Weighted Signed Graph Attention Networks

Course Project - ELL880

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Introduction

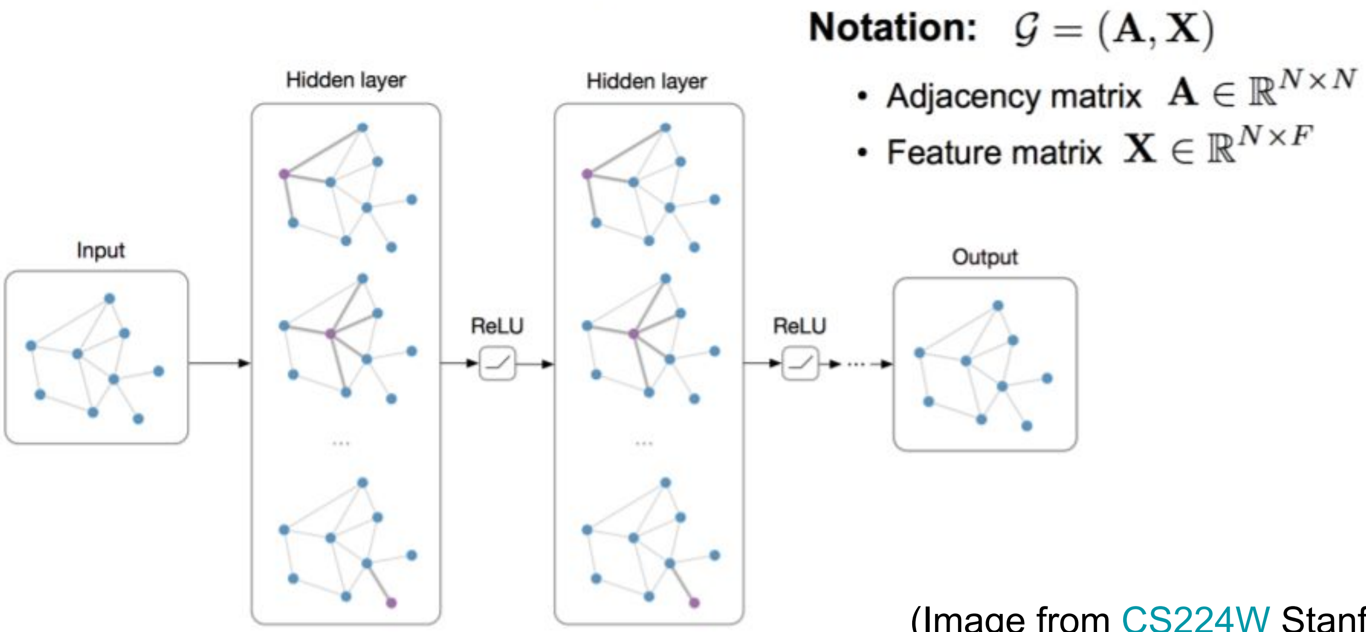
- The study of networks is a highly interdisciplinary field that draws ideas and inspiration from multiple disciplines including biology, computer science, economics, mathematics, physics, sociology, and statistics.
- However, when a network has like/dislike, love/hate, respect/disrespect, or trust/distrust relationships, such a representation is inadequate since it fails to encode the sign of a relationship. Hence, we need signed networks.

Problem Statement

- A Weighted Signed Network (WSN) is a undirected, weighted graph G = (V, E, W) where V is a set of nodes, $E \subseteq V \times V$ is a set of edges, and $W : E \rightarrow [-1, +1]$ is a mapping that assigns a value between -1 and +1 to each edge.
- W(u, v) can be thought of as assigning a degree of “likes”, “agrees” or “trust” score describing how much node u similar to a node v.
- Our goal is to predict the edge weights of the weighted signed network

