DPproject

March 31, 2019

0.1 Study of TUC policy for a simple network

```
In [1]: import numpy as np
        import copy
        import math
        import scipy.linalg as la
        from scipy.optimize import minimize
        from scipy.optimize import Bounds
        from scipy.optimize import LinearConstraint
        import matplotlib.pyplot as plt
        import seaborn as sns
```

```
In [2]: ### INITIALIZING NETWORK

        def init():

            # Setting size of the network
            j = 2 # number of junctions
            z = 3 # number of links per junction
            i = 2 # number of inflowing links per junction, has to be smaller than z
            o = 1 # number of outflow links

            # Setting time frame of the network
            C = np.full((j),120) # cycle time
            L = np.full((j),20) # lost time
            T = 5 # control cycle

            # Turning rates
            Tu0 = np.ones(j) * 0.3 # turners who leave the system

            Tu = np.ones((j,i,o))
            Tu[1,0] = 0.5 # t_AB QUantity of cars that go from link A to link B

            # Saturation flows
            S = np.full((j,z),40) # Saturation flow for each incoming link
            S[0,0]=30 # S_w1 incoming street
            S[0,1]=40 # S_w2 lateral streets smaller
            S[1,1]=40 # S_w3 lateral streets smaller
```

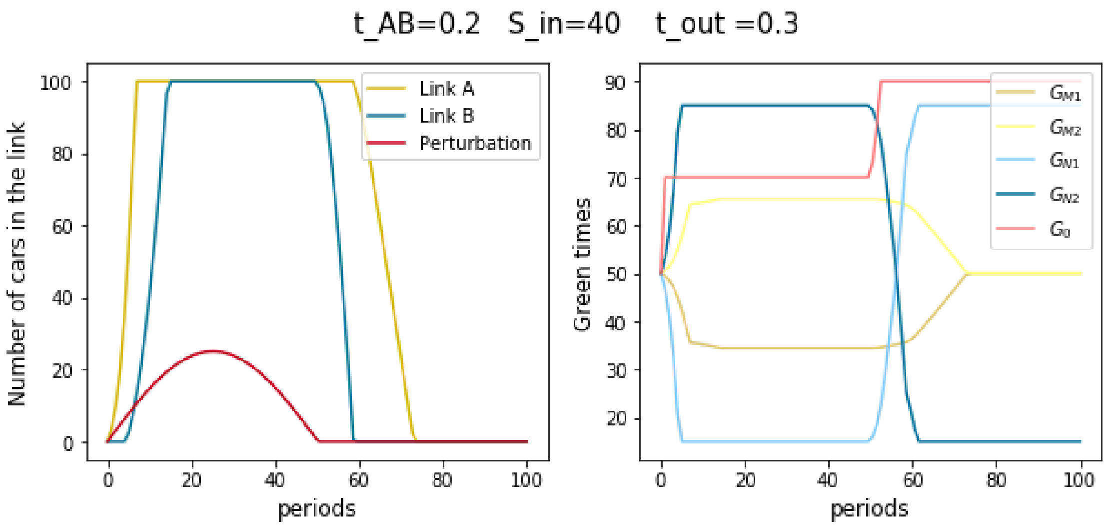
```python
    return (Tu0,Tu,C,S,L,T,j,o)


def compute_X_G(Tu0,Tu,C,S,L,T,j,o,time_var=True):

    # System matrices
    A1 = np.zeros(j) # initialise one A1 for each junction
    A2 = np.zeros(j) # initialise one A2 for each junction
    A3 = np.zeros(j) # initialise one A3 for each junction


    # Defining links and junction interactions
    for junction in range(j):
        A1[junction] = (1-Tu0[junction]) *
        (Tu[junction,0,0] * S[junction,0] / C[junction])

        A2[junction] = (1-Tu0[junction]) *
        (Tu[junction,1,0] * S[junction,1] / C[junction])

        A3[junction] = - (S[junction,2] / C[junction])


    # Riccati matrices
    A = np.identity(j)
    B = np.array([[A1[0], A2[0], A3[0]   ,0      ,  0     ],
                  [0     , 0     , A1[1]   ,A2[1] ,  A3[1]]])
    Q = np.identity(2)/100
    R = np.identity(5)*1e-4

    # Solving Riccati equations

    aux = la.solve_discrete_are(A,B,Q,R) # Solving Riccati equations
    L_ric = -la.inv(R + B.T.dot(aux).dot(B)) @ B.T.dot(aux).dot(A) # Finding L

    # Defining feasible green times boundaries
    bounds = ((15,85),(15,85),(15,85),(15,85),(70,90)) if time_var
    else ((50,50),(50,50),(50,50),(50,50),(70,70))

    # Constraint to ensure all signals stay within the defined cycle time.
    linear_constraint = ({'type': 'eq', 'fun': lambda x:  x[0]
     + x[1] + L[0] - C[0]}, # Link 1
                          {'type': 'eq', 'fun': lambda x: x[2]
                          + x[3] + L[1] - C[1]}) # Link 2
                          #{'type': 'eq', 'fun': lambda x: x[4] - 65}) # Exit link

    # Initializing green times and number of cars per link
    G = np.full((1,5),50)
```

