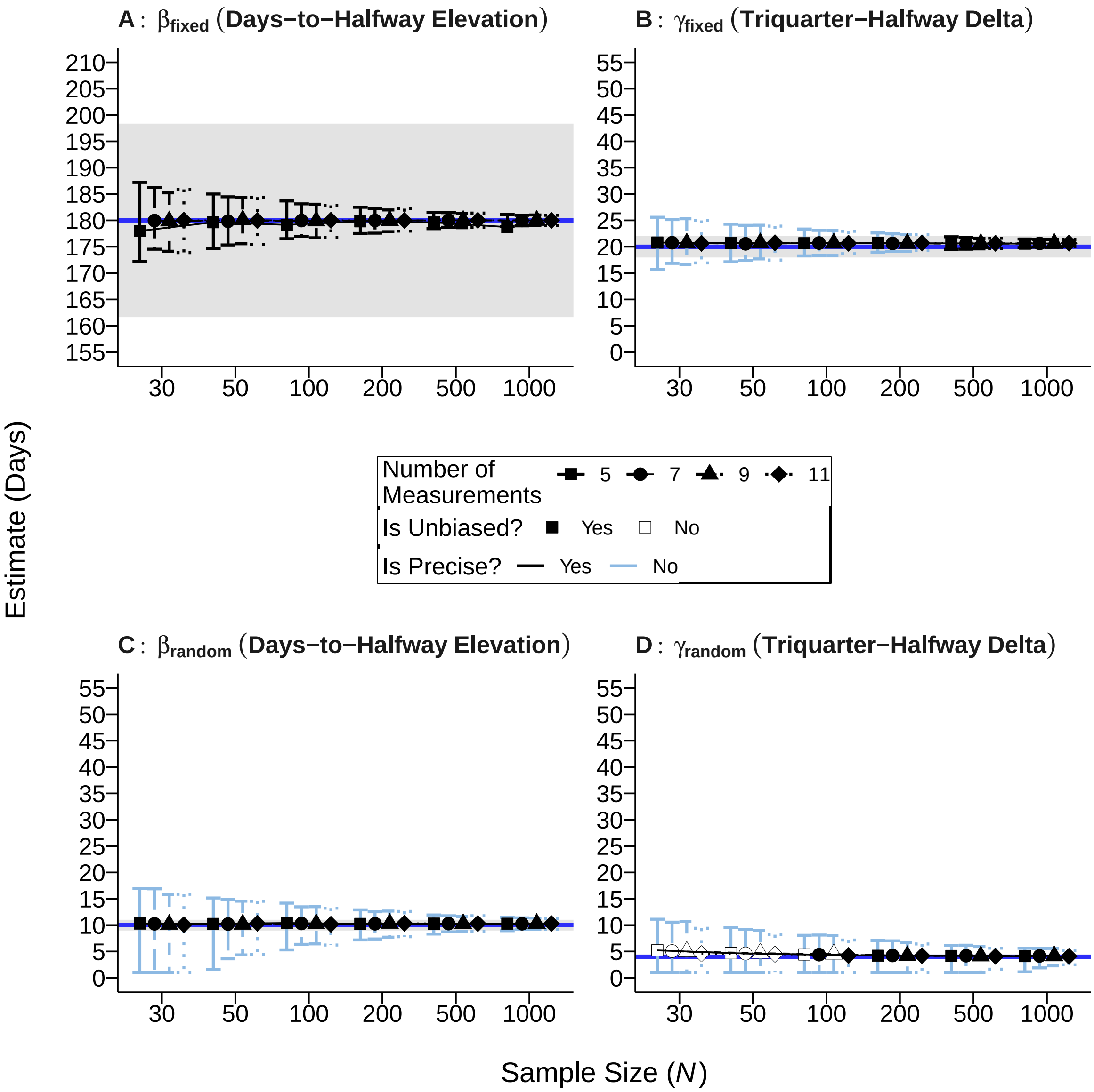
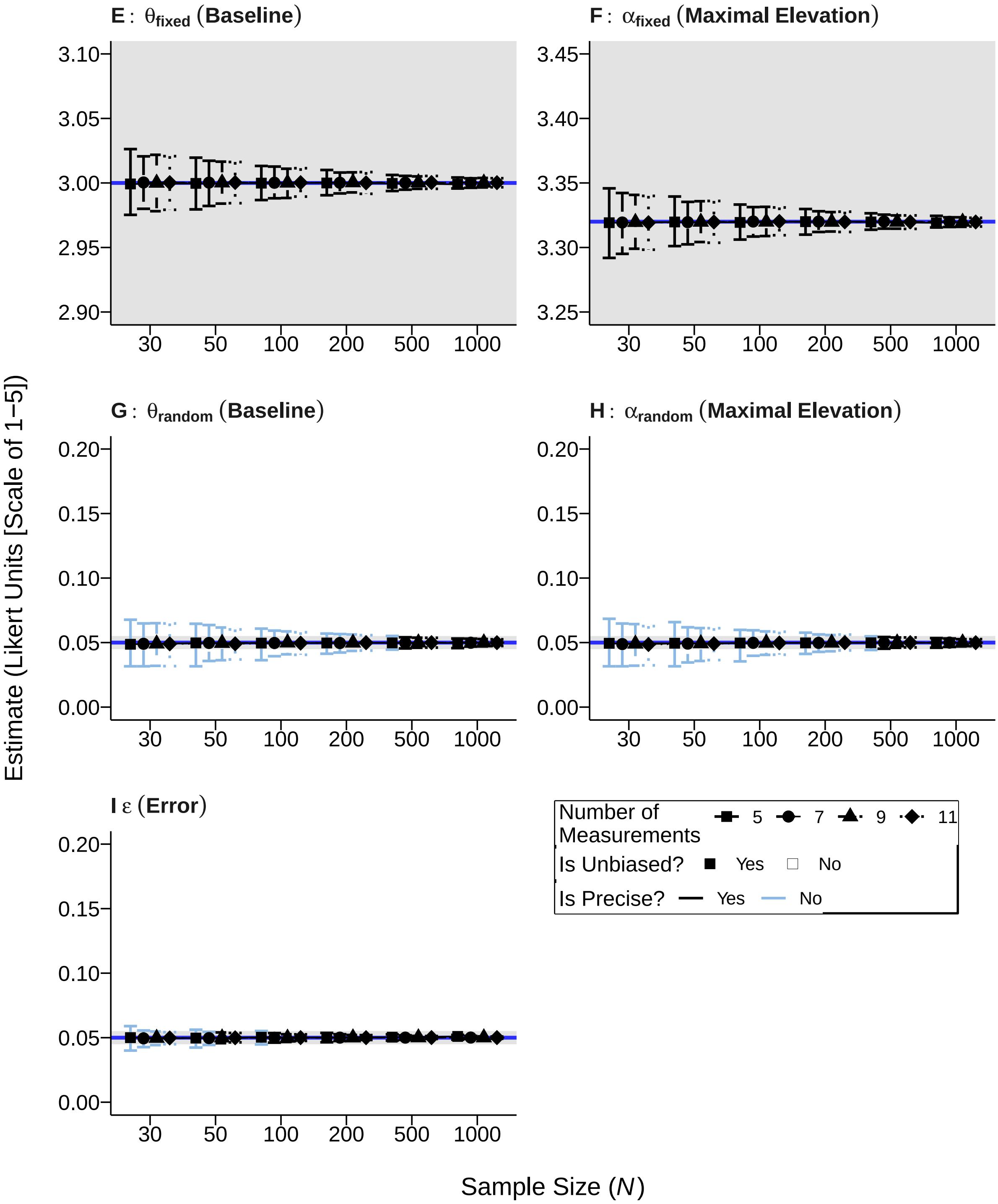


Figure F.8

Bias/Precision Plots for Day- and Likert-Unit Parameters With Middle-and-Extreme Spacing in Experiment 2 (continued)



Note. Panels A–B: Bias/precision plots for the fixed- and random-effect days-to-halfway elevation parameters, respectively (β_{fixed} and β_{random}). Panels C–D: Bias/precision plots for the fixed- and random-effect triquarter-halfway elevation parameters, respectively (γ_{fixed} and γ_{random}). Panels E–F: Bias/precision plots for the fixed- and random-effect baseline parameters, respectively (θ_{fixed} and θ_{random}).

Panels G–H: Bias/precision plots for the fixed- and random-effect maximal elevation parameters, respectively (α_{fixed} and α_{random}). Blue horizontal lines in each panel represent the population value for each parameter. Population values for each day-unit parameter are as follows: $\beta_{fixed} \in$ 80, 180, 280, β_{random} = 10.00, γ_{fixed} = 20.00, γ_{random} = 4.00, θ_{fixed} = 3.00, θ_{random} = 0.05, α_{fixed} = 3.32, α_{random} = 0.05, ϵ = 0.05. Gray bands indicate the $\pm 10\%$ margin of error for each parameter and unfilled dots indicate cells with average parameter estimates outside of the margin or biased estimates. Error bars represent the middle 95% of estimated values, with light blue error bars indicating imprecise estimation. I considered dots that fell outside the gray bands as biased and error bar lengths with at least one whisker length exceeding the 10% cutoff (i.e., or longer than the portion of the gray band underlying the whisker) as imprecise. Note that random-effect parameter units are in standard deviation units. See Table H.2 for specific values estimated for each parameter.