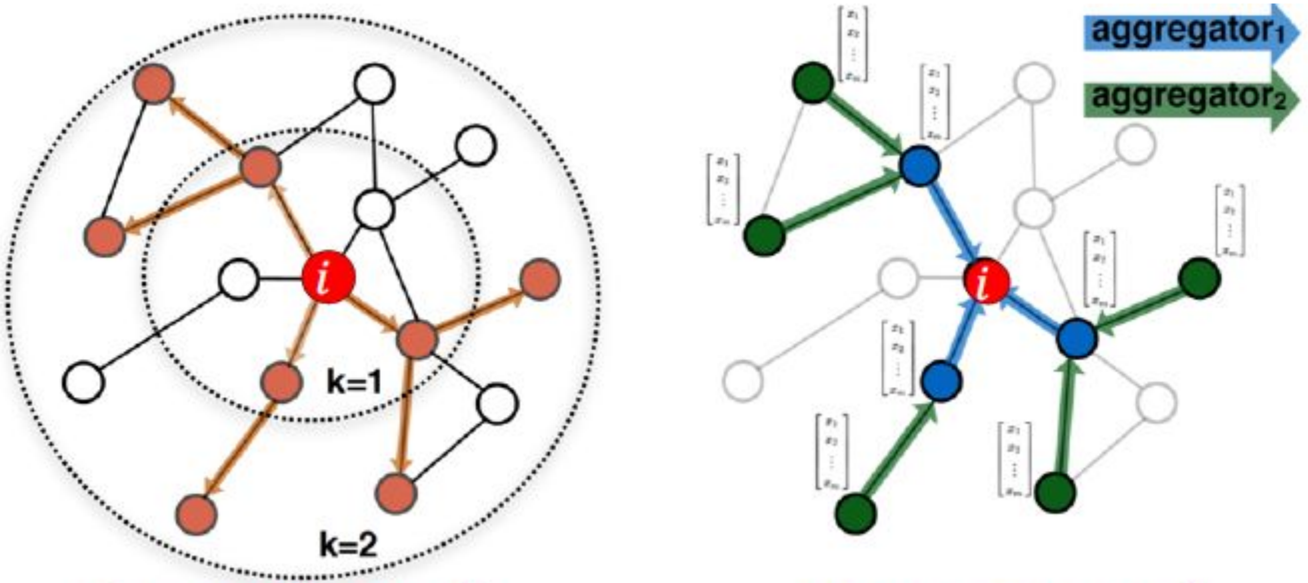
Graph Neural Networks

- There is an abundance of Graph structured Data which does not “live” on grids for ex. Social Networks,Citation Networks,Protein Interaction etc.
- GNNs(Graph Neural Networks) are most widely used model to capture/learn the dependence of the graphs using message passing scheme.



(Image from [CS224W](CS224W) Stanford)

