Traffic signal control – a discrete-time linear quadratic DP formulation with infinite-horizon

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Overview

1. Model
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   - DP Setting
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   - Constrained problem
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Motivation

- Saturated road conditions call for a constant improvement in the traffic control
- Limitiations include:
  - Constantly increasing traffic
  - Limited urban space
  - Cost of building new transportation junctions
- Traffic light control of existing infrustructe pays therefore a key role in the urban optimisation of traffic
- How can it be modelled? BY DYNAMIC PROGRAMMING techniques

Optimasation problem

The problem of traffic optimasation is two-stage:

1. Unconstrained LQ problem
   - Concentrated on speficying the urban network, which is represented as a directed graph with links (approaches) and junctions
   - Bases on specification of that network - in-flow, out-flow, links, junctions, traffic
   - Riccati equations deliver a direct solution
2. Constrained problem
   - LQ-methodology used in the first step disregards the control constraints
   - Those constraints are imposed and solved in real-time for each junction so as to specify feasible green times
   - With the newly set constraint, the aim is to reach for the optimum obtained in the unconstrained problem

LQ model - Dynamics

$$x_z(k+1) = x_z(k) + T\left[q_z(k) - s_z(k) + d_z(k) - u_z(k)\right] \tag{1}$$

Using traffic control variables:

$$\begin{aligned} x_z(k+1) &= x_z(k) + \\ &+ T\left[(1 - t_{z,0})\sum_{w\in I_M} \frac{t_{w,z}S_w\left(\sum_{i\in v_w}\Delta g_{M,i}(k)\right)}{C} + \Delta d_z(k) - \frac{S_z\left(\sum_{i\in v_z}\Delta g_{N,i}(k)\right)}{C}\right] \end{aligned} \tag{2}$$

Finally in matrix notation:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\Delta\mathbf{g}(k) + \mathbf{T}\Delta\mathbf{d}(k) \tag{3}$$

LQ model - Cost

$$\mathcal{J} = \frac{1}{2} \sum_{k=0}^{\infty} \left(\|\mathbf{x}(k)\|_{\mathbf{Q}}^2 + \|\Delta \mathbf{g}(k)\|_{\mathbf{R}}^2 \right) \tag{4}$$

Here **Q** and **R** are non-negative definite, diagonal weighting matrices

LQ model - Solution

The discrete-time dynamic Riccati equation of this problem:

$$X = Q + A^T XA - \left(A^T XB\right)\left(R + B^T XB\right)^{-1}\left(B^T XA\right) \tag{5}$$

Solution to this problem is therefore given by a matrix (called the control) **L**:

$$\mathbf{L} = \left(B^T XB + R\right)^{-1} B^T XA \tag{6}$$

Putting it into DP framework:

$$\Delta g^* = -\left(B^T XB + R\right)^{-1}\left(B^T XA\right) x_{k-1} \tag{7}$$

And can equivalently by written as:

$$\mathbf{g}(k) = \mathbf{g}^{\mathrm{N}} - \mathbf{L}\mathbf{x}(k) \tag{8}$$

where $\Delta \mathbf{g} = \mathbf{g}(k) - \mathbf{g}^{\mathrm{N}}$.

Constrained problem

$$\min G_{j,i} \sum_{i \in F_j} (g_{j,i} - G_{j,i})^2 \tag{9}$$

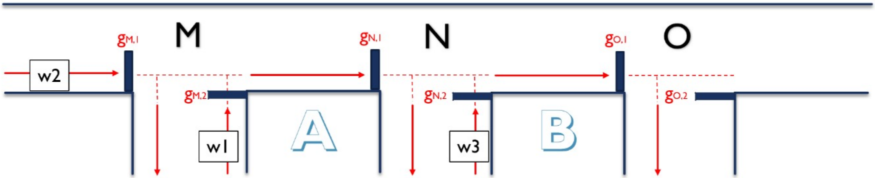
subject to

$$\sum_{i \in F_j} G_{j,i} + |L_j = C \tag{10}$$

$$G_{j,i} \in [g_{j,i,\min}, g_{j,i,\max}] \, \forall i \in F_j \tag{11}$$

Simulation - Network

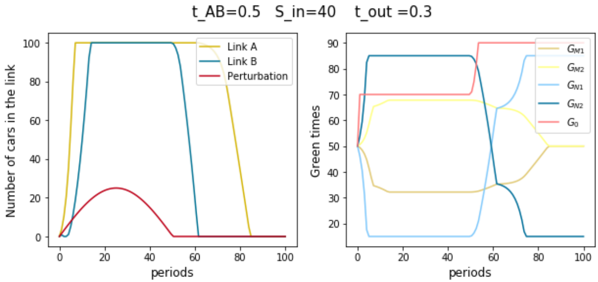
Figure: Network Graph



Simulated toy network of three junctions (O,M,N) with two connecting links (A,B) and 5 traffic lights

