EEEE 709 – Advanced Engineering Mathematics
MATLAB Project #3 Problem 2

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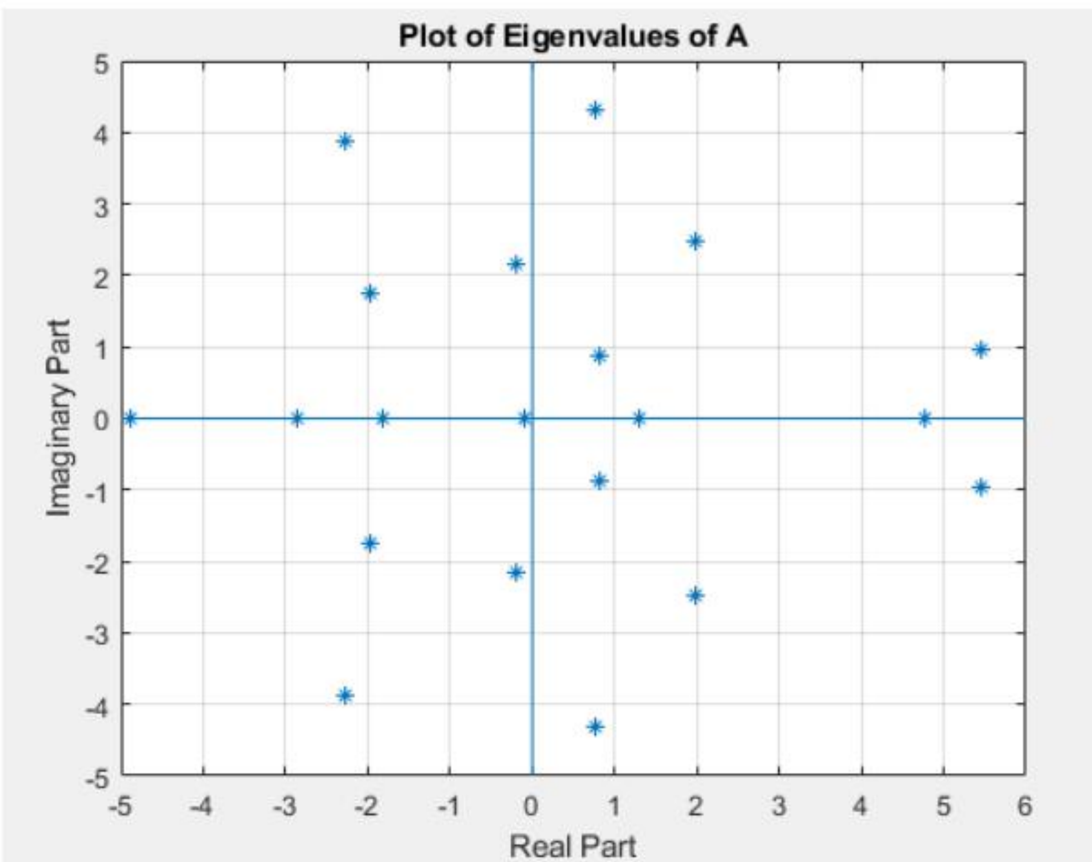
Date: 11/4/2021

Problem 2: Create a script in MATLAB to perform the following tasks:

a) Read in the grayscale image named "cameraman.tif" which is available in MATLAB [hint: imread()] and display it [hint: imshow()]
b) Compute its eigenvalues and eigenvectors [hint: eig()] and display its eigenvalues as shown in the figure below. You may need to use two separate figures to properly display all of the eigenvalues due to the scale difference. [Note: eig() will provide the eigenvalues generally sorted in descending order].
c) Plot the magnitude of the eigenvalues.
d) Re-synthesize the cameraman image (i.e. $Image' = MDM^{-1}$) by using only the first eigenvalue (the largest eigenvalue) and compute the absolute error between the original and synthesized image. (See EEEE-707 – Chapter 4: Eigenvalues and Eigenvectors). Display the re-synthesized image and the corresponding error.
e) Repeat part (d) by using the first and second largest eigenvalues, then first, second and third largest eigenvalue and so on until you have used all of the eigenvalues.
f) Display the synthesized image after using the largest 1, the largest 5 eigenvalues, the largest 50 eigenvalues, the largest 100 eigenvalues, the largest 200 eigenvalues and all of the eigenvalues.
g) Plot the absolute error as a function of the number of eigenvalues used.
h) Provide a short discussion of your observations.

