Quantitative Macroeconomics
Homework 4.

Sebastian Zalas

December 22, 2020

<u>Instructions on how to replicate results:</u> [to do]

II. 1. The recursive formulation of the problem.

<u>Question:</u> Formulate the problem of the agent recursively, i.e. write down Bellman's equation and derive the stochastic Euler equation.

<u>Answer:</u> Bellman equation is given by:

$$V(a,y) = \max_{c,a'}\{U(c) + \beta\mathbb{E}[V(a',y')]\} \tag{1}$$

subject to:

$$c + a' \geq (1+r)a + wy \tag{2}$$

First order condition of the problem:

$$U(c') + \beta\mathbb{E}[V'(a',s')](-1) = 0 \tag{3}$$

Envelope condition:

$$V'(a,s) = \beta\mathbb{E}[V'(a',s')](1+r) = 0 \tag{4}$$

Merge conditions 3 and 4 by substituting for $\beta\mathbb{E}[V'(a',s')]$:

$$V'(a,s) = U'(c)(1+r) \tag{5}$$

Iterate forward equation 5:

$$\mathbb{E}[V'(a,s)] = \mathbb{E}[U'(c')(1+r')] \tag{6}$$

Substitute in equation 6 for :

$$U'(c) = \beta\mathbb{E}[U'(c')(1+r')] \tag{7}$$

Hence I obtained Euler equation.

II. 2. The infinitely-lived households economy.

Code contained in file **partial**$_e q$

II. 3. The life-cycle economy.

Code contained in file **partial**$_e q$

II. 4. Partial Equilibrium.

<u>Question</u>: **Use the code from above to answer the following: Let $\sigma = 2$ and $\bar{c} = 100$, and the borrowing constraint equal to the natural borrowing limit.**

II.4.1. With certainty.

<u>Question</u>: **Fist, let $\gamma = 0$ and $\sigma_y = 0$, that is, there is no uncertainty.**

1. <u>Question</u>: **For $T = \infty$ plot the consumption function(s). On the x-axis should be a, on the y-axis c(a; y1) and c(a; y2) for both preference specifications. Also generate a time profile of consumption by choosing a0 as starting assets and by using the policy functions c(a; y) and a0(a; y).**

   <u>Answer</u>: **On Figure 3 I present requested plots. Comparing two cases of utility specification, plots do not reveal any significant differences.**

2. <u>Question</u>: **Do the same as in the previous question, but now with T = 45. For the consumption function plots pick two ages, say plot c5(a; y) and c40(a;y).**

   <u>Answer</u>: **On Figure 4 I present requested plots. The difference occur in initial draws, but then infinite case and with T = 45 converge to the same levels, comparing two utility specifications.**

II.4.2. With uncertainty.

<u>Question:</u> Now, let $\gamma = 0$ and $\sigma_y = 0.1$.

1. <u>Question:</u> Plot and compare the consumption functions (for each y plot c(a; y) against a) under certainty equivalence (quadratic case) with the consumption function derived in the presence of a precautionary saving motive. Are the differences more pronounced for T $=$ 1 or T $=$ 45 and e why? How do they compare to what you found in the case of certainty.
   <u>Answer:</u> On Figure 5 I present requested plots.

2. <u>Question:</u> Present and compare representative simulated time paths of consumption for the certainty equivalence and precautionary saving economy. On the x-axis should be time, on the y-axis the income shock and the consumption realization. You may limit yourself to the T $=$ 45 case.
   <u>Answer:</u> On Figure 6 I present requested plots.

3. <u>Question:</u> Increase prudence by increasing $\sigma = 2$ to $\sigma = 5$ and $\sigma = 20$. How much do your answers change and why?
   <u>Answer:</u> On Figures 7 and 8 I present requested plots.

4. <u>Question:</u> Increase the variance of the income shock from $\sigma_y = 0.1$ to $\sigma_y = 0,5$. What happens to the consumption function in the certainty equivalence case (you should know the theoretical answer to that question). Also plot the new consumption functions for the precautionary savings case. Are the differences between certainty equivalence and precautionary savings consumption functions bigger or smaller now? Explain. Support your explanations with simulated time paths of consumption. Again limit yourself to T $=$ 45. How much do your answers change and why?
   <u>Answer:</u> On Figure 9 I present requested plots.

5. <u>Question:</u> Increase the persistence of the income shocks from $= 0$ to $= 0.95$ (keep $\sigma_y = 0.5$ as well as all other parameters constant). How much do your answers change and why?.

   <u>Answer:</u> Results presented in Figure 10

II.5 General Equilibrium

II.5.1 Simple ABHI model.

For this exercise I used code ayiagari$_r$oot.