Graph Convolutional Networks

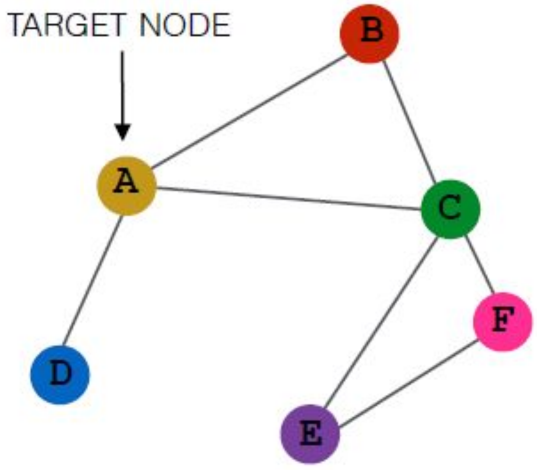


Determine node computation graph

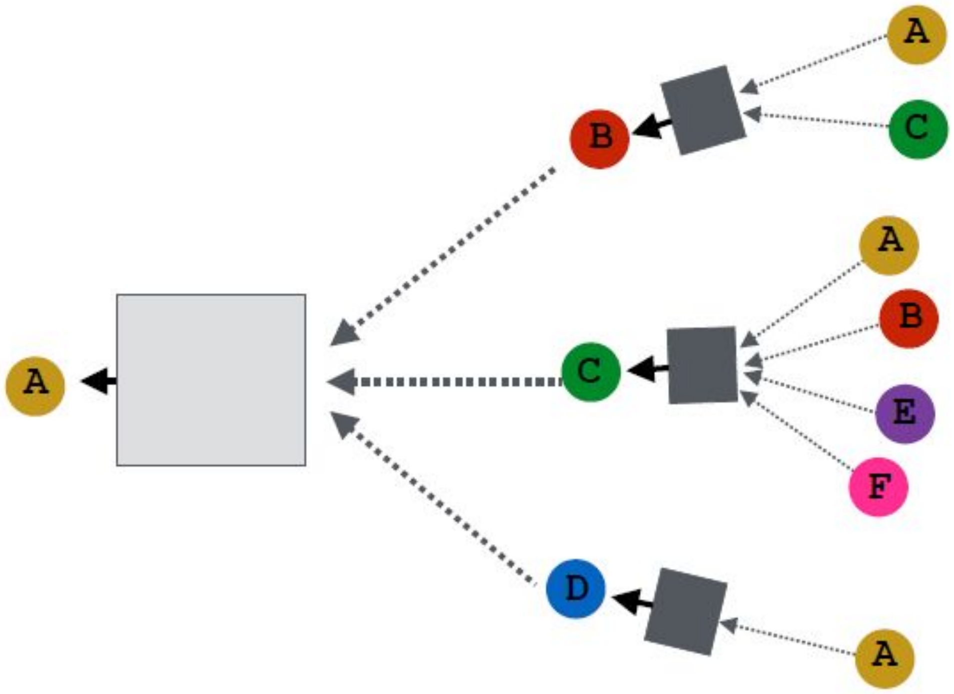
Propagate and transform information

Graph Convolutional Networks

- Generate Node Embeddings based on local network Neighbourhood

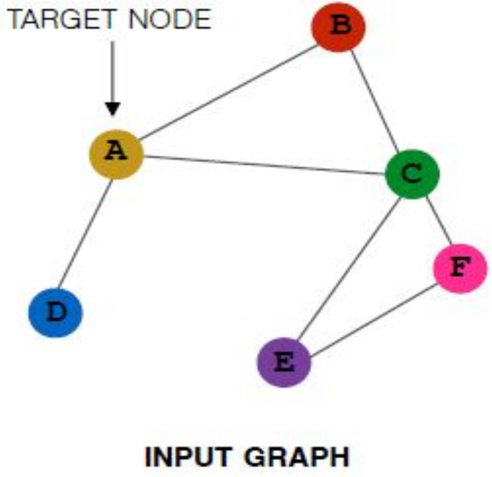


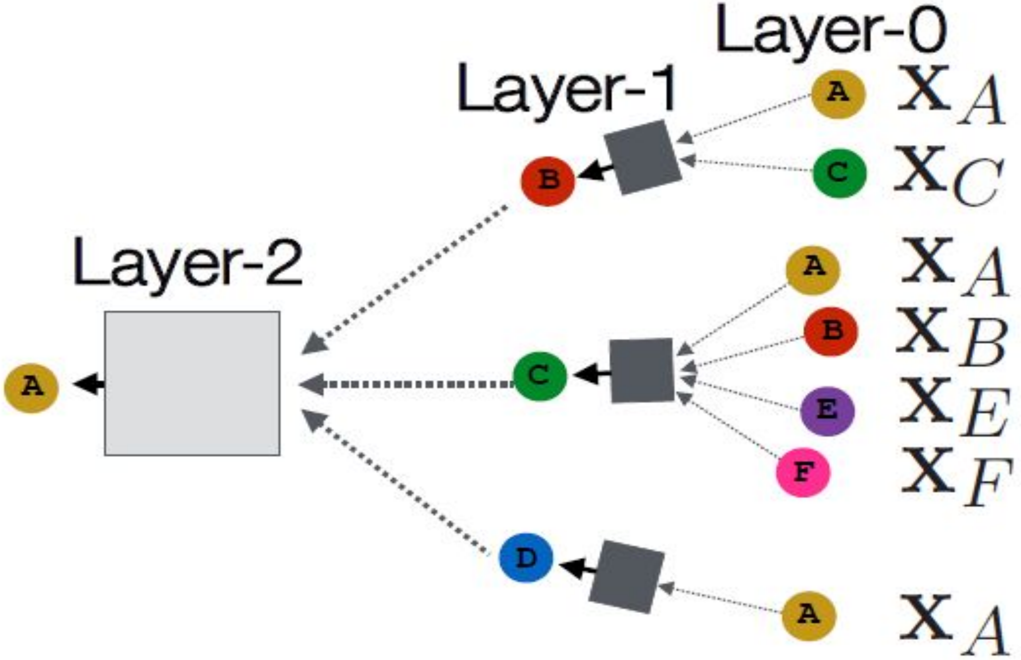
INPUT GRAPH



Graph Convolutional Networks

- Layer K embeddings will have information the nodes K hopes away





Graph Convolutional Networks

Initial 0-th layer embeddings are equal to node features

$$\boxed{\mathrm{h}_v^0 = \mathrm{x}_v}$$

embedding of v at layer l

$$\mathrm{h}_v^{(l+1)} = \boxed{\sigma}\left(\mathrm{W}_l \boxed{\sum_{u \in \mathrm{N}(v)} \frac{\mathrm{h}_u^{(l)}}{|\mathrm{N}(v)|}} + \mathrm{B}_l \boxed{\mathrm{h}_v^{(l)}}\right), \forall l \in \{0, \ldots, \boxed{L} - 1\}$$

$$\boxed{\mathrm{z}_v} = \mathrm{h}_v^{(L)}$$

Embedding after L layers of neighborhood aggregation

