Engineering Tripos Part IIB F-YA331-1

Complex Biological Synapses for Unsupervised Learning in Non-Stationary Environments

Summary on Preparatory Work

Name: Yuhao Wang
College: Peterhouse
CRSid: yw451

1 Learning Rule

The learning rule used here is the Oja (1982) rule, which is a Hebbian rule with dynamic constraint. It can be expressed as

$$\tau_w \frac{d\boldsymbol{w}}{dt} = v\boldsymbol{u} - \alpha v^2 \boldsymbol{w}, \tag{1}$$

where τ_w is a time constant, $\boldsymbol{u}$ and v are inputs and output of the neuron, respectively, $\boldsymbol{w}$ is the input synapse weight vector, and α is a positive constant. It can be shown that with this learning rule, $|\boldsymbol{w}|$ over time will relax to the value $1/\alpha$. $\boldsymbol{u}$ and $\boldsymbol{w}$ are N dimensional vectors where N is the number of input synapses connected to the neuron.

2 Input Statistics

The input vector at each moment is generated by sampling a random vector with multivariate Gaussian distribution $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{C})$. For the time being $\boldsymbol{\mu}$ is kept at $\mathbf{0}$. $\boldsymbol{C}$ is constructed using

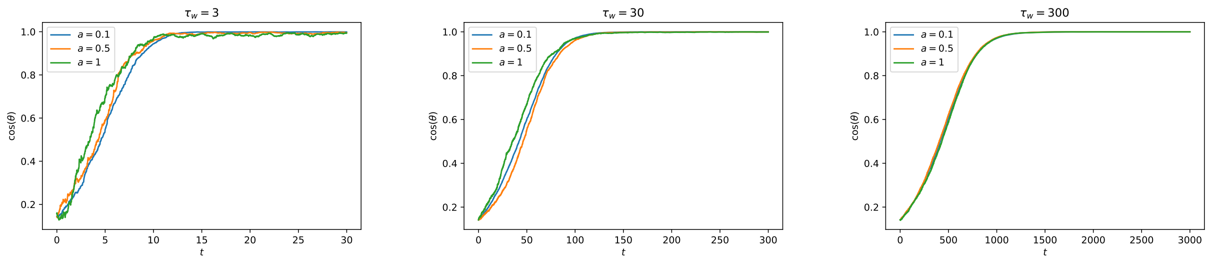
$$\boldsymbol{C} = \boldsymbol{e}_1^T \boldsymbol{e}_1 + a\boldsymbol{I}, \tag{2}$$

with $\boldsymbol{e}_1$ being some arbitrary unit (not necessary but for convenience) N-dimensional vector, a some constant and $\boldsymbol{I}$ the $N \times N$ identity matrix.

3 Results and Discussion

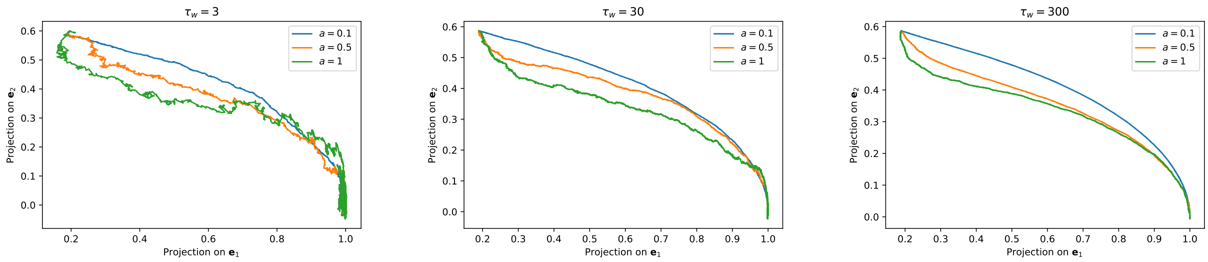
Figure 1 are plots of $\cos(\theta)$ against time, where θ is the angle between $\boldsymbol{w}$ and $\boldsymbol{e}_1$. Figure 2 are traces of $\boldsymbol{w}^T\boldsymbol{e}_1/|\boldsymbol{e}_1|$ plotted against $\boldsymbol{w}^T\boldsymbol{e}_2/|\boldsymbol{e}_2|$, where $\boldsymbol{e}_2$ is some arbitrary unit vector orthogonal to $\boldsymbol{e}_1$. Note that to generate each of these plots, $\boldsymbol{w}$ is started from the same initial condition at $t = 0$ and the input statistics is also kept constant both throughout each trial and across different trials. $\alpha = 1$ throughout the experiment.

As predicted by theoretical analysis (see Dayan and Abbott), these figures show that the direction of $\mathbb{E}\{\boldsymbol{w}(t)\}$ approaches that of $\boldsymbol{e}_1$ as $t \to \infty$. τ_w dominantly influences the speed of convergence (note the different scaling of time axes in Figure 1). a influences the variance of



(a) (b) (c)

Figure 1

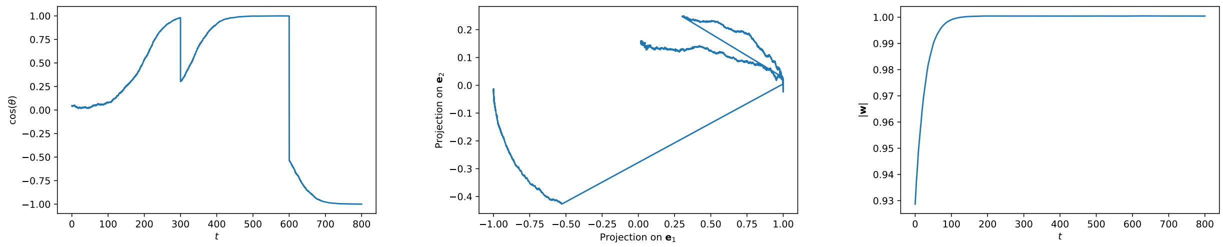


(a) (b) (c)

Figure 2

$\boldsymbol{w}$, as it changes the relative magnitude of eigenvalues of other eigenvectors compared to the eigenvalue of $\boldsymbol{e}_1$. To be specific, $\lambda_1 = 1 + a$, $\lambda_2 = \cdots = \lambda_N = a$. For the same τ_w, larger values of a gives greater variance.

τ_w also affects variance, as a slower dynamics more effectively filters out the higher frequency features of the input. The larger τ_w is, the more similar is the neuron's response to that generated by an averaged learning rule.



(a) (b) (c)

Figure 3

In the trial shown in Figure 3, $\boldsymbol{C}$ is no longer kept constant throughout the trial – $\boldsymbol{e}_1$ is randomly given a new value every T_e time units. The constants used are $\tau_w = 50$, $a = 1$, $T_e = 300$.

Figure 3(a)(b) are the same plots as explained before, and are intuitive to understand. In addition, $|\boldsymbol{w}|$ is plotted against time in Figure 3 (c) to show the effect of dynamic constraint discussed earlier in Section 1. It can be seen that this convergence of $|\boldsymbol{w}|$ to $1/\alpha$ is not disturbed by changes in input statistics.