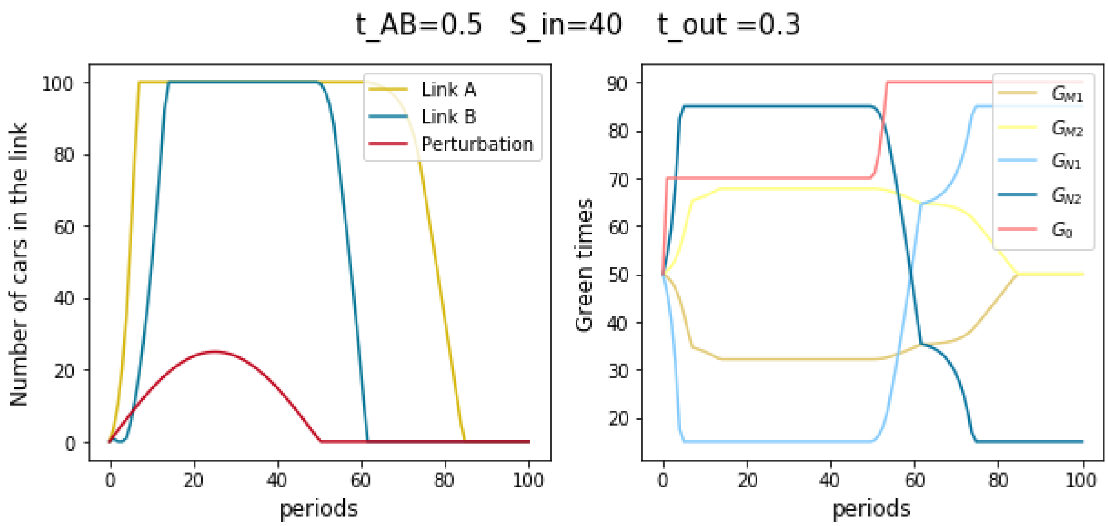
```python
    X = np.full((1,2),0)

    # Defining perturbation
    k = np.linspace(0,100,num=100)
    perturbations = 25*np.sin(6.25 * k/100)
    perturbations[perturbations < 0] = 0

    # Minimization to find feasible gs as close as possible to the optimal gs
    for k in range(100):

        _ = A @ X[-1] + (B @ G[-1,:]) + perturbations[k]
        _[_<0] = 0
        _[_>100] = 100

        X = np.vstack((X,_))
        G = np.vstack((G,G[-1:,]-L_ric @ (X[-1]-X[-2])))

        g = G[-1]

        min_f = minimize(greens, # function to be minimized
                         x0=g, # init point
                         args = g,
                         method='SLSQP', # Sequential Least Squares
                         jac = greens_grad, # fact_gradient of the function
                         hess = hess, # Hessian
                         constraints = linear_constraint, # Linear constraints
                         bounds = bounds) # Bounds

        G[-1] = min_f.x

    return (X,G,perturbations)

# Function to be minimized f
def greens(fact_g,*g):
    return (np.sum((g-fact_g)**2))

# fact_gradient of f
def greens_grad(fact_g,*g):
    return (-2*(g-fact_g))

# Hessian of f
def hess(fact_g):
    return (np.identity(5)*2)


def make_plots(X,G,params,perturbations):

    k = np.linspace(0,100,num=100)
```