F.3 Experiment 3

F.3.9 Time-Structured Data

Figure F.9

Bias/Precision Plots for Day- and Likert-Unit Parameters With Time-Structured Data in Experiment 3

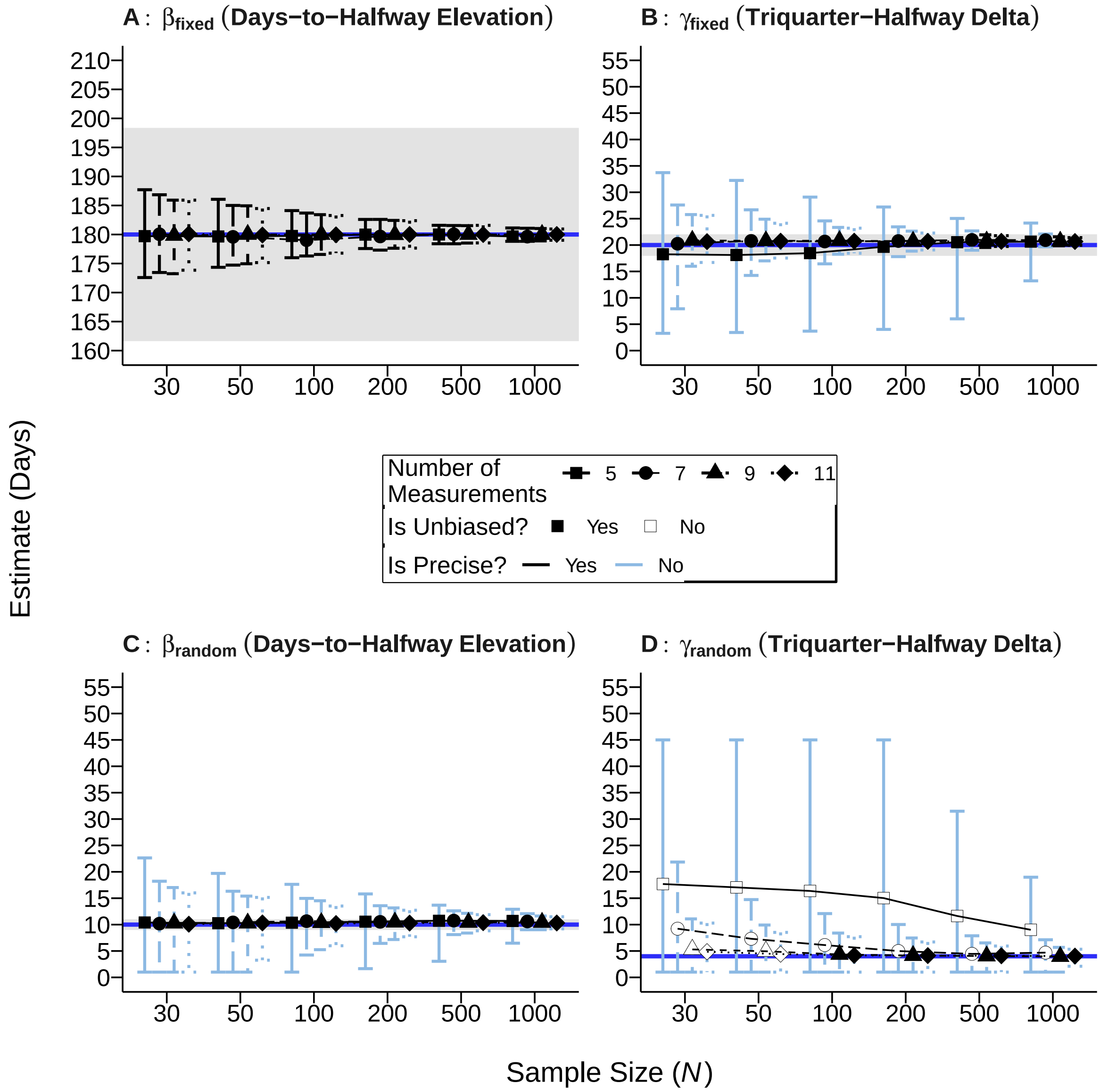
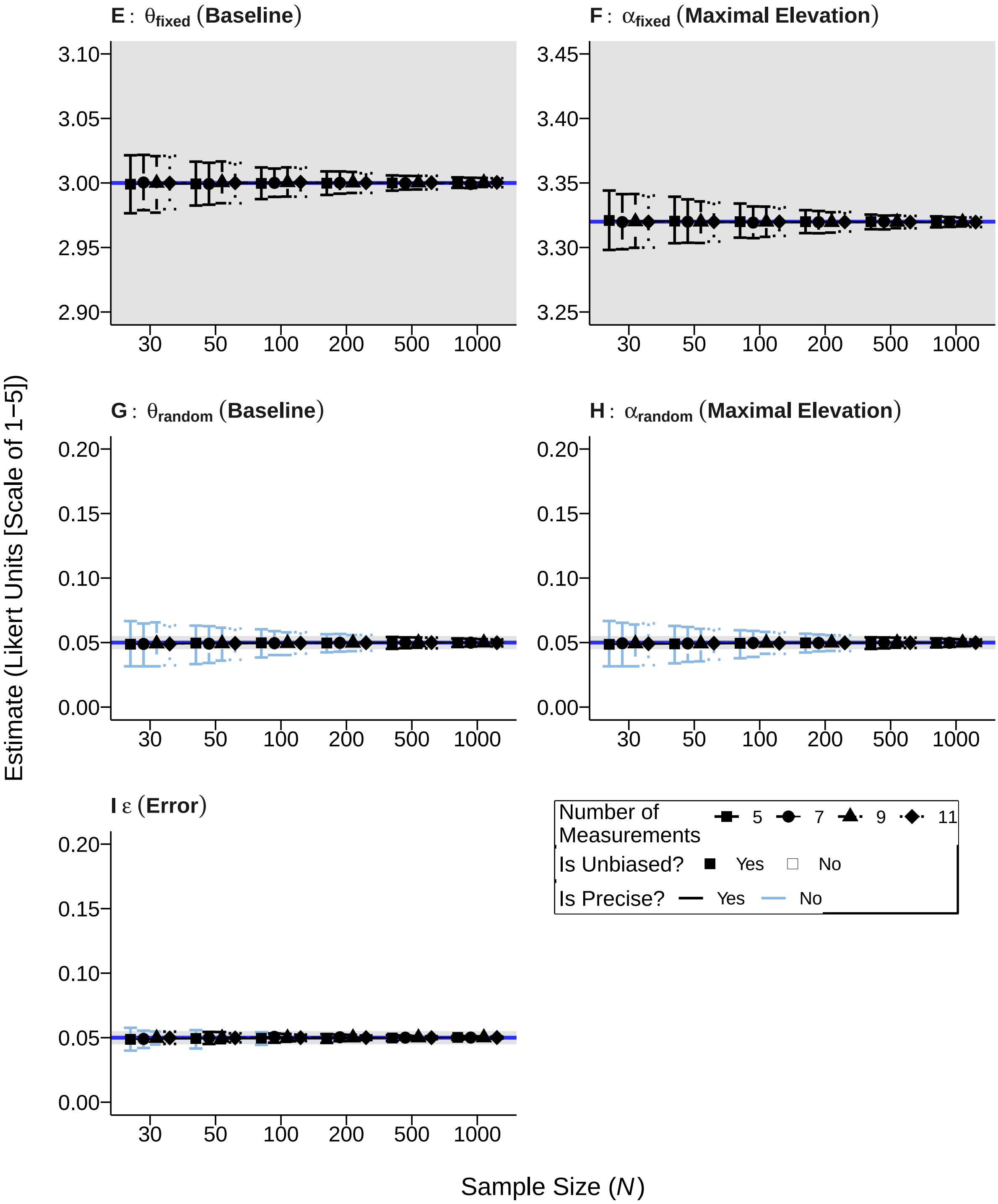


Figure F.9

Bias/Precision Plots for Day- and Likert-Unit Parameters With Time-Structured Data in Experiment 3 (continued)



Note. Panels A–B: Bias/precision plots for the fixed- and random-effect days-to-halfway elevation parameters, respectively (β_{fixed} and β_{random}). Panels C–D: Bias/precision plots for the fixed- and random-effect triquarter-halfway elevation parameters, respectively (γ_{fixed} and γ_{random}). Panels E–F: Bias/precision plots for the fixed- and random-effect baseline parameters, respectively (θ_{fixed} and θ_{random}).

Panels G–H: Bias/precision plots for the fixed- and random-effect maximal elevation parameters, respectively (α_{fixed} and α_{random}). Blue horizontal lines in each panel represent the population value for each parameter. Population values for each day-unit parameter are as follows: $\beta_{fixed} \in$ 80, 180, 280, β_{random} = 10.00, γ_{fixed} = 20.00, γ_{random} = 4.00, θ_{fixed} = 3.00, θ_{random} = 0.05, α_{fixed} = 3.32, α_{random} = 0.05, ϵ = 0.05. Gray bands indicate the $\pm 10\%$ margin of error for each parameter and unfilled dots indicate cells with average parameter estimates outside of the margin or biased estimates. Error bars represent the middle 95% of estimated values, with light blue error bars indicating imprecise estimation. I considered dots that fell outside the gray bands as biased and error bar lengths with at least one whisker length exceeding the 10% cutoff (i.e., or longer than the portion of the gray band underlying the whisker) as imprecise. Note that random-effect parameter units are in standard deviation units. See Table H.3 for specific values estimated for each parameter.

F.3.10 Time-Unstructured Data Characterized by a Fast Response Rate

Figure F.10

Bias/Precision Plots for Day- and Likert-Unit Parameters With Time-Unstructured Data Characterized by a Fast Response Rate in Experiment 3