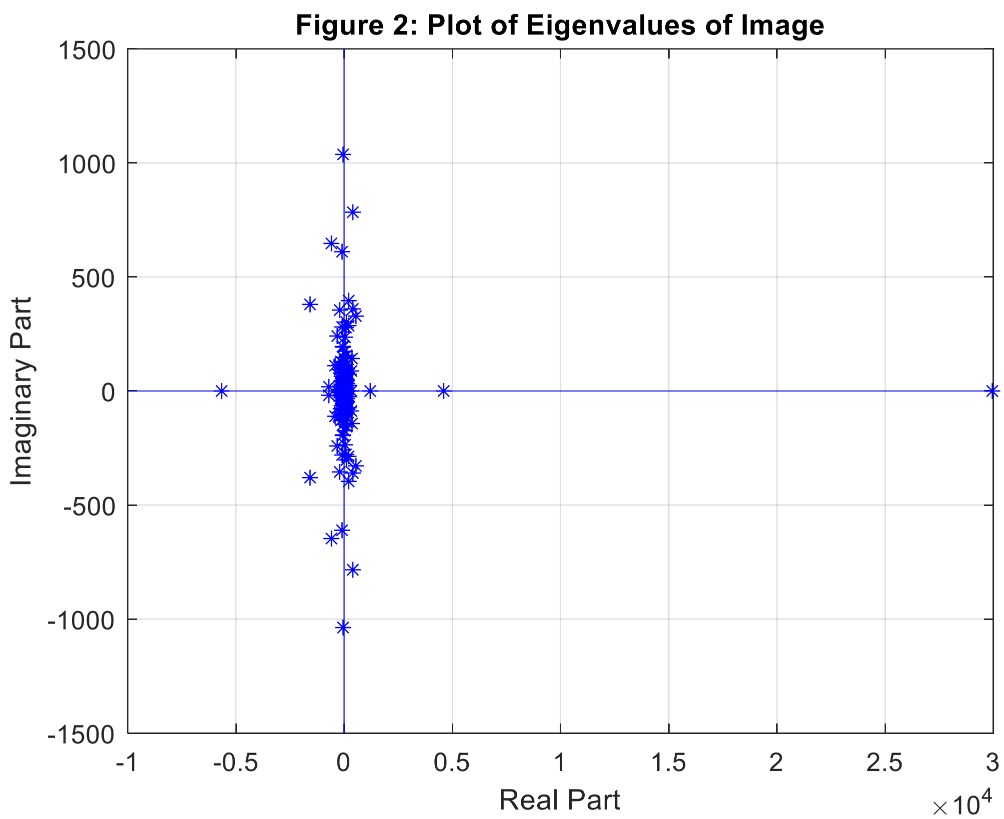
Note: Some of the eigenvalues of the above matrices will be complex. When displaying all eigenvalues, display them in rectangular format (i.e. imaginary part on y axis vs real part on x axis) as shown in the figure below



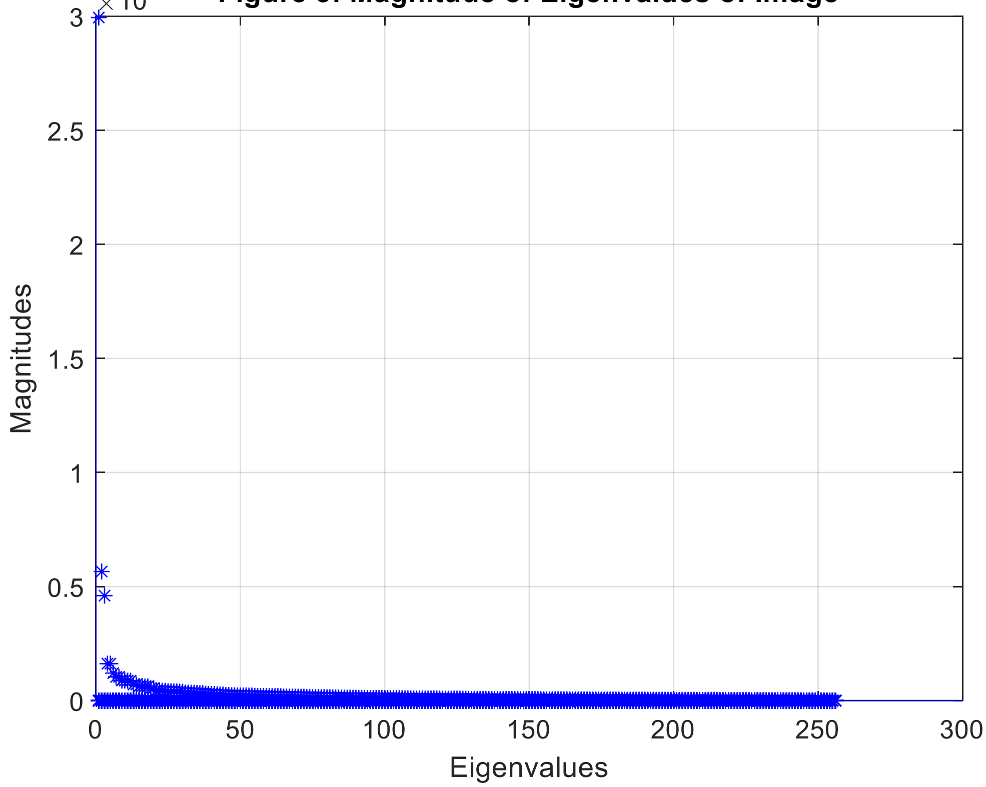
Problem 2 Solution:

a) **Figure 1: Original cameraman.tif Grayscale Image**

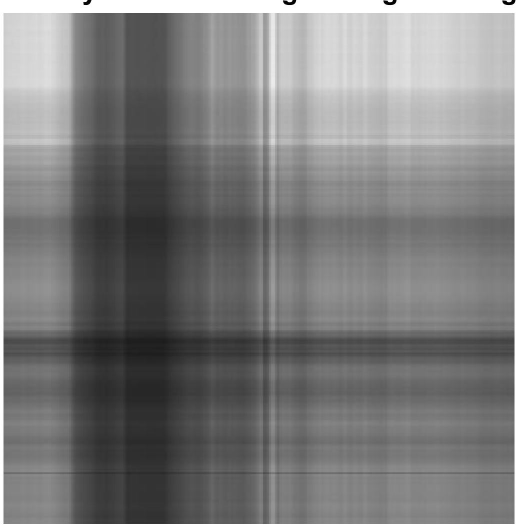
b)



c)



d) **Figure 4: Re-synthesized Image Using First Eigenvalue**



```
error =

   3.6503e+04
```

e) Completed. See Appendix 2 for corresponding code.

f) **Figure 5: Re-synthesized Image Using First Five Eigenvalues**

Figure 6: Re-synthesized Image Using First Fifty Eigenvalues

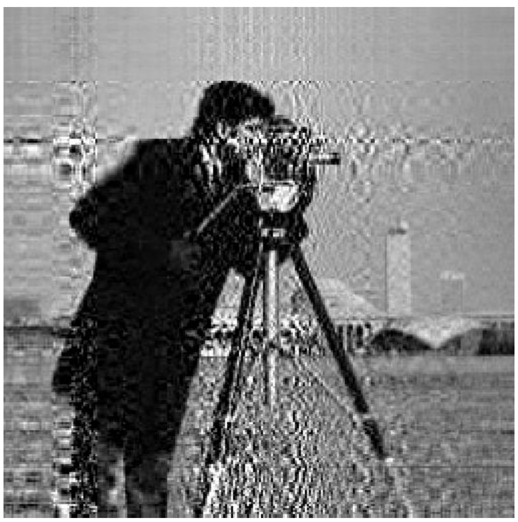
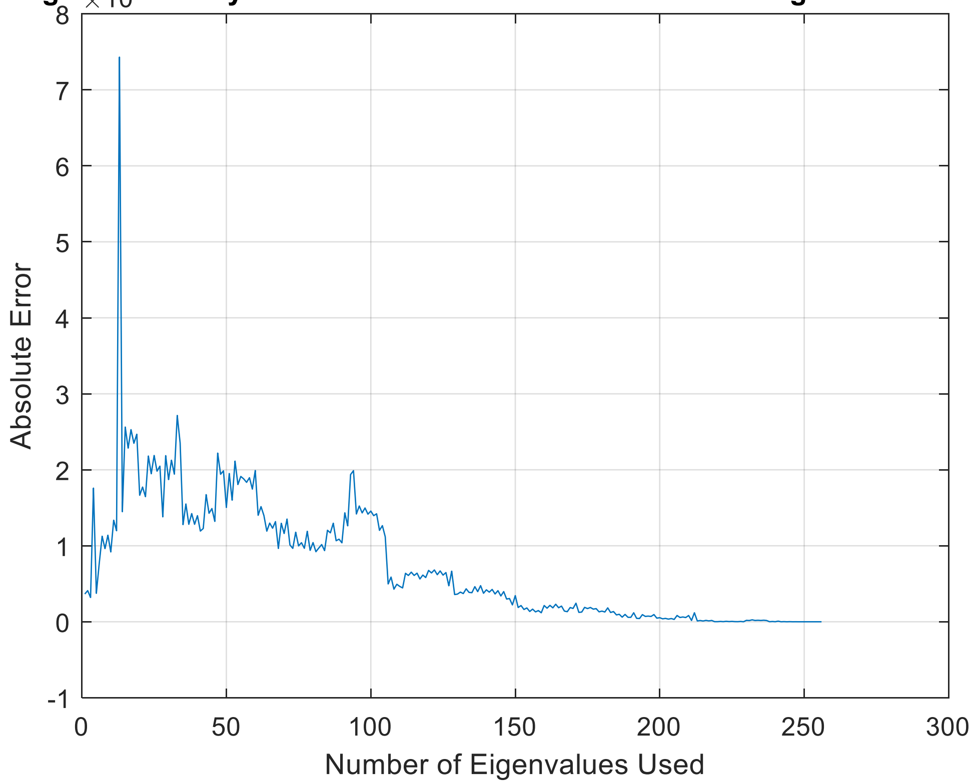


