Legged Robots

*Mini Project 1

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***Index Terms*—component, formatting, style, styling, insert**

I. Introduction

This report encompasses the forward kinematics of a three-link biped. The derivation of the forward kinematics allows for the eventual development of a controller that enables stable gait. The robot is assumed to have a no-slip condition and is only in contact with the ground at one point at a time. The forward kinematics were derived for the point mass of the two legs, the point mass of the hip, the point mass of the torso and the foot end location all of which are shown in Figure 1.

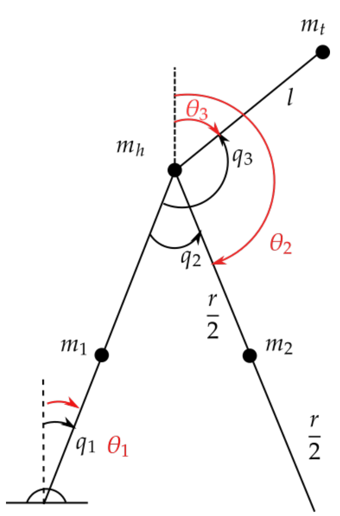

Fig. 1. Three Link Biped

II. Model Dynamics

A. Model Definition

For the three-link biped, there are 4 point masses depicted by the circles. Additionally, the angles can be defined as q_1 being the absolute angle of the stance leg, q_2 being the angle between the stance leg and the swing leg, and q_3 being the angle between the stance leg and the torso.We define these3 as the generalized co-ordinates of the system. The center of mass and the foot end position can then be defined as a function of the generalized co-ordinates. Additionally, the absolute angles for these links measured with respect to the positive vertical is taken to be as Θ

B. Forward Kinematics

The generalized co-ordinates is defined as vector $\boldsymbol{q}$,

$$\boldsymbol{q} = [q_1, q_2, q_3]$$

The absolute positions of the three links of the biped described in 1 Θ as a function of the generalized co-ordinates are shown below:

$$\Theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} q_1 \\ \pi - (q_2 - q_1) \\ \pi - (q_3 - q_1) \end{bmatrix} \tag{1}$$

The positions of the 4 center of masses are as follows,

$$\boldsymbol{P}_{m_1} = \begin{bmatrix} \frac{r}{2} * sin(q_1) \\ \frac{r}{2} * cos(q_1) \end{bmatrix} \tag{2}$$

$$\boldsymbol{P}_{m_h} = \begin{bmatrix} r * sin(q_1) \\ r * cos(q_1) \end{bmatrix} \tag{3}$$

$$\boldsymbol{P}_{m_2} = \boldsymbol{P}_{m_h} + \begin{bmatrix} \frac{r}{2} * sin(\theta_2) \\ \frac{r}{2} * cos(\theta_2) \end{bmatrix} \tag{4}$$

$$\boldsymbol{P}_{m_t} = \boldsymbol{P}_{m_h} + \begin{bmatrix} l * sin(\theta_3) \\ l * cos(\theta_3) \end{bmatrix} \tag{5}$$

$$\boldsymbol{P}_{f_2} = \boldsymbol{P}_{m_h} + \begin{bmatrix} r * sin(\theta_2) \\ r * cos(\theta_2) \end{bmatrix} \tag{6}$$

$$\boldsymbol{P}_{cm} = \frac{\Sigma m_i \boldsymbol{P}_i}{\Sigma m_i} \tag{7}$$

where i is index of each of the point masses.

The velocities can then be calculated with the help of the Jacobian relating the generalized velocities to the absolute velocities.

$$\boldsymbol{v}_{m_h} = \frac{\partial \boldsymbol{P}_{m_h}}{\partial \boldsymbol{q}} \left(\frac{d\boldsymbol{q}}{dt} \right) \tag{8}$$

$$\boldsymbol{v}_{m_t} = \frac{\partial \boldsymbol{P}_{m_t}}{\partial \boldsymbol{q}} \left(\frac{d\boldsymbol{q}}{dt} \right) \tag{9}$$

$$\boldsymbol{v}_{m_1} = \frac{\partial \boldsymbol{P}_{m_1}}{\partial \boldsymbol{q}} \left(\frac{d\boldsymbol{q}}{dt} \right) \tag{10}$$

$$\boldsymbol{v}_{m_2} = \frac{\partial \boldsymbol{P}_{m_2}}{\partial \boldsymbol{q}} \left(\frac{d\boldsymbol{q}}{dt} \right) \tag{11}$$

$$\boldsymbol{v}_{cm} = \frac{\Sigma m_i \boldsymbol{v}_i}{\Sigma m_i} \tag{12}$$

C. Lagrangian and dynamic equations of pinned model

To solve for the Lagrangian, the kinetic and potential energy of the system must be solved. To solve for the kinetic energy, the equation below is used:

$$K_{tot} = \sum_{i=1}^{N} K_i = \frac{1}{2} m_i (\boldsymbol{v}_i)^2 \tag{13}$$

and the potential energy can be solved using the equation:

$$V_{tot} = \sum_{i=1}^{N} V_i = m_i g(\boldsymbol{p}_i) \tag{14}$$

This can then be used to solve for the Lagrangian using the equation:

$$\mathcal{L} = K - V \tag{15}$$

The equations of motion can be then be written in the form

$$D(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - G(\boldsymbol{q}) = B\boldsymbol{u}, \tag{16}$$

where, D is the mass inertia tensor, C is the Coriolis matrix and G is the gravity vector. B is the vector that maps the external forces on to the generalized co-ordinates and $\boldsymbol{u}$ is a vector of inputs. If the full states are then defined as $\boldsymbol{x} = [\boldsymbol{q}, \dot{\boldsymbol{q}}]$, $\dot{\boldsymbol{x}}$ can be written in the following form,

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\boldsymbol{q}}_s \\ D^{-1}(C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - G(\boldsymbol{q}) + B\boldsymbol{u}) \end{bmatrix} \tag{17}$$

this can then be written as:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}$$

where,

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} \dot{q}_s \\ D^{-1}(C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - G(\boldsymbol{q}) \end{bmatrix} \tag{18}$$

$$\boldsymbol{g}(\boldsymbol{x}) = \begin{bmatrix} 0 \\ D^{-}1(B) \end{bmatrix} \tag{19}$$

To find $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ the D matrix must be solved for using the equation:

$$D = \sum_{i=1}^{N} m_i \frac{\partial \boldsymbol{P}_{m_i}}{\partial \boldsymbol{q}}^2$$

The Christoffel symbols

$$C_{k_j} = \sum_{i=1}^{\bar{N}} \frac{1}{2} \left(\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right) \dot{q}_i \tag{20}$$

The gravity vector $G(\boldsymbol{q})$ is calculated as;

$$G(\boldsymbol{q}) = \frac{\partial V(\boldsymbol{q})}{\partial \boldsymbol{q}}$$

D. Unpinned model with augmented states

To model the unpinned model, we augment the generalized co-ordinates q of the pinned model with an arbitrary fixed point on the three link biped, denoted by $\boldsymbol{p}_e$. The new co-ordinates are then defined as q_e. The dynamic model of this unpinned model is found by steps followed in II-C.