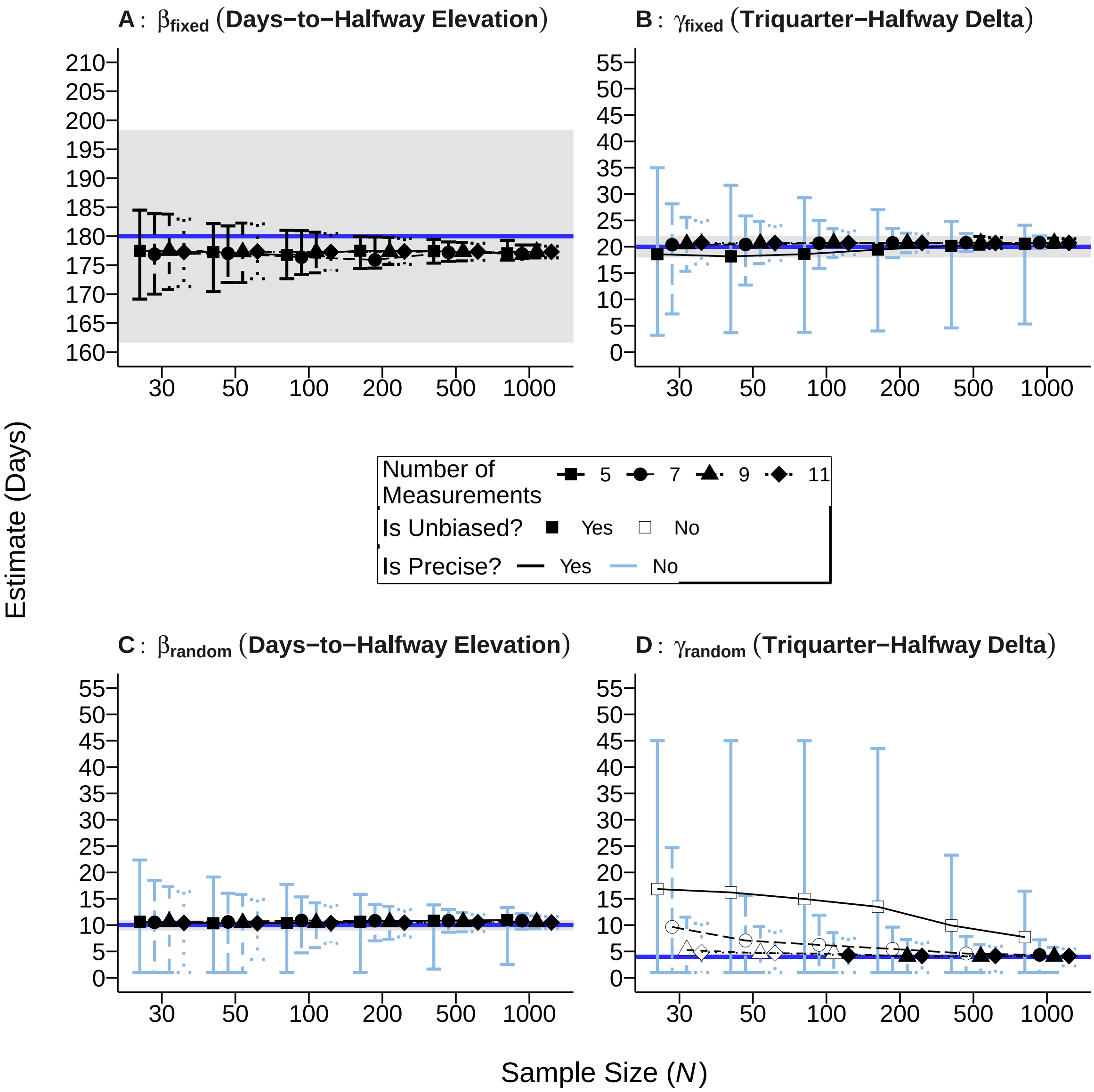
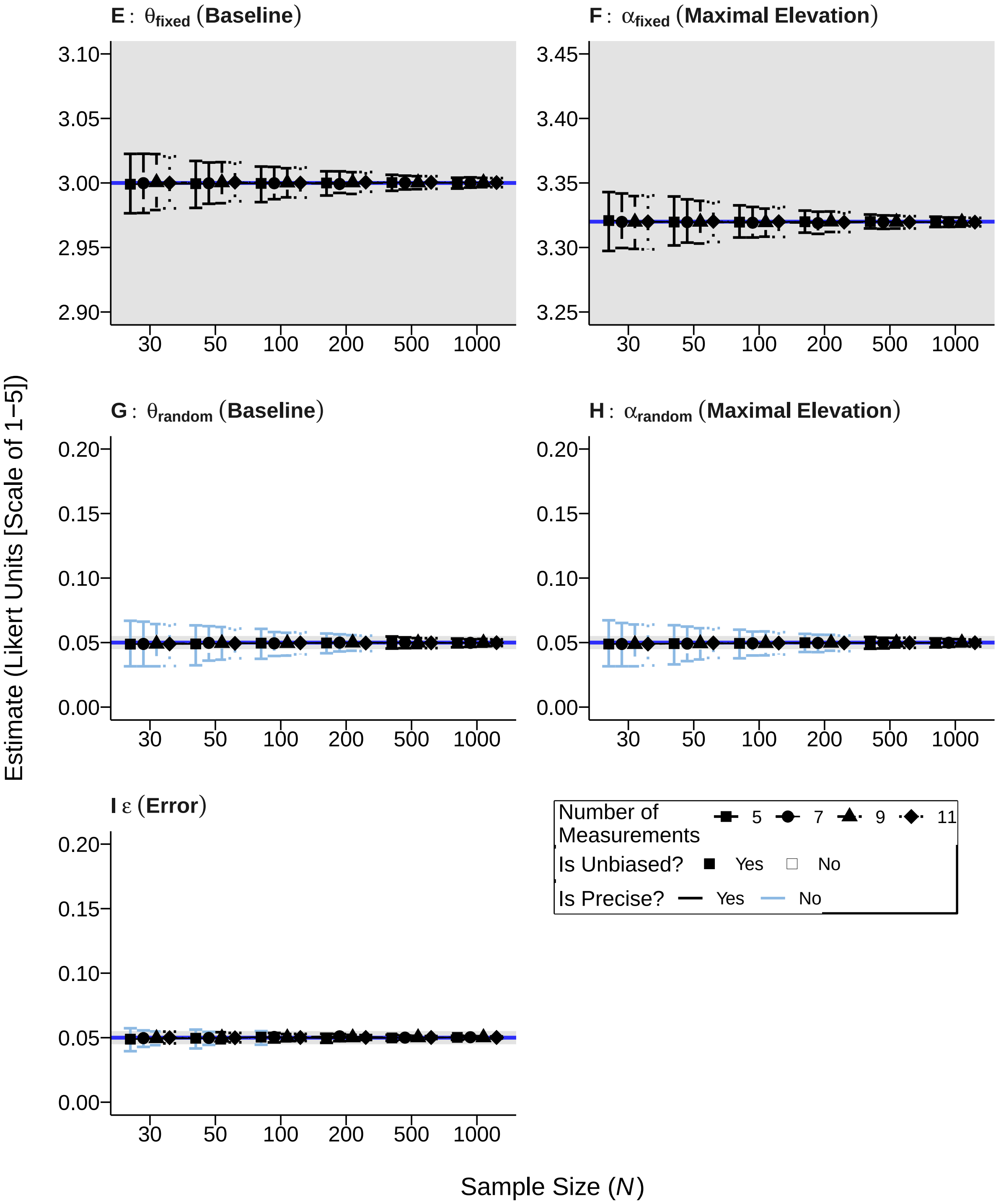


Figure F.10

Bias/Precision Plots for Day- and Likert-Unit Parameters With Time-Unstructured Data Characterized by a Fast Response Rate in Experiment 3 (continued)



Note. Panels A–B: Bias/precision plots for the fixed- and random-effect days-to-halfway elevation parameters, respectively (β_{fixed} and β_{random}). Panels C–D: Bias/precision plots for the fixed- and random-effect triquarter-halfway elevation parameters, respectively (γ_{fixed} and γ_{random}). Panels E–F: Bias/precision plots for the fixed- and random-effect baseline parameters, respectively (θ_{fixed} and θ_{random}).

Panels G–H: Bias/precision plots for the fixed- and random-effect maximal elevation parameters, respectively (α_{fixed} and α_{random}). Blue horizontal lines in each panel represent the population value for each parameter. Population values for each day-unit parameter are as follows: $\beta_{fixed} \in$ 80, 180, 280, β_{random} = 10.00, γ_{fixed} = 20.00, γ_{random} = 4.00, θ_{fixed} = 3.00, θ_{random} = 0.05, α_{fixed} = 3.32, α_{random} = 0.05, ϵ = 0.05. Gray bands indicate the $\pm 10\%$ margin of error for each parameter and unfilled dots indicate cells with average parameter estimates outside of the margin or biased estimates. Error bars represent the middle 95% of estimated values, with light blue error bars indicating imprecise estimation. I considered dots that fell outside the gray bands as biased and error bar lengths with at least one whisker length exceeding the 10% cutoff (i.e., or longer than the portion of the gray band underlying the whisker) as imprecise. Note that random-effect parameter units are in standard deviation units. See Table H.3 for specific values estimated for each parameter.

F.3.11 Time-Unstructured Data Characterized by a Slow Response Rate

Figure F.11

Bias/Precision Plots for Day- and Likert-Unit Parameters With Time-Unstructured Data Characterized by a Slow Response Rate in Experiment 3