Figure 7: Re-synthesized Image Using First 100 Eigenvalues

Figure 8: Re-synthesized Image Using First 200 Eigenvalues

Figure 9: Re-synthesized Image Using All Eigenvalues

g) **Figure 10: Re-synthesis Absolute Error vs Number of Eigenvalues Used**



h) This problem demonstrated not only that the re-synthesis absolute error decreases as more eigenvalues are used, but that eigenvalues of greater magnitude make a bigger impact on the absolute error and the rate at which the original image can be fully re-synthesized. The clarity of the image increased and the absolute error decreased as more eigenvalues were added to the re-synthesis algorithm. The visual difference between Figure 8 and Figure 9 is miniscule, to the point where throwing away low-magnitude eigenvalues could be seen as a crude image compression algorithm.

Appendix 2: MATLAB Code for Problem 2

```
%% Aharon Sebton - Advanced Engineering Mathematics Project 3 - Problem 2
%% Problem 2 - Part a
clear all; clc;                                          % Clear all variables and
command window
I=imread('cameraman.tif');                               % Store image from graphics
file as matrix of grayscale values
figure(1)                                                % Create new figure window
imshow(I)                                                % Display grayscale image
title('Figure 1: Original cameraman.tif Grayscale Image')   % Give the image a title
%% Problem 2 - Part b
[M,D]=eig(double(I));                                    % Compute the eigenvalues and
eigenvectors of image I
figure(2)                                                % Create new figure window
plot(real(D),imag(D),'b*')                               % Plot eigenvalues on the
complex plane
xline(0,'b')                                             % Plot line along y-axis
yline(0,'b')                                             % Plot line along x-axis
xlabel('Real Part')                                      % Label x-axis on plot
ylabel('Imaginary Part')                                 % Label y-axis on plot
title('Figure 2: Plot of Eigenvalues of Image')          % Give the plot a title
grid on                                                  % Turn on the grid
%% Problem 2 - Part c
figure(3)                                                % Create new figure window
plot(abs(D),'b*')                                        % Plot the magnitude of the
eigenvalues
xline(0,'b')                                             % Plot line along y-axis
yline(0,'b')                                             % Plot line along x-axis
xlabel('Eigenvalues')                                    % Label x-axis on plot
ylabel('Magnitudes')                                     % Label y-axis on plot
title('Figure 3: Magnitude of Eigenvalues of Image')     % Give the plot a title
grid on                                                  % Turn on the grid
%% Problem 2 - Part d
[maxEV,maxLinIndex]=max(abs(D),[],'all','linear');       % Find the largest eigenvalue
[maxRow,maxCol]=ind2sub(size(D),maxLinIndex);            % Translate linear index into
row and column indices
D1=zeros(size(D));                                       % Create new, empty
eigenvalue matrix
D1(maxRow,maxCol)=D(maxRow,maxCol);                      % Copy largest eigenvalue
from original D matrix
I1=M*D1*inv(M);                                          % Re-synthesize image matrix
using D matrix with only the largest eigenvalue
figure(4)                                                % Create new figure window
imshow(uint8(I1))                                        % Display re-synthesized
image
title('Figure 4: Re-synthesized Image Using First Eigenvalue')  % Give the plot a title
error=sum(abs(I1)-abs(double(I)),'all')                  % Elementwise-subtract the
absolute values of each matrix, and sum the differences
%% Problem 2 - Part e
newD=zeros(size(D));                                     % Create new, empty
eigenvalue matrix
error=zeros(1,length(D));                                % Create array to track
calculated absolute error as eigenvalues are added
for i=1:length(D)                                        % For each eigenvalue,
    [maxEV,maxLinIndex]=max(abs(D),[],'all','linear');   % Find the largest eigenvalue
remaining in D matrix
    [maxRow,maxCol]=ind2sub(size(D),maxLinIndex);        % Translate linear index into