Non-linearity (e.g., ReLU)

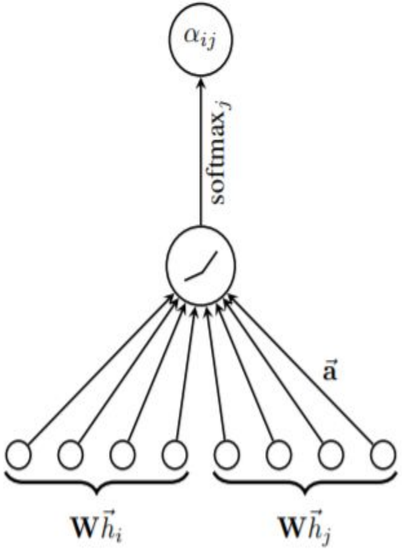
Average of neighbor's previous layer embeddings

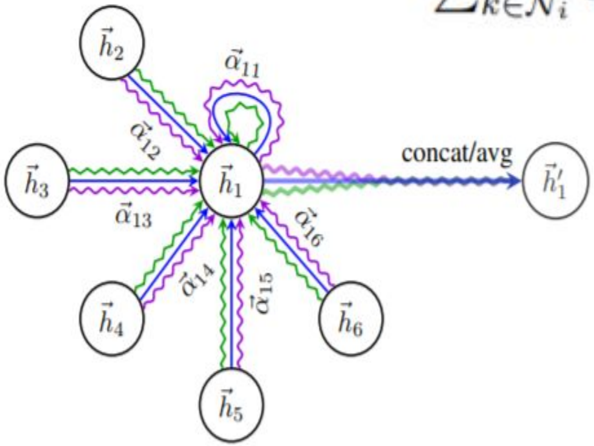
Total number of layers

Graph Attention Mechanism

- Instead of Simple neighbourhood aggregation we can do a weighted aggregation which gives different importance to different neighbours of each node in the graph.

$$\alpha_{ij} = \frac{\exp\left(\mathrm{LeakyReLU}\left(\vec{\mathbf{a}}^T[\mathbf{W}\vec{h}_i \| \mathbf{W}\vec{h}_j]\right)\right)}{\sum_{k\in\mathcal{N}_i}\exp\left(\mathrm{LeakyReLU}\left(\vec{\mathbf{a}}^T[\mathbf{W}\vec{h}_i \| \mathbf{W}\vec{h}_k]\right)\right)}$$



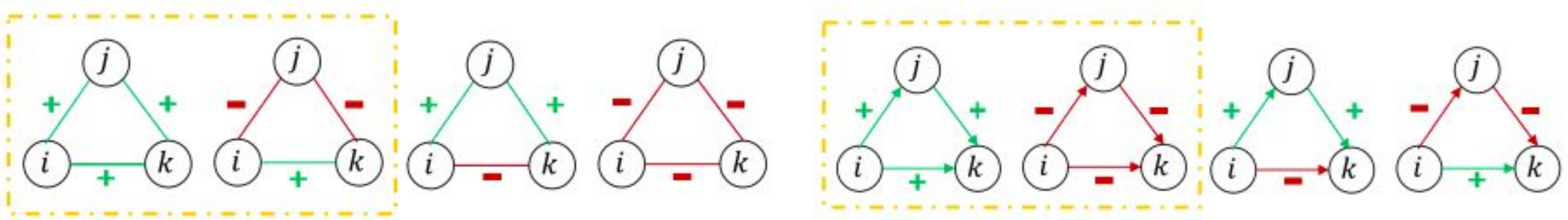


a - Attention Mechanism
W - Weight Matrix

Supporting Theories

Balance theory: It was developed for undirected signed graphs, where relationship among nodes is mutual. Its says that a friend of my friend is my friend and enemy of my enemy is my friend.