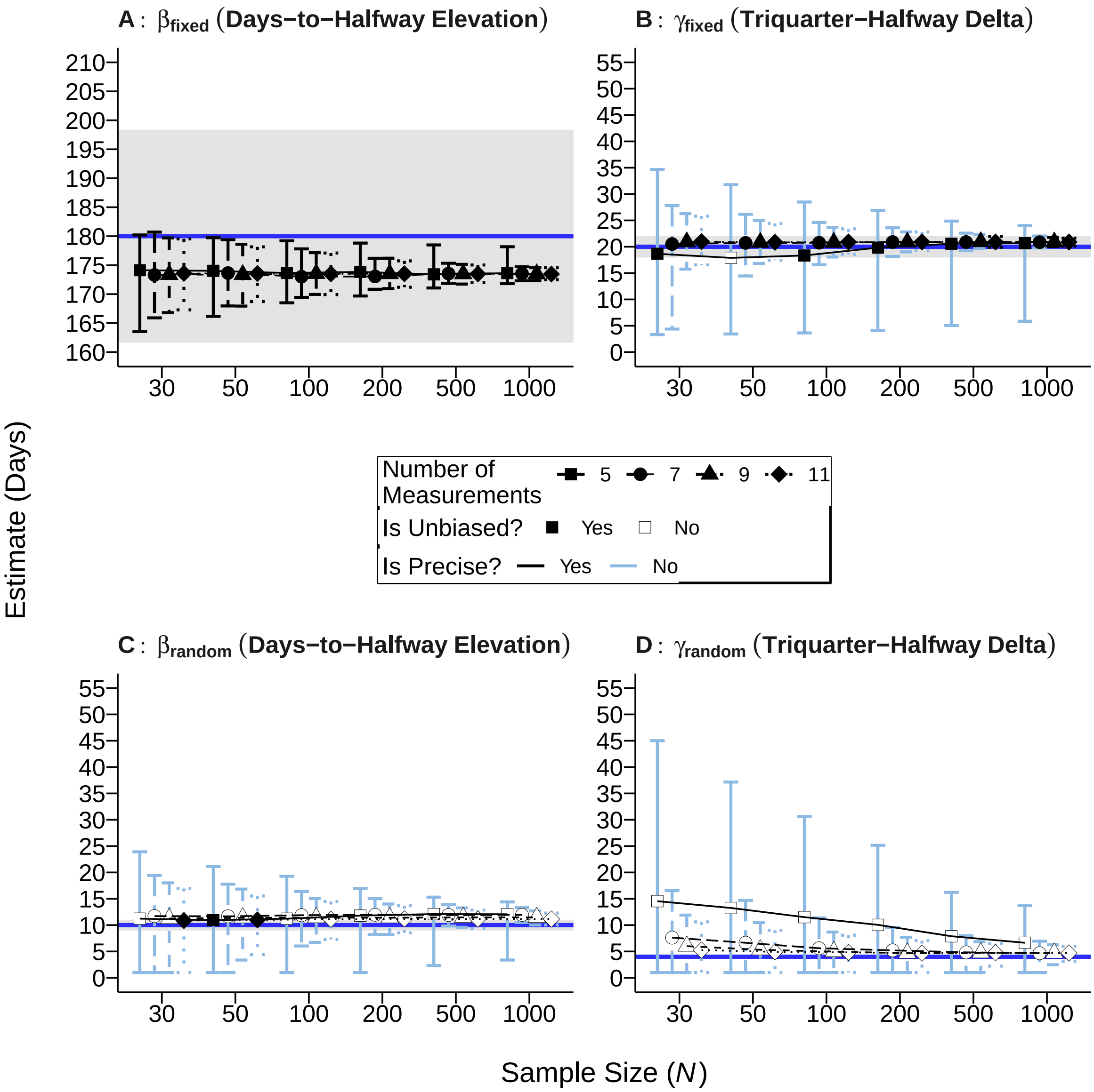
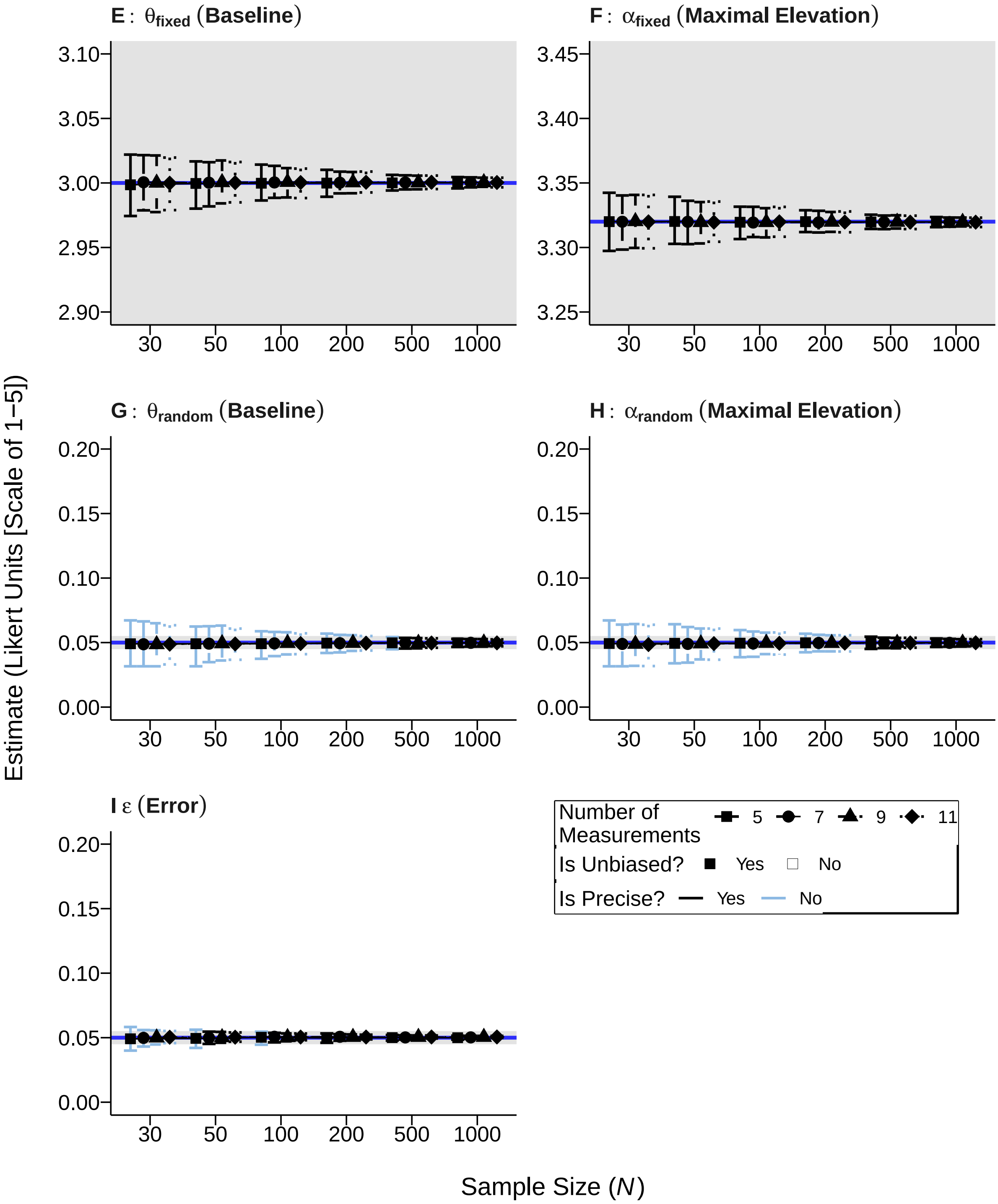


Figure F.11

Bias/Precision Plots for Day- and Likert-Unit Parameters With Time-Unstructured Data Characterized by a Slow Response Rate in Experiment 3 (continued)



Note. Panels A–B: Bias/precision plots for the fixed- and random-effect days-to-halfway elevation parameters, respectively (β_{fixed} and β_{random}). Panels C–D: Bias/precision plots for the fixed- and random-effect triquarter-halfway elevation parameters, respectively (γ_{fixed} and γ_{random}). Panels E–F: Bias/precision plots for the fixed- and random-effect baseline parameters, respectively (θ_{fixed} and θ_{random}).

Panels G–H: Bias/precision plots for the fixed- and random-effect maximal elevation parameters, respectively (α_{fixed} and α_{random}). Blue horizontal lines in each panel represent the population value for each parameter. Population values for each day-unit parameter are as follows: $\beta_{fixed} \in$ 80, 180, 280, β_{random} = 10.00, γ_{fixed} = 20.00, γ_{random} = 4.00, θ_{fixed} = 3.00, θ_{random} = 0.05, α_{fixed} = 3.32, α_{random} = 0.05, ϵ = 0.05. Gray bands indicate the $\pm 10\%$ margin of error for each parameter and unfilled dots indicate cells with average parameter estimates outside of the margin or biased estimates. Error bars represent the middle 95% of estimated values, with light blue error bars indicating imprecise estimation. I considered dots that fell outside the gray bands as biased and error bar lengths with at least one whisker length exceeding the 10% cutoff (i.e., or longer than the portion of the gray band underlying the whisker) as imprecise. Note that random-effect parameter units are in standard deviation units. See Table H.3 for specific values estimated for each parameter.

F.3.12 Time-Unstructured Data Characterized by a Slow Response Rate and Modelled with Definition Variables

Figure F.12

Bias/Precision Plots for Day- and Likert-Unit Parameters When Using Definition Variables To Model Time-Unstructured Data Characterized by a Slow Response Rate

