Legged Robots

*Mini Project 1

Kaushik Ventakesh
Northeastern University
360 Huntington Ave,
Boston, MA 02115
email address or ORCID

Sebastian Wicke
Northeastern University
360 Huntington Ave,
Boston, MA 02115
email address or ORCID

Ali Abbas
Northeastern University
360 Huntington Ave,
Boston, MA 02115
email address or ORCID

Ibrahim Abubakar
Northeastern University
360 Huntington Ave,
Boston, MA 02115
abubakar.i@northeastern.edu

***Index Terms*—component, formatting, style, styling, insert**

I. Introduction

This report encompasses the forward kinematics of a three-link biped. The derivation of the forward kinematics allows for the eventual development of a controller that enables stable gait. The robot is assumed to have a no-slip condition and is only in contact with the ground at one point at a time. The forward kinematics were derived for the point mass of the two legs, the point mass of the hip, the point mass of the torso and the foot end location all of which are shown in Figure 1.

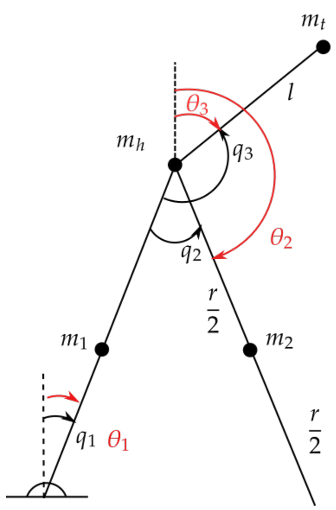

Fig. 1. Three Link Biped

II. Model Dynamics

A. Model Definition

For the three-link biped, there are 4 point masses depicted by the circles. Additionally, the angles can be defined as q_1 being the absolute angle of the stance leg, q_2 being the angle between the stance leg and the swing leg, and q_3 being the angle between the stance leg and the torso.We define these3 as the generalized co-ordinates of the system. The center of mass and the foot end position can then be defined as a function of the generalized co-ordinates. Additionally, the absolute angles for these links measured with respect to the positive vertical is taken to be as Θ

B. Forward Kinematics

The generalized co-ordinates is defined as vector q,

$$q = [q_1, q_2, q_3]$$

The absolute positions of the three links of the biped described in 1 Θ as a function of the generalized co-ordinates are shown below:

$$\Theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} q_1 \\ \pi - (q_2 - q_1) \\ \pi - (q_3 - q_1) \end{bmatrix} \tag{1}$$

The positions of the 4 center of masses are as follows,

$$P_{m_1} = \begin{bmatrix} \frac{r}{2} * sin(q_1) \\ \frac{r}{2} * cos(q_1) \end{bmatrix} \tag{2}$$

$$P_{m_h} = \begin{bmatrix} r * sin(q_1) \\ r * cos(q_1) \end{bmatrix} \tag{3}$$

$$P_{m_2} = P_{m_h} + \begin{bmatrix} \frac{r}{2} * sin(\theta_2) \\ \frac{r}{2} * cos(\theta_2) \end{bmatrix} \tag{4}$$

$$P_{m_t} = P_{m_h} + \begin{bmatrix} l * sin(\theta_3) \\ l * cos(\theta_3) \end{bmatrix} \tag{5}$$

$$P_{f_2} = P_{m_h} + \begin{bmatrix} r * sin(\theta_2) \\ r * cos(\theta_2) \end{bmatrix} \tag{6}$$

$$P_{cm} = \frac{\Sigma m_i P_i}{\Sigma m_i} \tag{7}$$

where i is index of each of the point masses.

The velocities can then be calculated with the help of the Jacobian relating the generalized velocities to the absolute velocities.

$$v_{m_h} = \frac{\partial P_{m_h}}{\partial q} \left(\frac{dq}{dt} \right) \tag{8}$$

$$v_{m_t} = \frac{\partial P_{m_t}}{\partial q}\left(\frac{dq}{dt}\right) \tag{9}$$

$$v_{m_1} = \frac{\partial P_{m_1}}{\partial q}\left(\frac{dq}{dt}\right) \tag{10}$$

$$v_{m_2} = \frac{\partial P_{m_2}}{\partial q}\left(\frac{dq}{dt}\right) \tag{11}$$

$$v_{cm} = \frac{\Sigma m_i v_i}{\Sigma v_i} \tag{12}$$