Status Theory: This generalises Balance theory for directed signed graphs. Its says that if there is a (+) edge from A to B, then A is friend of B and B is higher in status than A.



(a) Illustration of structural balance theory

(b) Illustration of status theory

Signed Graph Attention Networks

Since positive neighbors and negative neighbors should have different effects on the target node, they should obviously be treated separately and direction is also one of the important factor to be considered.

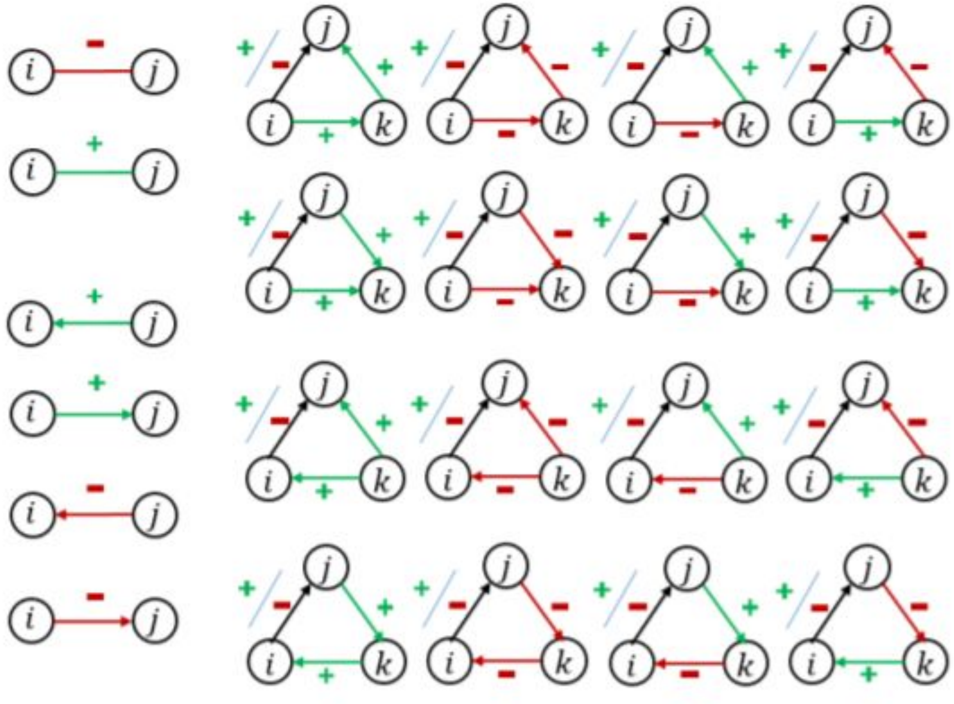


Fig: Different Motifs in SiGAT

Signed Graph Attention Networks

- Attention Scores and Node Features :