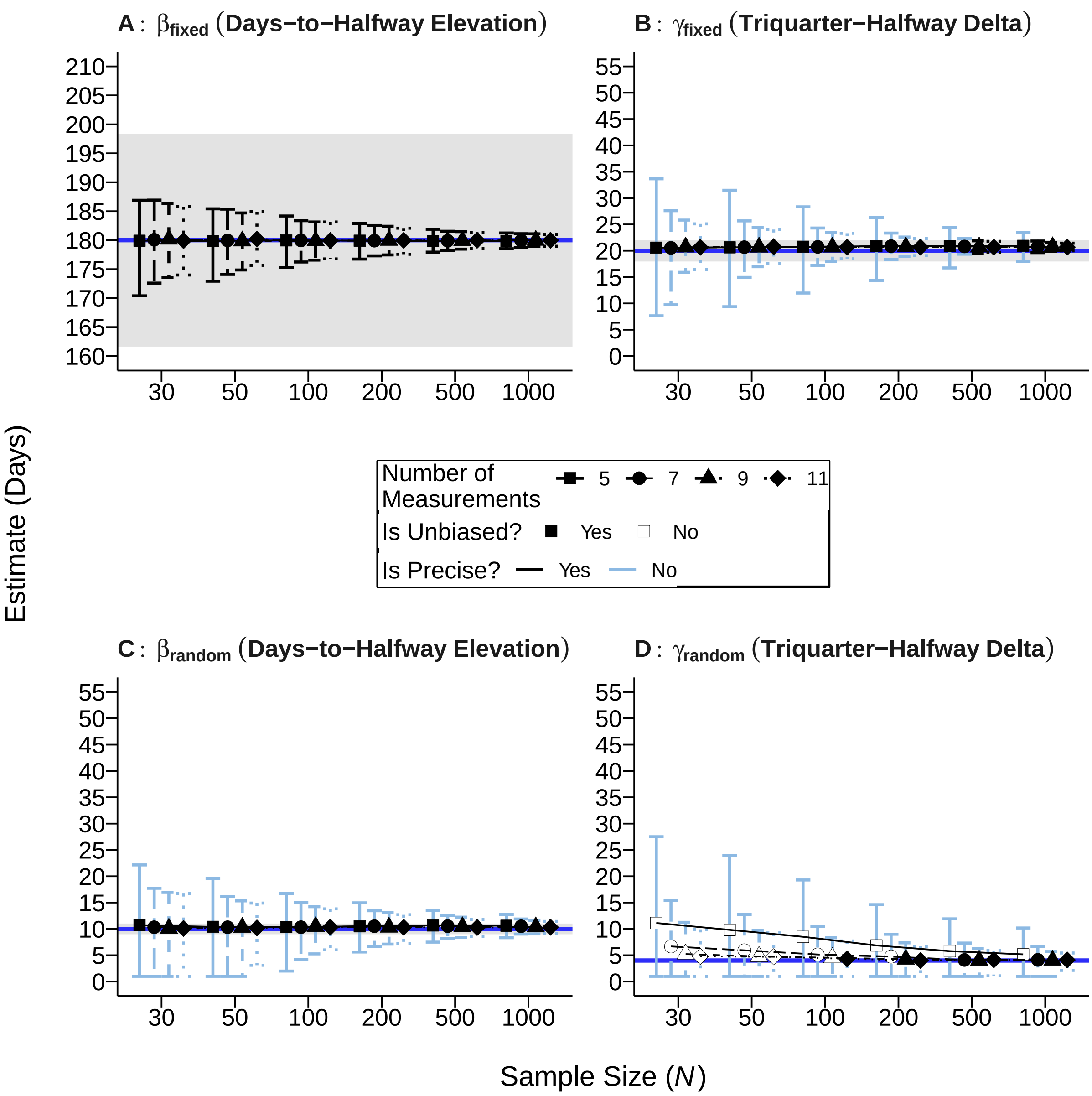
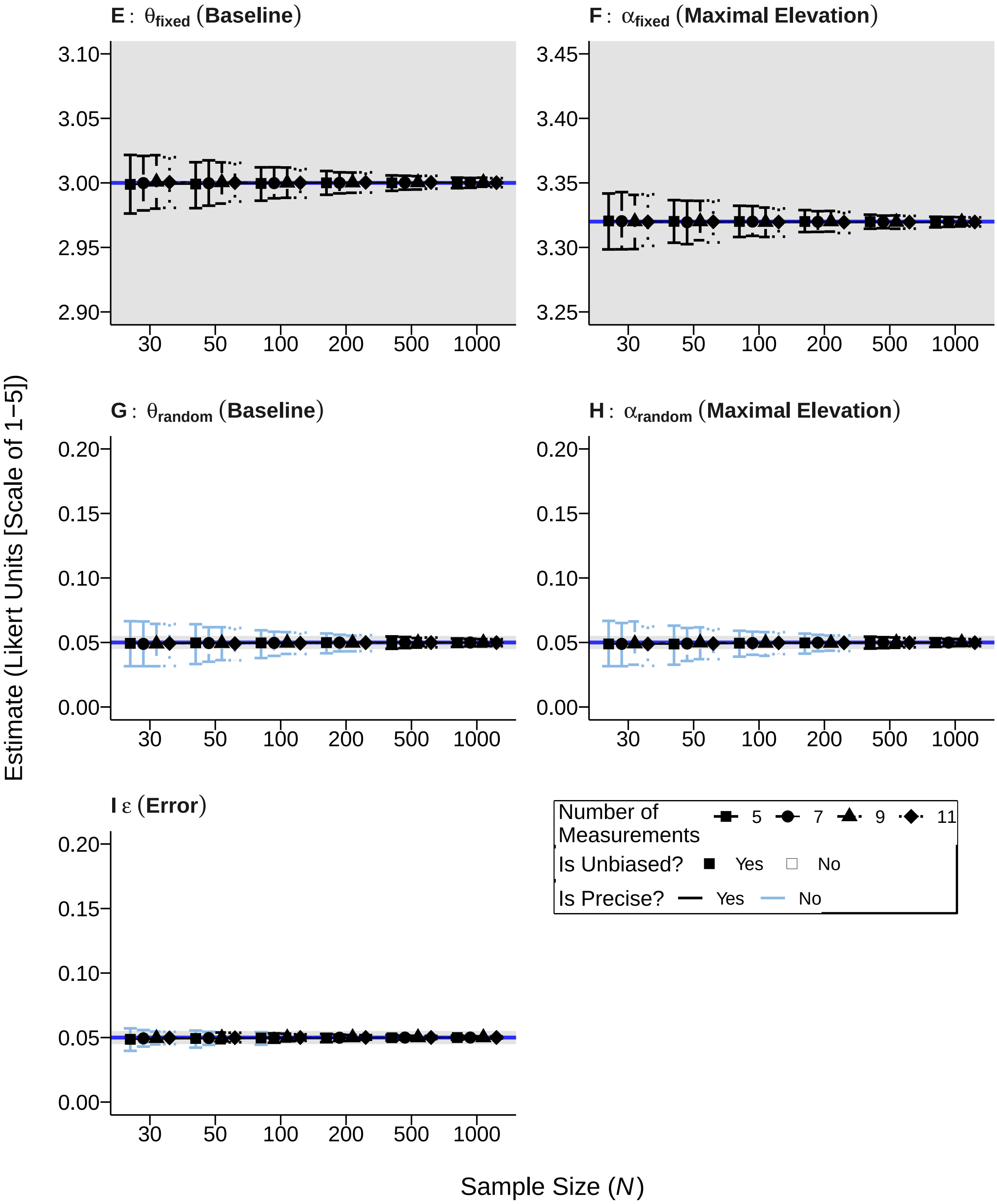


Figure F.12

Bias/Precision Plots for Day- and Likert-Unit Parameters When Using Definition Variables To Model Time-Unstructured Data Characterized by a Slow Response Rate (continued)



Note. Panels A–B: Bias/precision plots for the fixed- and random-effect days-to-halfway elevation parameters, respectively (β_{fixed} and β_{random}). Panels C–D: Bias/precision plots for the fixed- and random-effect triquarter-halfway elevation parameters, respectively (γ_{fixed} and γ_{random}). Panels E–F: Bias/precision plots for the fixed- and random-effect baseline parameters, respectively (θ_{fixed} and θ_{random}).

Panels G–H: Bias/precision plots for the fixed- and random-effect maximal elevation parameters, respectively (α_{fixed} and α_{random}). Blue horizontal lines in each panel represent the population value for each parameter. Population values for each day-unit parameter are as follows: $\beta_{fixed} \in$ 80, 180, 280, β_{random} = 10.00, γ_{fixed} = 20.00, γ_{random} = 4.00, θ_{fixed} = 3.00, θ_{random} = 0.05, α_{fixed} = 3.32, α_{random} = 0.05, ϵ = 0.05. Gray bands indicate the $\pm 10\%$ margin of error for each parameter and unfilled dots indicate cells with average parameter estimates outside of the margin or biased estimates. Error bars represent the middle 95% of estimated values, with light blue error bars indicating imprecise estimation. I considered dots that fell outside the gray bands as biased and error bar lengths with at least one whisker length exceeding the 10% cutoff (i.e., or longer than the portion of the gray band underlying the whisker) as imprecise. Note that random-effect parameter units are in standard deviation units. See Table H.3 for specific values estimated for each parameter.

Appendix G: Convergence Success Rates

G.1 Experiment 1

Table G.1

Convergence Success Rates in Experiment 1

		Days to Halfway Elevation		
Measurement Spacing	Number of Measurements	80	180	280
Equal	5	1.00	0.98	0.95
	7	1.00	1.00	0.99
	9	1.00	1.00	1.00
	11	1.00	1.00	1.00
Time-interval increasing	5	1.00	1.00	1.00
	7	1.00	1.00	1.00
	9	1.00	1.00	1.00
	11	1.00	1.00	1.00
Time-interval decreasing	5	1.00	0.96	0.82
	7	1.00	0.99	0.98
	9	1.00	1.00	1.00
	11	1.00	1.00	1.00

	5	1.00	0.96	0.86
Middle-and-extreme	7	1.00	1.00	1.00
	9	1.00	1.00	1.00
	11	1.00	1.00	1.00

Note. Cells shaded in gray indicate conditions where less than 90% of models converged.

G.2 Experiment 2

Table G.2

Convergence Success Rates in Experiment 2

		Sample Size (*N*)					
Measurement Spacing	Number of Measurements	30	50	100	200	500	1000
Equal	5	1.00	1.00	0.99	0.98	0.95	0.92
	7	1.00	1.00	1.00	1.00	0.99	0.98
	9	1.00	1.00	1.00	1.00	1.00	1.00
	11	1.00	1.00	1.00	1.00	1.00	1.00
Time-interval increasing	5	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00
	9	1.00	1.00	1.00	1.00	1.00	1.00
	11	1.00	1.00	1.00	1.00	1.00	1.00
Time-interval decreasing	5	1.00	0.99	0.98	0.95	0.93	0.88
	7	1.00	1.00	0.99	0.99	0.98	0.95
	9	1.00	1.00	1.00	1.00	1.00	0.99
	11	1.00	1.00	1.00	1.00	1.00	1.00
Middle-and-extreme	5	1.00	0.99	0.98	0.96	0.90	0.81
	7	1.00	1.00	1.00	1.00	1.00	1.00
	9	1.00	1.00	1.00	1.00	1.00	1.00
	11	1.00	1.00	1.00	1.00	1.00	1.00

Note. Cells shaded in gray indicate conditions where less than 90% of models converged.

G.3 Experiment 3

Table G.3
Convergence Success Rates in Experiment 3

Time Structuredness	Number of Measurements	Sample Size (*N*) 30	50	100	200	500	1000
Time structured	5	1.00	0.99	0.99	0.98	0.96	0.90
	7	1.00	1.00	1.00	1.00	0.99	0.98
	9	1.00	1.00	1.00	1.00	1.00	1.00
	11	1.00	1.00	1.00	1.00	1.00	1.00
Time unstructured (fast response)	5	1.00	1.00	0.98	0.99	0.96	0.90
	7	1.00	1.00	1.00	0.99	0.98	0.99
	9	1.00	1.00	1.00	1.00	1.00	1.00
	11	1.00	1.00	1.00	1.00	1.00	1.00
Time unstructured (slow response)	5	1.00	1.00	0.99	1.00	0.95	0.92
	7	1.00	1.00	1.00	0.99	0.99	0.98
	9	1.00	1.00	1.00	1.00	1.00	1.00
	11	1.00	1.00	1.00	1.00	1.00	1.00
Time unstructured (slow response) with definition variables	5	1.00	1.00	1.00	1.00	0.99	0.98
	7	1.00	1.00	1.00	1.00	1.00	0.99
	9	1.00	1.00	1.00	1.00	1.00	1.00
	11	1.00	1.00	1.00	1.00	1.00	1.00

Note. Cells shaded in gray indicate conditions where less than 90% of models converged.

Table G.4
Convergence Success in Experiment 3 With Definition Variables

Time Structuredness	Number of Measurements	Sample size (*N*) 30	50	100	200	500	1000

Time unstructured	5	1.00	1.00	1.00	1.00	0.99	0.98
(slow response)	7	1.00	1.00	1.00	1.00	1.00	0.99
with definition	9	1.00	1.00	1.00	1.00	1.00	1.00
variables	11	1.00	1.00	1.00	1.00	1.00	1.00

Note. Cells shaded in gray indicate conditions where less than 90% of models converged.