row and column indices
    newD(maxRow,maxCol)=D(maxRow,maxCol);                % Copy largest eigenvalue
from original D matrix
```

```
    D(maxRow,maxCol)=0;                                         % Remove largest eigenvalue
from original D matrix, allowing us to find the next-largest eigenvalue upon each loop
iteration
    newI=M*newD*inv(M);                                         % Re-synthesize image matrix
using D matrix with i largest eigenvalues
    error(i)=sum(abs(newI)-abs(double(I)),'all');               % Elementwise-subtract the
absolute values of each matrix, and sum the differences
    switch i                                                    % Use switch cases to extract
only select re-synthesized images
        case 1                                                  % If only the largest
eigenvalue was used
            I1=uint8(newI);                                     % Convert the image values
and store them before they are overwritten
        case 5                                                  % If the largest five
eigenvalues were used
            I5=uint8(newI);                                     % Convert the image values
and store them before they are overwritten
        case 50                                                 % If the largest fifty
eigenvalues were used
            I50=uint8(newI);                                    % Convert the image values
and store them before they are overwritten
        case 100                                                % If the largest 100
eigenvalues were used
            I100=uint8(newI);                                   % Convert the image values
and store them before they are overwritten
        case 200                                                % If the largest 200
eigenvalues were used
            I200=uint8(newI);                                   % Convert the image values
and store them before they are overwritten
        case length(D)                                          % If all of the eigenvalues
were used
            Ifull=uint8(newI);                                  % Convert the image values
and store them before they are overwritten
    end
end
%% Problem 2 - Part f
figure(5)                                                       % Create new figure window
imshow(I5)                                                      % Display re-synthesized
image
title('Figure 5: Re-synthesized Image Using First Five Eigenvalues')    % Give the plot a
title
figure(6)                                                       % Create new figure window
imshow(I50)                                                     % Display re-synthesized
image
title('Figure 6: Re-synthesized Image Using First Fifty Eigenvalues')   % Give the plot a
title
figure(7)                                                       % Create new figure window
imshow(I100)                                                    % Display re-synthesized
image
title('Figure 7: Re-synthesized Image Using First 100 Eigenvalues')     % Give the plot a
title
figure(8)                                                       % Create new figure window
imshow(I200)                                                    % Display re-synthesized
image
title('Figure 8: Re-synthesized Image Using First 200 Eigenvalues')     % Give the plot a
title
figure(9)                                                       % Create new figure window
imshow(Ifull)                                                   % Display re-synthesized
image
title('Figure 9: Re-synthesized Image Using All Eigenvalues')           % Give the plot a
title
```

```
%% Problem 2 - Part g
numvalues=1:length(D);                                          % Create array of integers
from 1 to the total number of eigenvalues
figure(10)                                                      % Create new figure window
plot(numvalues,error)                                           % Plot the absolute error as
a function of the number of eigenvalues used
xlabel('Number of Eigenvalues Used')                            % Label x-axis on plot
ylabel('Absolute Error')                                        % Label y-axis on plot
title('Figure 10: Re-synthesis Absolute Error vs Number of Eigenvalues Used')   % Give
the plot a title
grid on                                                         % Turn on the grid
```