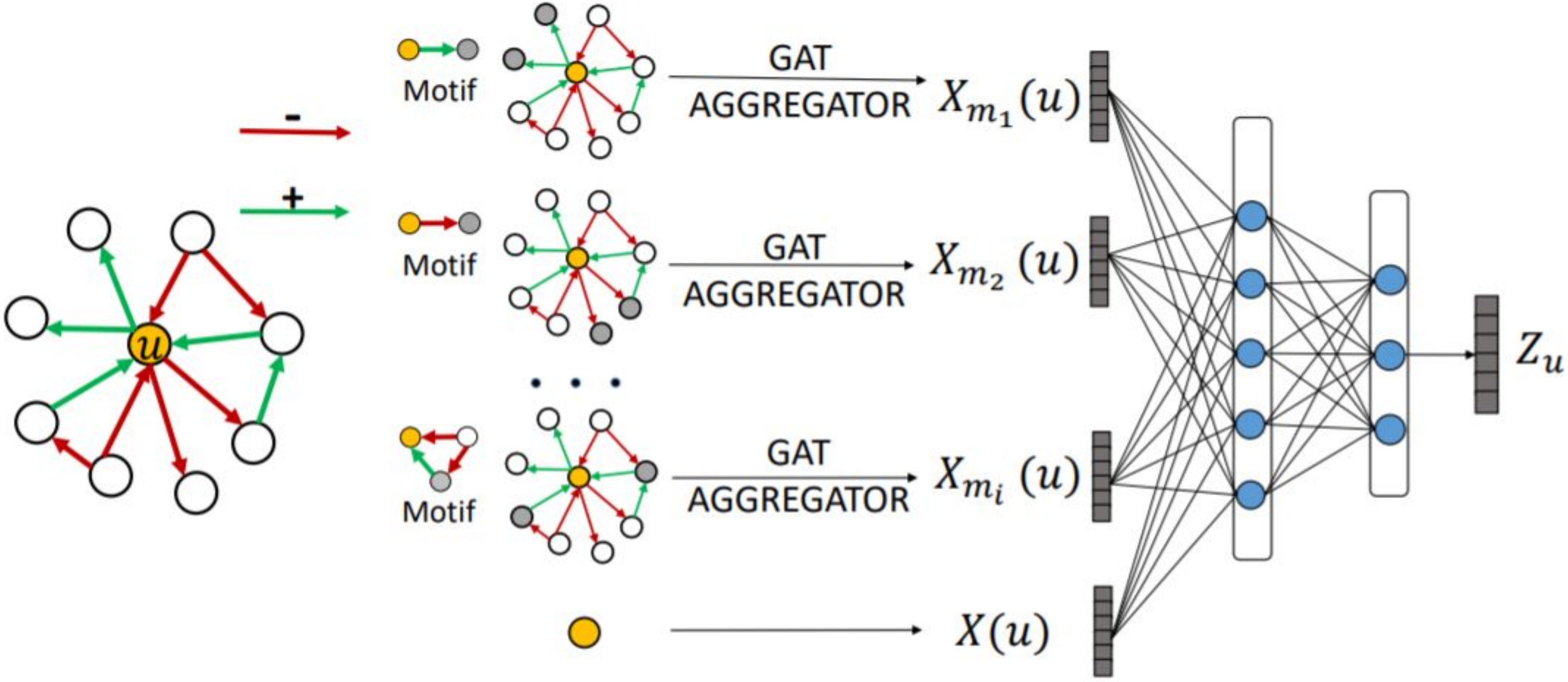
$$\alpha_{uv}^{m_i} = \frac{\exp\left(\mathrm{LeakyReLU}\left(\mathbf{a}_{m_i}^T[\mathbf{W}_{m_i}\mathbf{X}(u)\|\mathbf{W}_{m_i}\mathbf{X}(v)]\right)\right)}{\sum_{k\in\mathcal{N}_{m_i}(u)}\exp\left(\mathrm{LeakyReLU}\left(\mathbf{a}_{m_i}^T[\mathbf{W}_{m_i}\mathbf{X}(u)\|\mathbf{W}_{m_i}\mathbf{X}(k)]\right)\right)},$$

$$\mathbf{X}_{m_i}(u) = \sum_{v\in\mathcal{N}_{m_i}(u)} \alpha_{uv}^{m_i}\mathbf{W}_{m_i}\mathbf{X}(v).$$

- Cost Function

$$J_{\mathcal{G}}(\mathbf{Z}_u) = -\sum_{v^+\in\mathcal{N}(u)^+}\log\left(\sigma(\mathbf{Z}_u^\top\mathbf{Z}_{v^+})\right) - Q\sum_{v^-\in\mathcal{N}(u)^-}\log\left(\sigma(-\mathbf{Z}_u^\top\mathbf{Z}_{v^-})\right)$$