Appendix H: Parameter Estimate Tables

H.1 Experiment 1

Table H.1

Parameter Values Estimated for Day- and Likert-Unit Parameters in Experiment 1

		β_{fixed} (Days to halfway elevation)			β_{random} (Days to halfway elevation) Pop value = 10.00			γ_{fixed} (Triquarter-halfway delta) Pop value = 20.00			γ_{random} (Triquarter-halfway delta) Pop value = 4.00		
Measurement Spacing	Number of Measurements	80	180	280	80	180	280	80	180	280	80	180	280
Equal spacing	5	79.73	179.78	279.81□	10.14	10.40	10.08	19.37	19.49	19.71	7.41□	14.53□	8.11□
	7	80.21	178.99	279.55□	10.16	10.55	10.13	20.67	20.83	20.60	4.37	5.14□	4.41□
	9	80.00	179.94	279.99□	10.29	10.37	10.34	20.77	20.76	20.67	4.24	4.14	4.30
	11	80.03	180.01	279.88□	10.27	10.29	10.32	20.64	20.70	20.64	4.13	4.08	4.18
Time-interval increasing	5	79.88	180.10	274.37□	10.32	9.73	13.04□	20.71	20.39	18.32	4.57□	4.99□	6.20□
	7	80.19	179.82	279.86□	10.42	10.47	10.14	20.66	20.79	19.78	4.29	4.87□	7.03□
	9	79.59	179.06	279.70□	10.07	10.22	10.20	20.33	20.66	20.72	4.17	4.25	4.32
	11	79.89	179.84	279.62□	10.38	10.30	10.47	20.78	20.75	20.68	4.23	4.18	4.13
Time-interval decreasing	5	70.67	179.92	279.63□	15.28□	9.80	10.22	16.63	20.07	20.55	5.48□	5.17□	4.59□
	7	78.23	178.22	279.84□	10.08	10.46	10.39	19.38	20.59	20.69	6.80□	5.09□	4.24
	9	79.95	179.34	278.98□	10.03	10.20	10.05	20.42	20.54	20.28	4.37	4.32	4.19
	11	79.42	179.70	279.52□	10.38	10.13	10.06	20.75	20.45	20.31	4.17	4.16	4.17
Middle-and-extreme spacing	5	71.95	179.61	287.73□	16.78□	10.26	16.74□	15.59	20.61	17.09	6.54□	4.24	8.61□
	7	80.45	180.00	279.15□	13.93□	10.25	13.69□	20.71	20.58	20.61	5.21□	4.16	4.98□
	9	80.28	180.05	279.63□	10.42	10.24	10.24	20.91	20.65	20.85	4.74□	4.26	4.72□
	11	80.19	179.96	279.86□	10.27	10.28	10.15	20.71	20.70	20.71	4.14	4.08	4.16

Table H.1

Parameter Values Estimated for Day- and Likert-Unit Parameters in Experiment 1 (continued)

		θ_{fixed} (Baseline) Pop value = 3.00			θ_{random} (Baseline) Pop value = 0.05			α_{fixed} (Maximal elevation) Pop value = 3.32			α_{random} (Maximal elevation) Pop value = 0.05			ϵ(error) Pop value = 0.03		
Measurement Spacing	Number of Measurements	80	180	280	80	180	280	80	180	280	80	180	280	80	180	280
Equal spacing	5	3.00	3.00	3.00	0.05	0.05	0.05	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.00	3.00	3.00	0.05	0.05	0.05	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.00	3.00	3.00	0.05	0.05	0.05	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.00	3.00	3.00	0.05	0.05	0.05	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
Time-interval increasing	5	3.00	3.00	3.00	0.05	0.05	0.05	3.32	3.32	3.33	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.00	3.00	3.00	0.05	0.05	0.05	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.00	3.00	3.00	0.05	0.05	0.05	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.00	3.00	3.00	0.05	0.05	0.05	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
Time-interval decreasing	5	2.99	3.00	3.00	0.05	0.05	0.05	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.00	3.00	3.00	0.05	0.05	0.05	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.00	3.00	3.00	0.05	0.05	0.05	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.00	3.00	3.00	0.05	0.05	0.05	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
Middle-and-extreme spacing	5	2.99	3.00	3.00	0.05	0.05	0.05	3.32	3.32	3.33	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.00	3.00	3.00	0.05	0.05	0.05	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.00	3.00	3.00	0.05	0.05	0.05	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.00	3.00	3.00	0.05	0.05	0.05	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05

Note. Cells shaded in light blue indicate cells where estimation is imprecise (i.e., lower and/or upper whisker lengths exceeding 10% of the parameter's population value. Empty superscript squares ($^{\square}$) indicate biased estimates (i.e., bias exceeding 10% of parameter's population value). Importantly, bias and precision cutoff values for the days-to-halfway elevation parameter (β_{fixed}) are based on a value of 180.00.

H.2 Experiment 2

Table H.2
Parameter Values Estimated in Experiment 2

Measurement Spacing	Number of Measurements	β_{fixed} (Days to halfway elevation) Pop value = 180.00						β_{random} (Days to halfway elevation) Pop value = 10.00					
		30	50	100	200	500	1000	30	50	100	200	500	1000
Equal spacing	5	179.71	179.82	179.53	180.00	179.99	179.64	10.40	10.36	10.04	10.51	10.65	10.74
	7	180.05	179.65	179.53	179.75	179.76	179.99	10.18	10.59	10.49	10.54	10.60	10.58
	9	179.84	180.07	179.94	180.00	180.02	180.03	10.28	10.20	10.30	10.40	10.39	10.36
	11	180.11	180.11	180.01	180.03	179.98	179.98	10.08	10.04	10.28	10.29	10.38	10.29
Time-interval increasing	5	181.81	181.16	181.14	180.27	179.78	179.57	11.24□	10.24	9.93	9.59	9.91	10.22
	7	179.99	179.96	179.73	179.77	179.79	179.83	10.26	10.43	10.50	10.43	10.47	10.47
	9	179.33	179.18	178.99	179.07	179.11	179.13	10.15	10.10	10.17	10.18	10.21	10.29
	11	179.81	179.79	179.86	179.88	179.81	179.82	9.99	10.19	10.32	10.27	10.30	10.30
Time-interval decreasing	5	177.01	178.48	179.13	179.23	179.86	180.37	10.95	11.38□	9.97	9.55	10.36	10.11
	7	178.98	179.68	179.12	179.53	180.07	179.75	10.07	10.31	10.48	10.37	10.46	10.51
	9	179.65	179.01	178.46	179.47	179.64	179.75	10.11	10.16	10.20	10.17	10.28	10.26
	11	179.48	179.68	179.70	179.65	179.64	179.68	9.85	9.98	10.03	10.12	10.13	10.11
Middle-and-extreme spacing	5	177.99	179.65	179.15	179.83	179.61	178.74	10.30	10.24	10.40	10.24	10.28	10.26
	7	179.96	179.82	179.97	179.98	180.02	179.98	10.25	10.20	10.32	10.26	10.29	10.27
	9	179.88	180.07	179.89	179.98	179.98	179.99	10.12	10.16	10.24	10.30	10.24	10.29
	11	180.02	179.96	180.01	179.98	180.01	179.99	10.08	10.35	10.15	10.35	10.30	10.28

Table H.2

Parameter Values Estimated for Day- and Likert-Unit Parameters in Experiment 2 (continued)

		γ_{fixed} (Triquarter-halfway delta) Pop value = 20.00						γ_{random} (Triquarter-halfway delta) Pop value = 4.00					
Measurement Spacing	Number of Measurements	30	50	100	200	500	1000	30	50	100	200	500	1000
Equal spacing	5	18.25	18.11	18.27	19.59	20.27	20.60	17.69□	16.95□	16.41□	15.19□	12.19□	8.51□
	7	20.25	20.53	20.66	20.75	20.81	20.74	9.22□	7.70□	5.77□	4.89□	4.98□	4.34
	9	20.88	20.72	20.73	20.76	20.75	20.73	5.30□	4.99□	4.44□	4.27	4.03	4.00
	11	20.65	20.66	20.73	20.70	20.69	20.71	4.86□	4.49□	4.20	4.10	4.02	4.07
Time-interval increasing	5	18.81	19.11	19.56	20.25	20.80	20.92	6.18□	5.88□	5.25□	4.94□	4.68□	4.42□
	7	20.74	20.74	20.94	20.83	20.83	20.82	7.38□	6.31□	5.45□	5.06□	4.66□	4.45□
	9	20.72	20.65	20.69	20.65	20.63	20.65	5.15□	4.83□	4.44□	4.26	4.16	4.23
	11	20.80	20.69	20.84	20.76	20.78	20.76	4.84□	4.43□	4.25	4.26	4.17	4.14
Time-interval decreasing	5	19.21	18.50	19.21	19.90	20.50	20.79	7.17□	6.01□	5.18□	5.12□	4.91□	4.66□
	7	20.36	20.49	20.57	20.69	21.03	20.76	6.98□	6.18□	5.43□	5.20□	4.67□	4.68□
	9	20.69	20.60	20.55	20.62	20.70	20.63	5.48□	5.12□	4.72□	4.52□	4.72□	4.83□
	11	20.49	20.53	20.38	20.41	20.47	20.41	4.66□	4.57□	4.34	4.20	4.18	4.17
Middle-and-extreme spacing	5	20.80	20.69	20.65	20.67	20.64	20.59	5.21□	4.68□	4.43□	4.18	4.15	4.11
	7	20.76	20.55	20.70	20.63	20.60	20.63	5.07□	4.60□	4.39	4.23	4.19	4.15
	9	20.68	20.71	20.67	20.63	20.58	20.63	4.99□	4.67□	4.49□	4.17	4.13	4.15
	11	20.64	20.74	20.67	20.70	20.66	20.68	4.57□	4.47□	4.22	4.19	4.09	4.07

Table H.2

Parameter Values Estimated for Day- and Likert-Unit Parameters in Experiment 2 (continued)

		θ_{fixed} (Baseline) Pop value = 3.00						θ_{random} (Baseline) Pop value = 0.05					
Measurement Spacing	Number of Measurements	30	50	100	200	500	1000	30	50	100	200	500	1000
Equal spacing	5	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
Time-interval increasing	5	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
Time-interval decreasing	5	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
Middle-and-extreme spacing	5	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05

Table H.2

Parameter Values Estimated for Day- and Likert-Unit Parameters in Experiment 2 (continued)