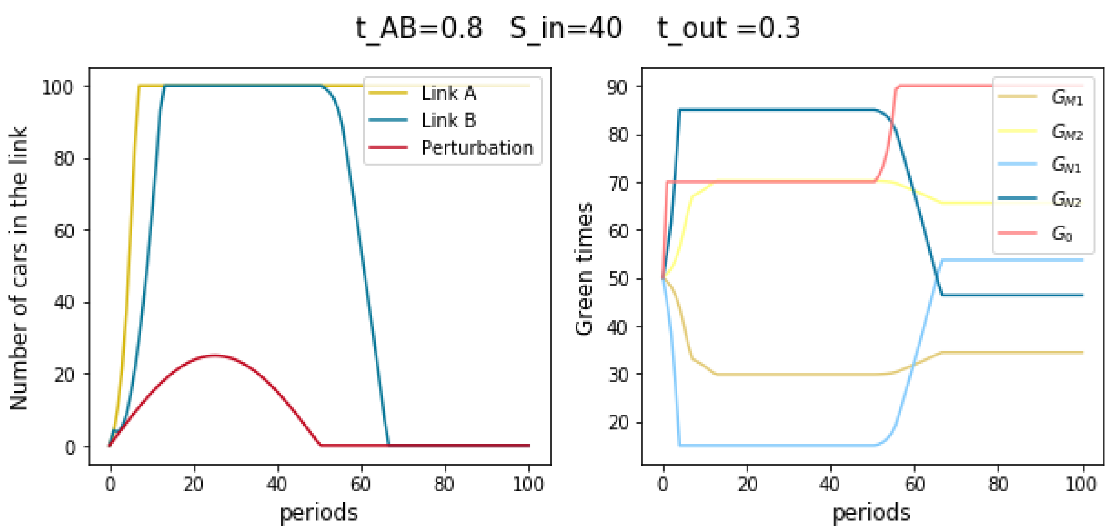
```
    colors_links = [sns.xkcd_rgb["dark yellow"],sns.xkcd_rgb["sea blue"],
    sns.xkcd_rgb["scarlet"]]
    colors_lights = [sns.xkcd_rgb["sand"],sns.
    xkcd_rgb["pale yellow"],sns.xkcd_rgb["sky"],
    sns.xkcd_rgb["ocean blue"],sns.xkcd_rgb["salmon pink"]]

    fig,axes = plt.subplots(nrows=1,ncols=2,figsize = (10,4))
    axes[0].set_prop_cycle('color',colors_links)
    axes[0].plot(k,X[:-1,:])
    axes[0].plot(k,perturbations)
    axes[0].set_xlabel("periods",fontsize=12)
    axes[0].set_ylabel("Number of cars in the link",fontsize=12)
    axes[0].legend(["Link A","Link B","Perturbation"],loc=1)


    axes[1].set_prop_cycle('color',colors_lights)
    axes[1].plot(k,G[:-1,:])
    axes[1].set_xlabel("periods",fontsize=12)
    axes[1].set_ylabel("Green times",fontsize=12)
    axes[1].legend(["$G_{M1}$","$G_{M2}$","$G_{N1}$","$G_{N2}$","$G_{0}$"],loc=1)

    fig.suptitle(f"t_AB={params[0]}   S_in={params[1]}    t_out ={params[2]}",
    fontsize = 15)
    fig.savefig(f"t_AB={params[0]}   S_out={params[1]}    t_out ={params[2]}",
    bbox_inches='tight',format="png")

def make_plots2(X1,X2,params,perturbations):

    k = np.linspace(0,100,num=100)
    colors_links = [sns.xkcd_rgb["dark yellow"],sns.xkcd_rgb["sea blue"],sns.
    xkcd_rgb["scarlet"]]
    colors_lights = [sns.xkcd_rgb["sand"],sns.xkcd_rgb["pale yellow"],sns.
    xkcd_rgb["sky"],sns.xkcd_rgb["ocean blue"],sns.xkcd_rgb["salmon pink"]]

    fig,axes = plt.subplots(nrows=1,ncols=2,figsize = (10,4))
    axes[0].set_prop_cycle('color',colors_links)
    axes[0].plot(k,X1[:-1,:])
    axes[0].plot(k,perturbations)
    axes[0].set_xlabel("periods",fontsize=12)
    axes[0].set_ylabel("Number of cars in the link",fontsize=12)
    axes[0].set_title("Variable r.o.w time")
    axes[0].legend(["Link A","Link B","Perturbation"],loc=1)

    axes[1].set_prop_cycle('color',colors_links)
    axes[1].plot(k,X2[:-1,:])
    axes[1].plot(k,perturbations)
    axes[1].set_xlabel("periods",fontsize=12)
    axes[1].set_ylabel("Number of cars in the link",fontsize=12)
```

```
    axes[1].set_title("Fixed r.o.w time")
    axes[1].legend(["Link A","Link B","Perturbation"],loc=1)

    fig.savefig(f"t_AB={params[0]}-S_out={params[1]}-t_out ={params[2]}
    _var_not_var",bbox_inches='tight',format="png")
```

