Hernandez Paredes Sebastian

1

Obtener los numeros $z \in C$ que satisfacen la ecuacion

$$z^3 = \frac{(z_1)(z_2 - z_3)}{2(z_4)^{21}(z_5)^2}$$

$$z^3 = \frac{(4\operatorname{cis}90°)(\sqrt{32}\operatorname{cis}135° - (-4-i))}{2(e^{5/2\pi i})^{21}(-i)^2}$$

$\sqrt{32}\operatorname{cis}135 \to \sqrt{32}\cos 135 + \sqrt{32}\operatorname{sen}135 i$

$\sqrt{32}\left(-\frac{\sqrt{2}}{2}\right) + (\sqrt{32})\left(\frac{\sqrt{2}}{2}\right)$

$\sqrt{32}\operatorname{cis}135° = -4 + 4i$

$$z^3 = \frac{(4i)(-4+4i - (-4-i))}{2(e^{5/2\pi i})^{21}(-i)^2} = \frac{(4i)(5i)}{2(e^{5/2\pi i})^{21}(-i)^2} = \frac{20i^2}{2(e^{5/2\pi i})^{21}(-1)}$$

$$z^3 = \frac{20i^2}{-2(e^{5/2\pi i})^{21}} = \frac{-20}{-2(\operatorname{cis}90)^{21}}$$

$$e^{5/2} = \frac{180(5)}{2} - 360 = 90$$

$$z^3 = \frac{-20}{-2(i)^{21}} = \frac{-20}{-2(i^1)} = \frac{-20}{-2i}$$

$$z^3 = \frac{20\operatorname{cis}180}{2\operatorname{cis}270}$$

$$z^3 = \frac{20}{2}\operatorname{cis} -90$$

$$z^3 = 10\operatorname{cis}270°$$

$$z = \sqrt[3]{10\operatorname{cis}270°} =$$

$\sqrt[3]{10}\operatorname{cis}90°$
$\sqrt[3]{10}\operatorname{cis}210°$
$\sqrt[3]{10}\operatorname{cis}330°$

Hernández Paredes Sebastián

(2)

$$\frac{z^{3/4}}{z_2 z_1} = \frac{1}{z_2 + z_3}$$

$z_2 = -2+2i \quad z_1 = 1 + \sqrt{3}i$

$$\frac{z^{3/4}}{(-2+2i)(1+\sqrt{3}i)} = \frac{1}{(-2-2i)(2i)}$$

$$\frac{z^{3/4}}{-2\sqrt{2}\operatorname{cis}(315°)\ 2\operatorname{cis}(60°)} = \frac{1}{-2\sqrt{2}\operatorname{cis}(45°)\ 2\operatorname{cis}(90°)}$$

$$\frac{z^{3/4}}{-4\sqrt{2}\operatorname{cis}(15°)} = \frac{1}{-4\sqrt{2}\operatorname{cis}(135)} \rightarrow z^{3/4} = \frac{\operatorname{cis}(15°)}{\operatorname{cis}(135°)}$$

$$z^{3/4} = \operatorname{cis}(240°) \rightarrow z = \sqrt[3]{[\operatorname{cis}(240°)]^4}$$

$$z = \sqrt[3]{\operatorname{cis}(240°)}$$

$$\boxed{\begin{array}{l} z_1 = \operatorname{cis}\left(\frac{240°}{3}\right) \rightarrow z_1 = \operatorname{cis}(80°) \\ z_2 = \operatorname{cis}\left(\frac{240° + 360°}{3}\right) \rightarrow z_2 = \operatorname{cis}(200°) \\ z_3 = \operatorname{cis}\left(\frac{240° + 2 \cdot 360°}{z}\right) \rightarrow z_3 = \operatorname{cis}(320°) \end{array}}$$

$z_3 = 2e^{\pi/2\, i}$

$2\operatorname{cis}(90°) \quad 2i$

$-2(1-i) = \theta = tg^{-1}(-9) = 315°$

$1+\sqrt{3}i = r = 2$

$\theta = 60$

$2(1+i) = r = \sqrt{2}$

$2i = r = 2$

$\theta = 90°$

3

Hernández Paredes Sebastián

a) $q(x) = x^4 + x^3 - 11x^2 - 9x + 18$

$q(x) = (-x)^4 + (-x)^3 - 11(-x)^2 - 9(-x) + 18$

$q(x) = x^4 - x^3 - 11x^2 + 9x + 18$

Raices

$\alpha_1 = 1$

$\alpha_2 = -2$

$\alpha_3 = 3$

$\alpha_4 = -3$

$PN = \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$PD = \pm 1$

$P.R.R. = \pm 1, \pm 2, \pm 3 \pm 6, \pm 9, \pm 18$

b) $q(x) = x^4 + x^3 - 11x^2 - 9x + 18$

1	1	-11	-9	+18	1
	1	2	-9	-18	
1	2	-9	-18	0	

$q(x) = (x-1)(x^3 + 2x^2 - 9x - 18)$

1	2	-9	-18	
	-2	0	+18	-2
1	0	-9	0	

$q(x) = (x-1)(x+2)(x^2-9)$

$q(x) = (x-1)(x+2)(x+3)(x-3$

Las raices son

$\alpha = 1$

$\alpha = -2$

$\alpha = -3$

$\alpha = 3$

4

Hernández Paredes Sebastián

a) $g(x) = x^2(x^4 + 8x^3 + 17x^2 + 4x - 156)$

$g(x) = (x-6)(x-6)(x^4 + 8x^3 + 17x^2 + 4x - 156)$

Posibles raíces: ±1 ±2 ±3 ±4 ±6 ±12 ±13 ±26 ±39 ±52

1	8	17	4	−156	
	2	20	74	156	2
1	10	37	78	0	

$g(x) = (x-6)(x-6)(x-2)(x^3 + 10x^2 + 37x + 78)$

1	10	37	78	
	−6	−24	−78	−6
1	4	13	0	

$g(x) = (x-6)(x-6)(x-2)(x+6)(x^2 + 4x + 13)$

$$\alpha_{5,6} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2(a)}$$

$$\alpha_{5,6} = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(13)}}{2(1)}$$

$$\alpha_{5,6} = \frac{-4 \pm \sqrt{16 - 52}}{2}$$

$$\alpha_{5,6} = \frac{-4 \pm \sqrt{-36}}{2} = \frac{-4 \pm \sqrt{36}\sqrt{-1}}{2}$$

$$\alpha_{5,6} = \frac{-4 \pm 6i}{2}$$

$$\alpha_5 = \frac{-4 + 6i}{2} \rightarrow \alpha_5 = -2 + 3i$$

$$\alpha_6 = \frac{-4 - 6i}{2} = \alpha_6 = -2 - 3i$$

$$\boxed{\begin{array}{l} \text{raíces} \\ \alpha_1 = \alpha_2 = 6 \\ \alpha_3 = -2 \\ \alpha_4 = 6 \\ \alpha_5 = -2 + 3i \\ \alpha_6 = -2 - 3i \end{array}}$$