Signed Graph Attention Networks



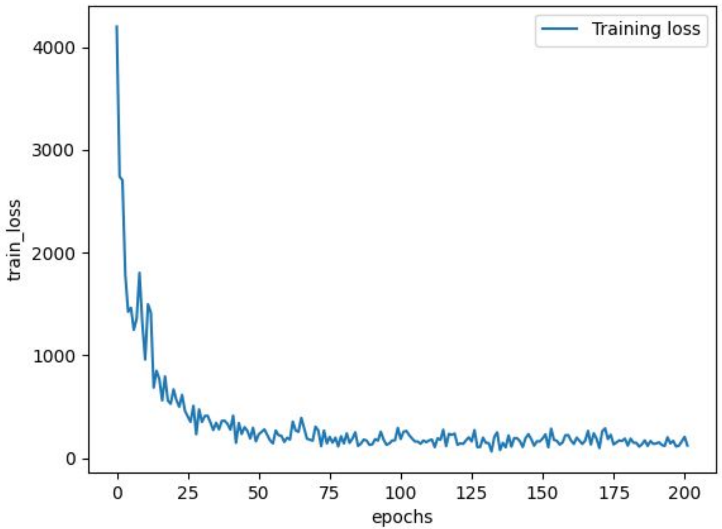
Proposed Model

- We proposed to consider the trust score (edge weights) as better signal for learning the embeddings,so we reformulated the loss function as :

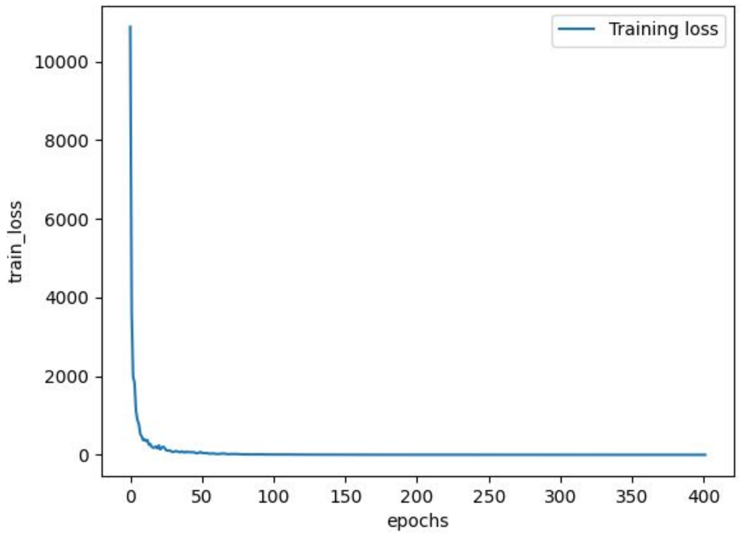
$$J(Z_u) = -\sum_{v^+ \in \mathcal{N}(u)^+} \log\left(\sigma(\mathrm{W}_{E^+} Z_u^\top Z_{v^+})\right) - Q \sum_{v^- \in \mathcal{N}(u)^-} \log\left(\sigma(\mathrm{W}_{E^-} Z_u^\top Z_{v^-})\right).$$

- $\mathbf{W_E}$ are the given edge weights for the dataset , Q is the weight for the respective term.
- We also replaced the Adjacency matrix to a Weighted adjacency matrix.