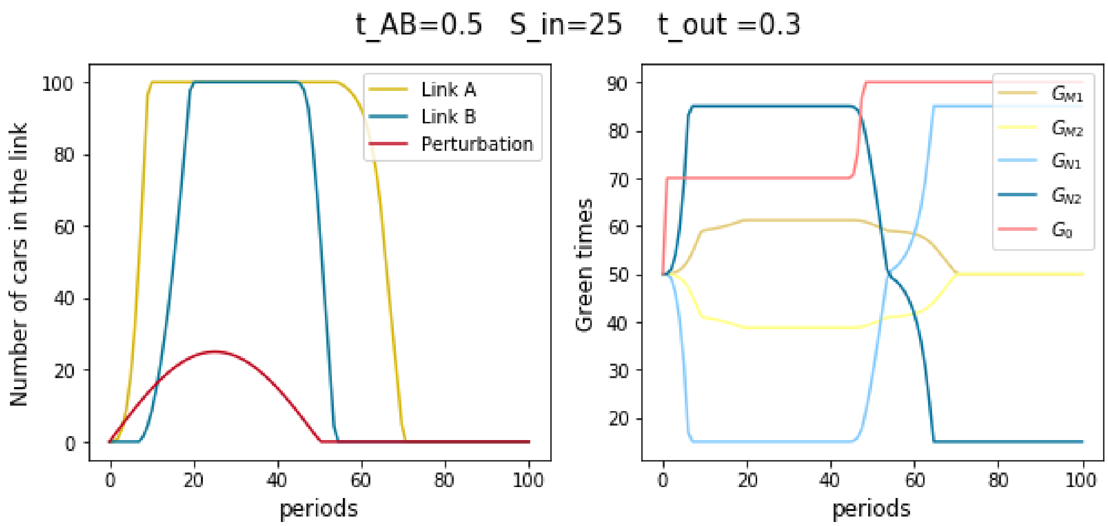
```
In [3]: # Modifying turnig rates AB
        Tu0,Tu,C,S,L,T,j,o = init()

        params =[0.2,0.5,0.8]
        for param in params:

            Tu[1,0] = param

            X,G,perturbations = compute_X_G(Tu0,Tu,C,S,L,T,j,o)
            make_plots(X,G,[Tu[1,0][0],S[0,1],Tu0[0]],perturbations);
```