Results - Base scenario

1. Initially, the green times are all set to 50 seconds, all links are empty
2. 50% of cars in link A turn to link B
3. The saturation flow for lateral link is 40
4. 30% of cars reach their destination at every link i.e. exit the network
5. Perturbation persists until period 50

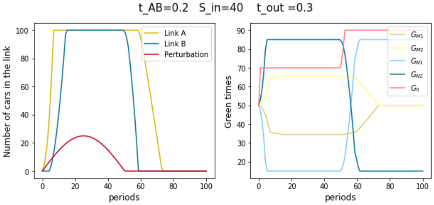


Both links become saturated quickly, and remain so until the perturbations subside. Link A remains saturated for longer.

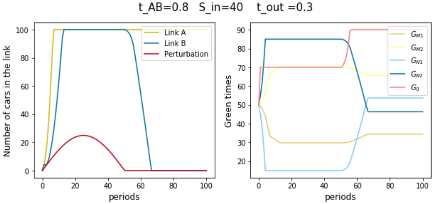
Results - changing turning rate

Increasing **the turning rate** from A to B is equivalent to increasing the percentage of cars that flow from link A to link B.

Low turning rate:



High turning rate:

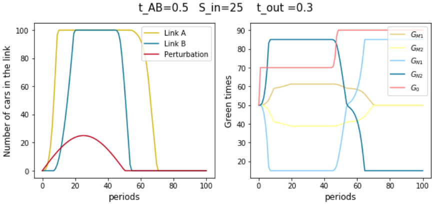


Clearly, the lower the turning rate, the lower the number of cars in link A waiting to pass to B.

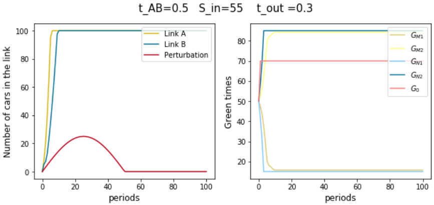
Results - changing saturation inflow

Modifying **the saturation flow** is equivalent to changing the number of cars that enter the network from the lateral roads.

Low saturation flow:



High saturation flow:

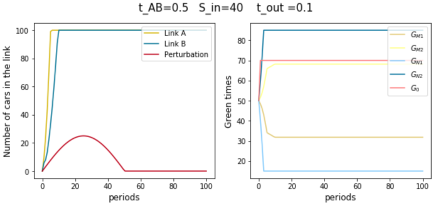


When the saturation coefficient goes beyond 50, both links congest. The r.o.w times of the lateral roads increase until their feasible maximum.

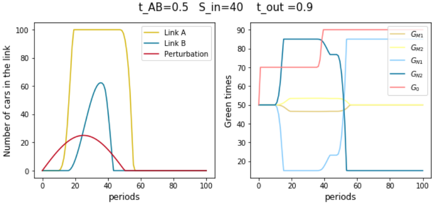
Results - changing outflowing traffic

Modifying **the outflowing traffic** to assess the effect of letting more cars out the system from the lateral roads.

Low outflow:



High outflow:

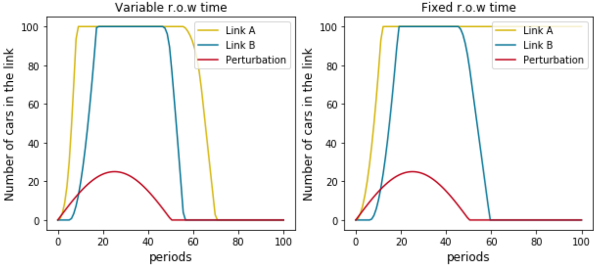


When the total amount of cars leaving the system is less than 10% the network gets congested and the r.o.w times do not fluctuate.

Results - comparison varying vs fixed r.o.w times

One may wonder if varying r.o.w time as a function of the link occupancy really improves network efficiency.

Figure: Variable vs. fixed green times



The differences in number of cars for each period for both policies are apparent. The TUC strategy is more efficient than setting the lights to a fixed time. Time varying policy empties the link A before the 100th period. Fixed policy struggles to do so.

Conclusions

1. The control matrix L derived in the TUC strategy provides a control law with a robust gating feature to protect down-stream links from oversaturation.
2. The higher the number of vehicles within a particular link the lower the green times of the links that lead into it are set.
3. The regulator has a reactive rather than anticipatory behaviour, no forecast is taken into account
4. The time-varing policy improves network efficiency, whereas fixed-time policy struggles to empty the link.