Experiments and Results - I



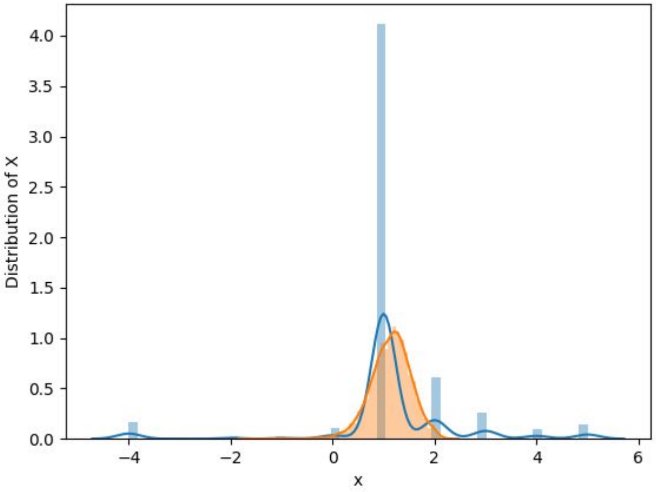
SiGAT



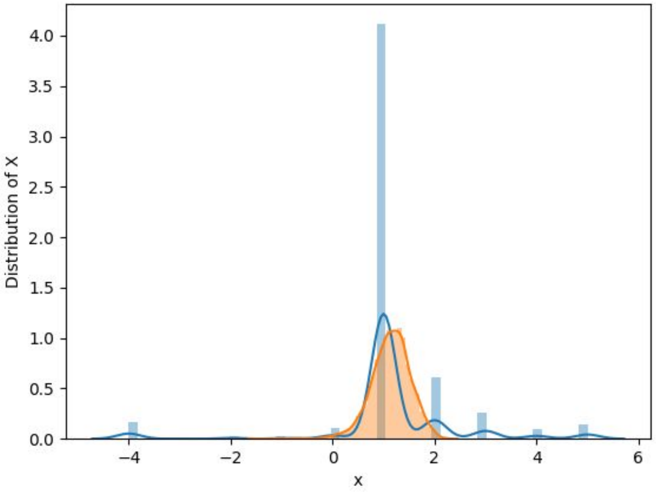
Modified SiGAT

Experiments and Results - II

The Embeddings in our model are able to predict values other than mode as well

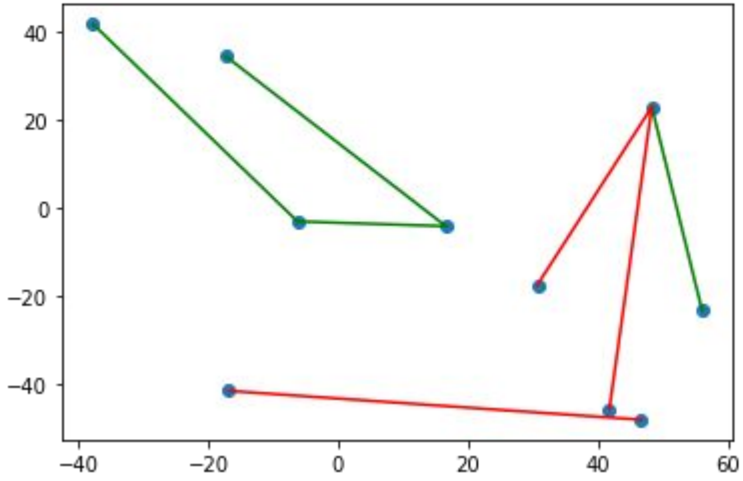


SiGAT -
Test RMSE - 1.549

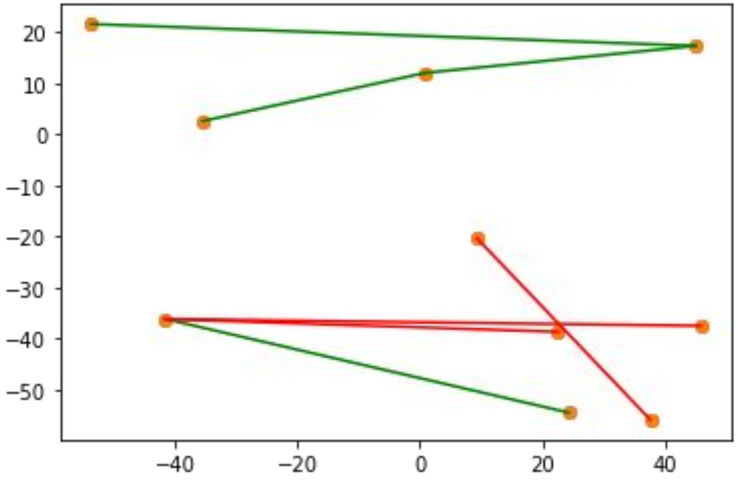


Modified SiGAT -
Test RMSE - 1.545

Experiments and Results - III



SiGAT



Modified SiGAT

Future Work

- Contrastive Learning based approach to bring the friend's embedding closer and enemy's embedding farther at the same time.

$$\ell_n = -\log \frac{\exp(\mathrm{sim}(z_{n,i}, z_{n,j})/\tau)}{\sum_{n'=1, n' \neq n}^{N} \exp(\mathrm{sim}(z_{n,i}, z_{n',j})/\tau)},$$

- Test on other datasets and compare with the other existing methods.

References

- Derr, Tyler, Y. Ma, and Jiliang Tang (2018). “Signed Graph ConvolutionalNetwork”. In:ArXivabs/1808.06354.
- Huang, Junjie et al. (2019). “Signed Graph Attention Networks”. In:ArXivabs/1906.10958.
- Kumar, Srijan et al. (2016). “Edge Weight Prediction in Weighted Signed Net-works”. In:2016 IEEE 16th International Conference on Data Mining (ICDM),pp. 221–230.

THANK YOU!!