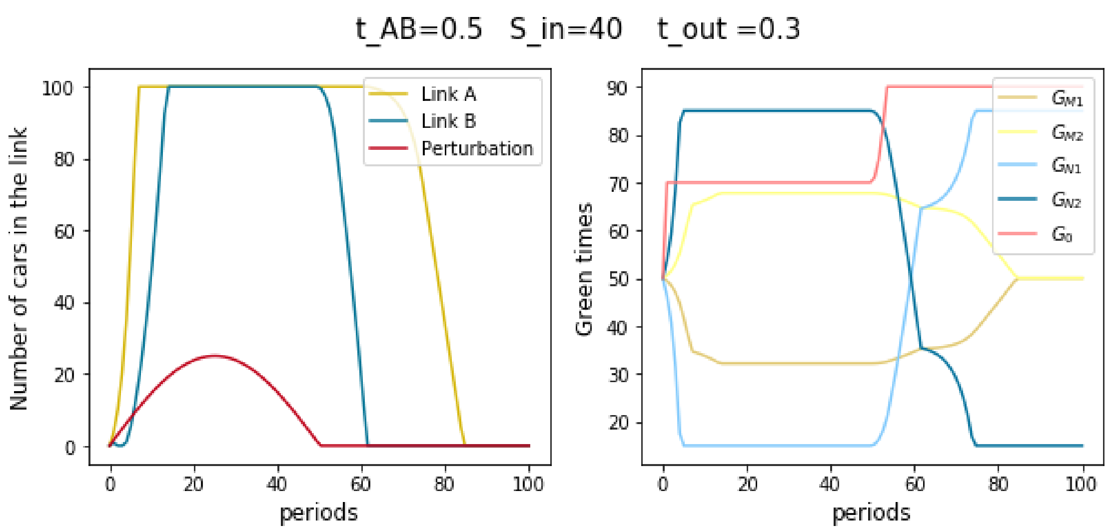


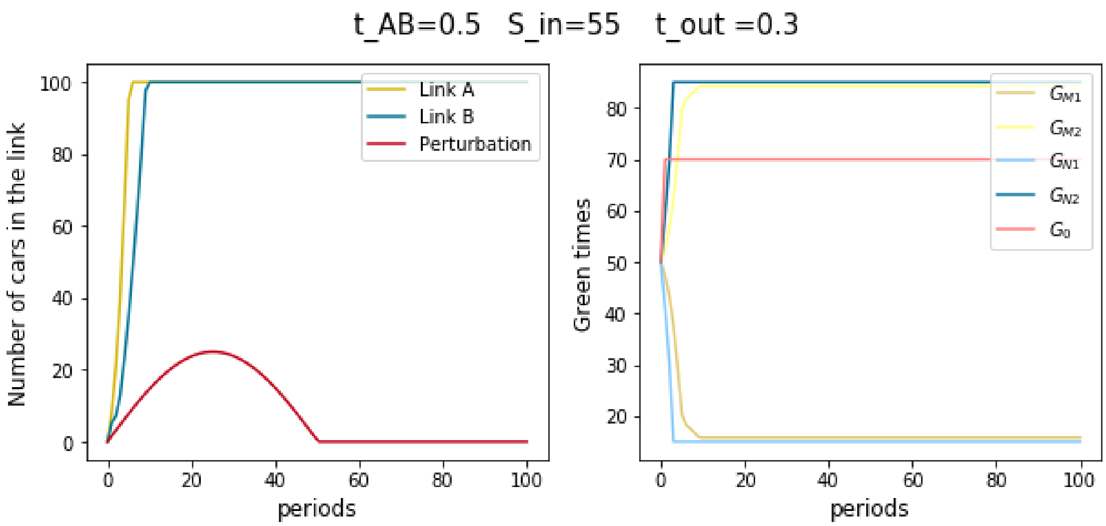




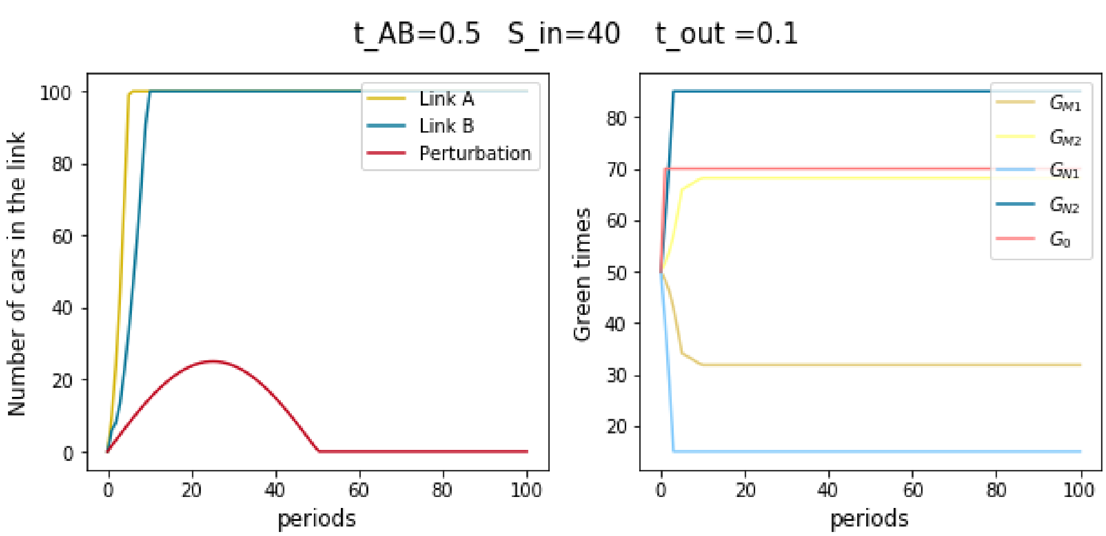
In [4]:
```
# Modifying lateral streets saturation coef
Tu0,Tu,C,S,L,T,j,o = init()

params =[25,40,55]
for param in params:

    S[0,1]=param # S_w2 lateral streets smaller
    S[1,1]=param # S_w3 lateral streets smaller = param

    X,G,perturbations = compute_X_G(Tu0,Tu,C,S,L,T,j,o)
    make_plots(X,G,[Tu[1,0][0],S[0,1],Tu0[0]],perturbations);
```