		α_{fixed} (Maximal elevation) Pop value = 3.32						α_{random} (Maximal elevation) Pop value = 0.05					
Measurement Spacing	Number of Measurements	30	50	100	200	500	1000	30	50	100	200	500	1000
Equal spacing	5	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
Time-interval increasing	5	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
Time-interval decreasing	5	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
Middle-and-extreme spacing	5	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05

Table H.2

Parameter Values Estimated for Day- and Likert-Unit Parameters in Experiment 2 (continued)

		ϵ(error)					
		Pop value = 0.03					
Measurement Spacing	Number of Measurements	30	50	100	200	500	1000
Equal spacing	5	0.05	0.05	0.05	0.05	0.05	0.05
	7	0.05	0.05	0.05	0.05	0.05	0.05
	9	0.05	0.05	0.05	0.05	0.05	0.05
	11	0.05	0.05	0.05	0.05	0.05	0.05
Time-interval increasing	5	0.05	0.05	0.05	0.05	0.05	0.05
	7	0.05	0.05	0.05	0.05	0.05	0.05
	9	0.05	0.05	0.05	0.05	0.05	0.05
	11	0.05	0.05	0.05	0.05	0.05	0.05
Time-interval decreasing	5	0.05	0.05	0.05	0.05	0.05	0.05
	7	0.05	0.05	0.05	0.05	0.05	0.05
	9	0.05	0.05	0.05	0.05	0.05	0.05
	11	0.05	0.05	0.05	0.05	0.05	0.05
Middle-and-extreme spacing	5	0.05	0.05	0.05	0.05	0.05	0.05
	7	0.05	0.05	0.05	0.05	0.05	0.05
	9	0.05	0.05	0.05	0.05	0.05	0.05
	11	0.05	0.05	0.05	0.05	0.05	0.05

Note. Cells shaded in light blue indicate cells where estimation is imprecise (i.e., lower and/or upper whisker lengths exceeding 10% of the parameter's population value. Empty superscript squares ($^{\square}$) indicate biased estimates (i.e., bias exceeding 10% of parameter's population value).

H.3 Experiment 3

Table H.3

Parameter Values Estimated for Day- and Likert-Unit Parameters in Experiment 3

		β_{fixed} (Days to halfway elevation) Pop value = 180.00					
Time Structuredness	Number of Measurements	30	50	100	200	500	1000
Time structured	5	179.71	179.67	179.75	179.98	180.00	179.66
	7	180.05	179.59	179.02	179.66	180.03	179.63
	9	179.84	180.01	180.01	179.97	180.01	180.00
	11	180.11	179.91	179.94	180.00	180.00	180.00
Time unstructured (fast response)	5	177.48	177.24	176.74	177.50	177.42	177.06
	7	176.89	177.03	176.37	175.92	177.20	176.95
	9	177.54	177.28	177.27	177.31	177.34	177.33
	11	177.25	177.35	177.27	177.37	177.35	177.30
Time unstructured (slow response)	5	174.13	174.02	173.65	173.85	173.41	173.63
	7	173.31	173.63	173.01	173.06	173.55	173.55
	9	173.37	173.37	173.54	173.52	173.50	173.49
	11	173.58	173.56	173.50	173.51	173.49	173.47
Time unstructured (slow response) with definition variables	5	179.92	179.87	179.97	179.92	179.87	179.88
	7	180.07	179.96	179.96	179.92	179.91	179.94
	9	180.17	179.86	179.88	179.97	179.95	179.98
	11	179.93	180.20	179.94	179.97	179.99	179.99

Table H.3

Parameter Values Estimated for Day- and Likert-Unit Parameters in Experiment 3 (continued)

		β_{random} (Days to halfway elevation) Pop value = 10.00					
Time Structuredness	Number of Measurements	30	50	100	200	500	1000
Time structured	5	10.40	10.27	10.37	10.56	10.73	10.69
	7	10.18	10.42	10.65	10.52	10.76	10.60
	9	10.28	10.28	10.37	10.46	10.42	10.41
	11	10.08	10.32	10.21	10.29	10.36	10.31
Time unstructured (fast response)	5	10.65	10.36	10.38	10.65	10.85	10.96
	7	10.53	10.60	10.88	10.83	10.84	10.84
	9	10.66	10.43	10.44	10.61	10.65	10.59
	11	10.41	10.37	10.37	10.45	10.52	10.51
Time unstructured (slow response)	5	11.23□	10.93	11.22□	11.80□	12.10□	12.07□
	7	11.71□	11.67□	11.88□	11.97□	11.91□	11.94□
	9	11.26□	11.38□	11.42□	11.40□	11.47□	11.46□
	11	10.87	10.98	11.12□	11.18□	11.14□	11.16□
Time unstructured (slow response) with definition variables	5	10.70	10.40	10.35	10.50	10.66	10.61
	7	10.32	10.32	10.33	10.52	10.53	10.50
	9	10.12	10.26	10.43	10.32	10.40	10.38
	11	10.11	10.20	10.34	10.31	10.27	10.32

Table H.3

Parameter Values Estimated for Day- and Likert-Unit Parameters in Experiment 3 (continued)

		γ_{fixed} (Triquarter-halfway delta) Pop value = 20.00						γ_{random} (Triquarter-halfway delta) Pop value = 4.00					
Time Structuredness	Number of Measurements	30	50	100	200	500	1000	30	50	100	200	500	1000
Time structured	5	18.25	18.11	18.46	19.67	20.55	20.65	17.69□	17.05□	16.38□	15.03□	11.63□	9.02□
	7	20.25	20.79	20.67	20.77	20.98	20.93	9.22□	7.32□	6.12□	4.99□	4.45□	4.69□
	9	20.88	20.79	20.84	20.69	20.74	20.71	5.30□	4.95□	4.34	4.13	4.05	3.96
	11	20.65	20.74	20.73	20.69	20.71	20.67	4.86□	4.41□	4.17	4.13	4.09	4.03
Time unstructured (fast response)	5	18.57	18.16	18.59	19.45	20.15	20.58	16.85□	16.21□	14.96□	13.48□	9.94□	7.72□
	7	20.39	20.44	20.67	20.73	20.77	20.77	9.65□	7.07□	6.25□	5.47□	4.61□	4.34
	9	20.54	20.66	20.75	20.71	20.72	20.74	5.27□	4.68□	4.59□	4.08	4.06	4.05
	11	20.77	20.70	20.72	20.70	20.71	20.73	4.85□	4.68□	4.29	4.14	4.16	4.14
Time unstructured (slow response)	5	18.66	17.88	18.34	19.83	20.57	20.67	14.54□	13.26□	11.51□	10.05□	7.89□	6.65□
	7	20.51	20.73	20.75	20.89	20.89	20.86	7.62□	6.65□	5.61□	5.21□	4.83□	4.67□
	9	20.91	20.82	20.82	20.89	20.94	20.89	6.00□	5.32□	4.97□	4.67□	4.74□	4.70□
	11	20.98	20.85	20.90	20.92	20.90	20.90	5.26□	4.92□	4.83□	4.69□	4.75□	4.71□
Time unstructured (slow response) with definition variables	5	20.58	20.64	20.76	20.86	20.90	20.94	11.12□	9.82□	8.51□	6.86□	5.78□	5.17□
	7	20.55	20.68	20.73	20.87	20.81	20.78	6.68□	5.93□	5.14□	4.74□	4.11	4.12
	9	20.69	20.68	20.69	20.74	20.70	20.73	5.22□	4.77□	4.53□	4.24	4.05	4.05
	11	20.66	20.77	20.69	20.69	20.67	20.69	4.79□	4.72□	4.32	4.01	4.14	4.11

Table H.3

Parameter Values Estimated for Day- and Likert-Unit Parameters in Experiment 3 (continued)

		θ_{fixed} (Baseline) Pop value = 3.00						θ_{random} (Baseline) Pop value = 0.05					
Time Structuredness	Number of Measurements	30	50	100	200	500	1000	30	50	100	200	500	1000
Time structured	5	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
Time unstructured (fast response)	5	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
Time unstructured (slow response)	5	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
Time unstructured (slow response) with definition variables	5	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.00	3.00	3.00	3.00	3.00	3.00	0.05	0.05	0.05	0.05	0.05	0.05

Table H.3

Parameter Values Estimated for Day- and Likert-Unit Parameters in Experiment 3 (continued)

		α_{fixed} (Maximal elevation) Pop value = 3.32						α_{random} (Maximal elevation) Pop value = 0.05					
Time Structuredness	Number of Measurements	30	50	100	200	500	1000	30	50	100	200	500	1000
Time structured	5	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
Time unstructured (fast response)	5	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
Time unstructured (slow response)	5	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
Time unstructured (slow response) with definition variables	5	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	7	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	9	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05
	11	3.32	3.32	3.32	3.32	3.32	3.32	0.05	0.05	0.05	0.05	0.05	0.05

Table H.3

Parameter Values Estimated for Day- and Likert-Unit Parameters in Experiment 3 (continued)

		ϵ(error) Pop value = 0.03					
Time Structuredness	Number of Measurements	30	50	100	200	500	1000
Time structured	5	0.05	0.05	0.05	0.05	0.05	0.05
	7	0.05	0.05	0.05	0.05	0.05	0.05
	9	0.05	0.05	0.05	0.05	0.05	0.05
	11	0.05	0.05	0.05	0.05	0.05	0.05
Time unstructured (fast response)	5	0.05	0.05	0.05	0.05	0.05	0.05
	7	0.05	0.05	0.05	0.05	0.05	0.05
	9	0.05	0.05	0.05	0.05	0.05	0.05
	11	0.05	0.05	0.05	0.05	0.05	0.05
Time unstructured (slow response)	5	0.05	0.05	0.05	0.05	0.05	0.05
	7	0.05	0.05	0.05	0.05	0.05	0.05
	9	0.05	0.05	0.05	0.05	0.05	0.05
	11	0.05	0.05	0.05	0.05	0.05	0.05
Time unstructured (slow response) with definition variables	5	0.05	0.05	0.05	0.05	0.05	0.05
	7	0.05	0.05	0.05	0.05	0.05	0.05
	9	0.05	0.05	0.05	0.05	0.05	0.05
	11	0.05	0.05	0.05	0.05	0.05	0.05

Note. Cells shaded in light blue indicate cells where estimation is imprecise (i.e., lower and/or upper whisker lengths exceeding 10% of the parameter's population value. Empty superscript squares ($^{\square}$) indicate biased estimates (i.e., bias exceeding 10% of parameter's population value).