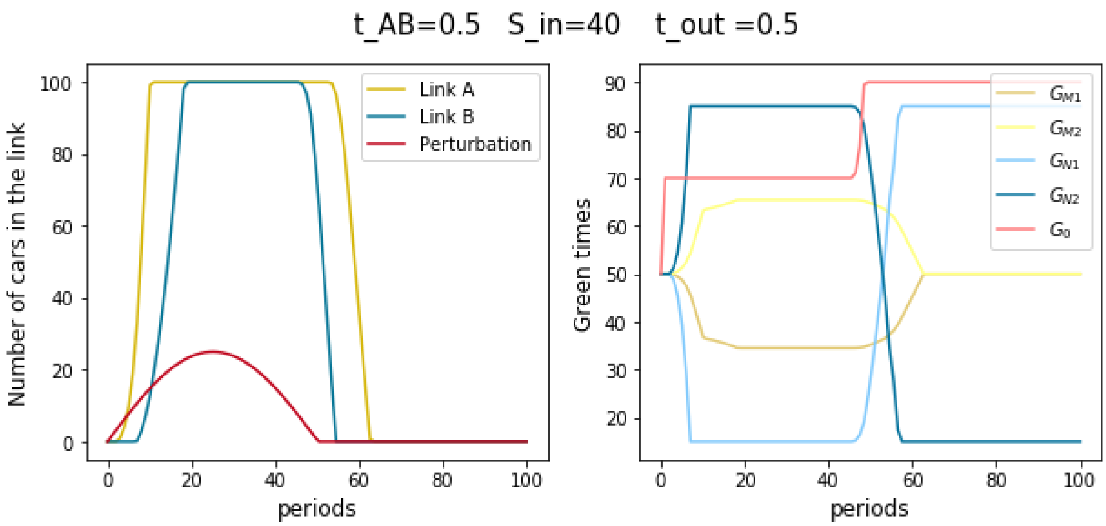

t_AB=0.5 S_in=25 t_out =0.3
Number of cars in the link
periods
Link A
Link B
Perturbation
Green times
periods
G_{M1}
G_{M2}
G_{N1}
G_{N2}
G_0

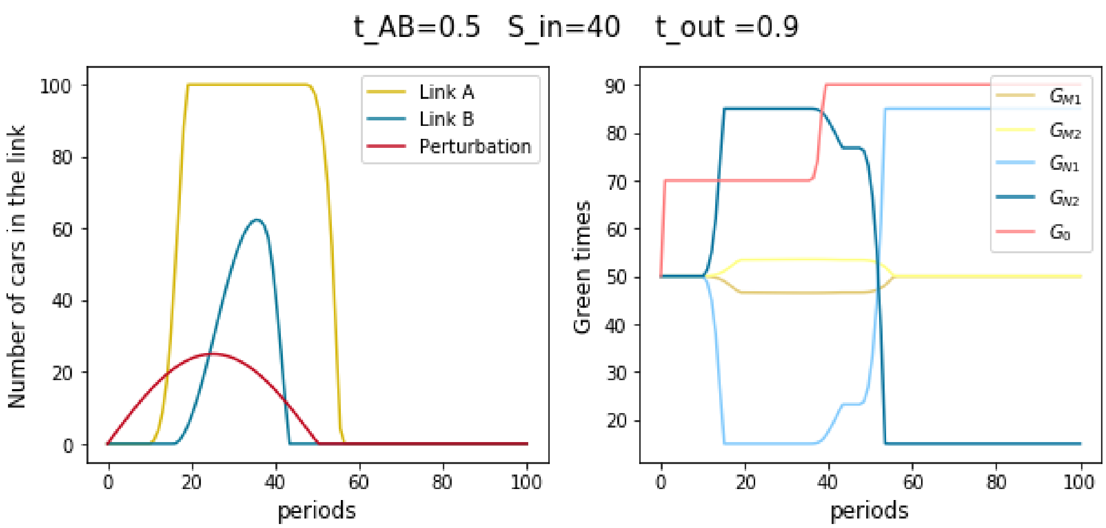

t_AB=0.5 S_in=40 t_out =0.3
Number of cars in the link
periods
Link A
Link B
Perturbation
Green times
periods
G_{M1}
G_{M2}
G_{N1}
G_{N2}
G_0



In [5]:
```
# Modifying exit rate
Tu0,Tu,C,S,L,T,j,o = init()

params =[0.1,0.5,0.9]
for param in params:

    Tu0= np.ones(len(Tu0)) * param #  Exit rate

    X,G,perturbations = compute_X_G(Tu0,Tu,C,S,L,T,j,o)
    make_plots(X,G,[Tu[1,0][0],S[0,1],Tu0[0]],perturbations);
```








```
In [6]: # Checking efficiency variable time r.o.w vs fixed r.o.w time

        Tu0,Tu,C,S,L,T,j,o = init()

        params =[30]
        for param in params:

            S[0,1]=param # S_w2 lateral streets smaller
            S[1,1]=param # S_w3 lateral streets smaller = param

            X1,G1,perturbations = compute_X_G(Tu0,Tu,C,S,L,T,j,o)
```

```
X2,G2,perturbations = compute_X_G(Tu0,Tu,C,S,L,T,j,o,time_var=False)
make_plots2(X1,X2,[Tu[1,0][0],S[0,1],Tu0[0]],perturbations);
```