Appendix I: Partial Omega-Squared Values for all Independent Variables in Each Simulation Experiment

Table I.1

Partial ω^2 Values for all Manipulated Variables in Experiment 1

	Effect						
Parameter	NM	NC	MS	NM x NC	NM x MS	NC x MS	NM x NC x MS
β_{fixed} (Figure ??A, ??A, ??A, ??A)	0.42	0.10	0.10	0.21	0.20	0.17	0.33
β_{random} (Figure ??B, ??B, ??B, ??B)	0.25	0.06	0.08	0.03	0.06	0.12	0.11
γ_{fixed} (Figure ??C, ??C, ??C, ??C)	0.58	0.13	0.02	0.18	0.12	0.24	0.30
γ_{random}(Figure ??D, ??D, ??D, ??D)	0.24	0.00	0.03	0.01	0.14	0.09	0.09

Note. NM = number of measurements $\in \{5, 7, 9, 11\}$, NC = nature of change (population value set for $\beta_{fixed} \in \{80, 180, 280\}$), MS = measurement spacing (equal, time interval increasing, time interval decreasing), NM x NC = interaction between number of measurements and nature of change, NM x MS = interaction between number of measurements and measurement spacing, NC x MS = interaction between nature of change and measurement spacing, NM x NC x MS = interaction between number of measurements, nature of change, and measurement spacing. β_{fixed} = fixed-effect days-to-halfway elevation parameter, γ_{fixed} = fixed-effect triquarter-halfway delta parameter, β_{random} = random-effect days-to-halfway elevation parameter, and γ_{random} = random-effect triquarter-halfway delta parameter.

Table I.2

Partial ω^2 Values for Independent Variables in Experiment 2

	Effect						
Parameter	NM	S	MS	NM x S	NM x MS	S x MS	NM x S x MS
β_{fixed} (Figure ??A, ??A, ??A, ??A)	0.07	0.07	0.03	0.02	0.05	0.01	0.01
β_{random} (Figure ??B, ??B, ??B, ??B)	0.21	0.31	0.06	0.02	0.07	0.01	0.01
γ_{fixed} (Figure ??C, ??C, ??C, ??C)	0.19	0.15	0.04	0.05	0.07	0.01	0.02
γ_{random}(Figure ??D, ??D, ??D, ??D)	0.14	0.11	0.11	0.00	0.18	0.00	0.01

Note. NM = number of measurements $\in \{5, 7, 9, 11\}$, MS = measurement spacing (equal, time interval increasing, time interval decreasing), S = sample size $\in \{30, 50, 100, 200, 500, 1000\}$, NM x NC = interaction between number of measurements and nature of change, NM x MS = interaction between number of measurements and measurement spacing, NC x MS = interaction between nature of change and measurement spacing, NM x NC x MS = interaction between number of measurements, nature of change, and measurement spacing. β_{fixed} = fixed-effect days-to-halfway elevation parameter, γ_{fixed} = fixed-effect triquarter-halfway delta parameter, β_{random} = random-effect days-to-halfway elevation parameter, and γ_{random} = random-effect triquarter-halfway delta parameter.

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Table I.3

Partial ω^2 Values for Manipulated Variables in Experiment 3

	Effect						
Parameter	NM	S	TS	NM x S	NM x TS	S x TS	NM x S x TS
β_{fixed} (Figure ??A, ??A, ??A)	0.01	0.06	0.00	0.00	0.00	0.00	0.00
β_{random} (Figure ??A, ??A, ??A)	0.31	0.47	0.00	0.08	0.00	0.00	0.00
γ_{fixed}(Figure ??B, ??B, ??B)	0.48	0.28	0.00	0.17	0.00	0.00	0.00
γ_{random} (Figure ??C, ??C, ??C)	0.33	0.10	0.01	0.02	0.01	0.00	0.00

Note. NM = number of measurements $\in$ {5, 7, 9, 11}, S = sample size $\in$ {30, 50, 100, 200, 500, 1000}, TS = time structuredness (time structured, fast response, slow response), NM x S = interaction between number of measurements and sample size, NM x TS = interaction between number of measurements and time structuredness, S x TS = interaction between sample size and time structuredness, NM x S x TS = interaction between number of measurements, sample size, and time structuredness.

Appendix J: OpenMx Code for Structured Latent Growth Curve Model With Definition Variables

The code that I used to model logistic pattern of change using definition variables (see definition variables) is shown in Code Block J.1. Note that, the code is largely excerpted from the `run_exp_simulations()` and `create_definition_model()` functions from the `nonlinSims` package, and so readers interested in obtaining more information should consult the source code of this package. One important point to mention is that the model specified in Code Block J.1 can accurately model time-unstructured data because it uses definition variables.

Code Block J.1

OpenMx Code for Structured Latent Growth Curve Model With Definition Variables

```
#Now convert data to wide format (needed for OpenMx)
data_wide <- data[ , c(1:3, 5)] %>%
    pivot_wider(names_from = measurement_day, values_from = c(obs_score, actual_measurement_day))

#Definition variable (data. prefix tells OpenMx to use recorded time of observation for each person's data)
obs_score_days <- paste('data.', extract_obs_score_days(data = data_wide), sep = '')

#Remove . from column names so that OpenMx does not run into error (this occurs because, with some spacing schedules, measurement days are not integer values.)
names(data_wide) <- str_replace(string = names(data_wide), pattern = '\\.', replacement = '_')

#Latent variable names (theta = baseline, alpha = maximal elevation, beta = days-to-halfway elevation, gamma = triquarter-halfway elevation)
latent_vars <- c('theta', 'alpha', 'beta', 'gamma')

def_growth_curve_model <- mxModel(
  model = model_name,
  type = 'RAM', independent = T,
  mxData(observed = data_wide, type = 'raw'),

  manifestVars = manifest_vars,
  latentVars = latent_vars,

  #Residual variances; by using one label, they are assumed to all be equal (homogeneity of variance). That is, there is no complex error structure.
  mxPath(from = manifest_vars,
         arrows=2, free=TRUE,  labels='epsilon', values = 1, lbound = 0),

  #Latent variable covariances and variances (note that only the variances are estimated. )
  mxPath(from = latent_vars,
         connect='unique.pairs', arrows=2,
         free = c(TRUE,FALSE, FALSE, FALSE,
```

```
                  TRUE, FALSE, FALSE,
                  TRUE, FALSE,
                  TRUE),
         values=c(1, NA, NA, NA,
                  1, NA, NA,
                  1, NA,
                  1),
         labels=c('theta_rand', 'NA(cov_theta_alpha)', 'NA(cov_theta_beta)',
                  'NA(cov_theta_gamma)',
                  'alpha_rand','NA(cov_alpha_beta)', 'NA(cov_alpha_gamma)',
                  'beta_rand', 'NA(cov_beta_gamma)',
                  'gamma_rand'),
         lbound = c(1e-3, NA, NA, NA,
                    1e-3, NA, NA,
                    1, NA,
                    1),
         ubound = c(2, NA, NA, NA,
                    2, NA, NA,
                    90^2, NA,
                    45^2)),

  # Latent variable means (linear parameters). Note that the parameters of beta and gamma do not have estimated means because they are nonlinear parameters (i.e., the logistic function's first-order partial derivative with respect to each of those two parameters contains those two parameters)
  mxPath(from = 'one', to = c('theta', 'alpha'), free = c(TRUE, TRUE), arrows = 1,
         labels = c('theta_fixed', 'alpha_fixed'), lbound = 0, ubound = 7,
         values = c(1, 1)),

  #Functional constraints (needed to estimate mean values of fixed-effect parameters)
  mxMatrix(type = 'Full', nrow = length(manifest_vars), ncol = 1, free = TRUE,
           labels = 'theta_fixed', name = 't', values = 1, lbound = 0,  ubound = 7),
  mxMatrix(type = 'Full', nrow = length(manifest_vars), ncol = 1, free = TRUE,
           labels = 'alpha_fixed', name = 'a', values = 1, lbound = 0,  ubound = 7),
  mxMatrix(type = 'Full', nrow = length(manifest_vars), ncol = 1, free = TRUE,
           labels = 'beta_fixed', name = 'b', values = 1, lbound = 1, ubound = 360),
  mxMatrix(type = 'Full', nrow = length(manifest_vars), ncol = 1, free = TRUE,
           labels = 'gamma_fixed', name = 'g', values = 1, lbound = 1, ubound = 360),

  #Definition variables set for loadings (accounts for time-unstructured data)
  mxMatrix(type = 'Full', nrow = length(obs_score_days), ncol = 1, free = FALSE,
  labels = obs_score_days, name = 'time'),

  #Algebra specifying first-order partial derivatives;
  mxAlgebra(expression = 1 - 1/(1 + exp((b - time)/g)), name="Tl"),
  mxAlgebra(expression = 1/(1 + exp((b - time)/g)), name = 'Al'),
  mxAlgebra(expression = -((a - t) * (exp((b - time)/g) * (1/g))/(1 + exp((b -
  time)/g))^2), name = 'Bl'),
  mxAlgebra(expression =  (a - t) * (exp((b - time)/g) * ((b - time)/g^2))/(1 + exp((b
  -time)/g))^2, name = 'Gl'),

  #Factor loadings; all fixed and, importantly, constrained to change according to their partial derivatives (i.e., nonlinear functions)
  mxPath(from = 'theta', to = manifest_vars, arrows=1, free=FALSE,
         labels = sprintf(fmt = 'Tl[%d,1]', 1:length(manifest_vars))),
  mxPath(from = 'alpha', to = manifest_vars, arrows=1, free=FALSE,
         labels = sprintf(fmt = 'Al[%d,1]', 1:length(manifest_vars))),
  mxPath(from='beta', to = manifest_vars, arrows=1,  free=FALSE,
         labels =  sprintf(fmt = 'Bl[%d,1]', 1:length(manifest_vars))),
  mxPath(from='gamma', to = manifest_vars, arrows=1,  free=FALSE,
         labels =  sprintf(fmt = 'Gl[%d,1]', 1:length(manifest_vars))),

  #Fit function used to estimate free parameter values.
  mxFitFunctionML(vector = FALSE)
)
```

```
#Fit model using mxTryHard(). Increases probability of convergence by attempting model convergence by randomly shifting starting values.
model_results <- mxTryHard(def_growth_curve_model)
```