II.5.2 Ayiagari 1994

For this exercise I used code **ayiagari$_r$oot7**.$WhenIwasreplicatingparametrizationfromAyiagaripapaer,$

1 Appendix

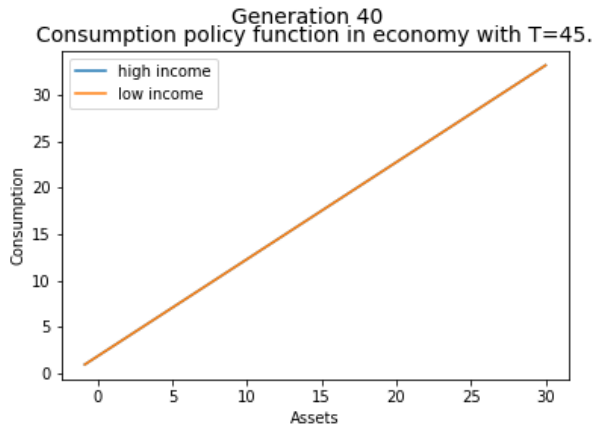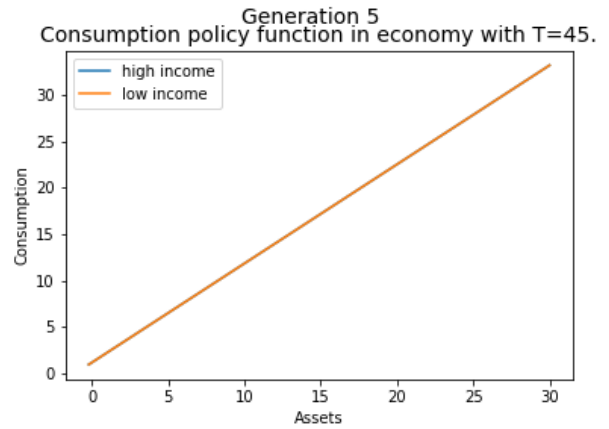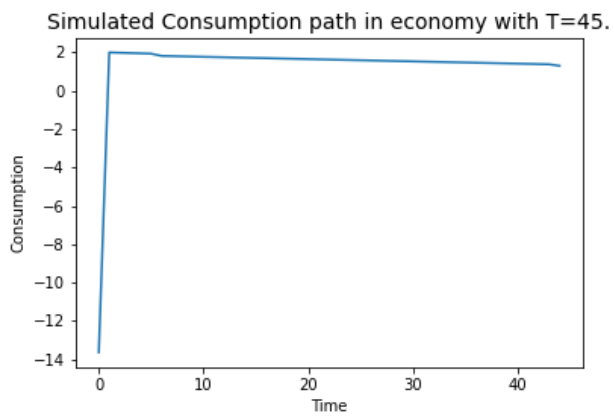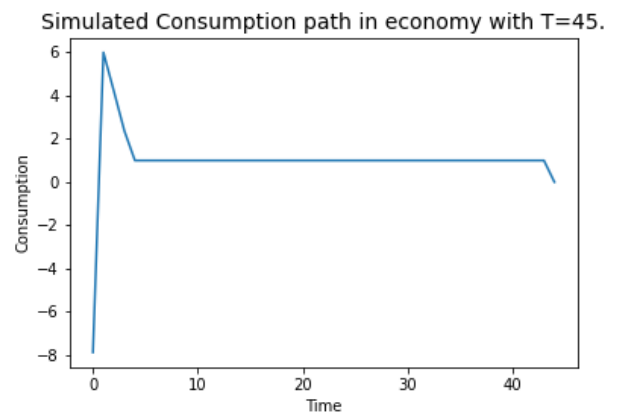Figure 3: Plots for II.4.1 point 1.
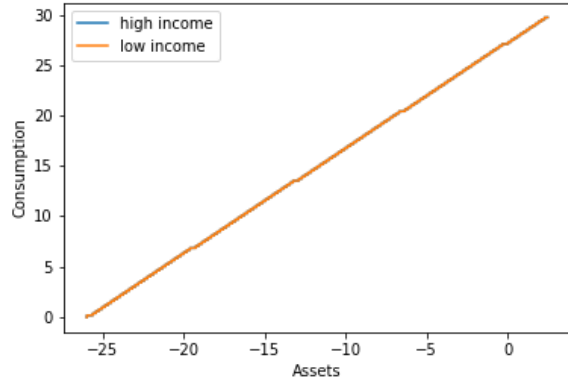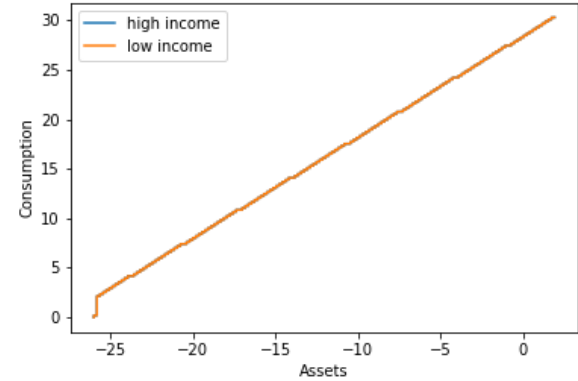
(a) Consumption policy function for CRRA case
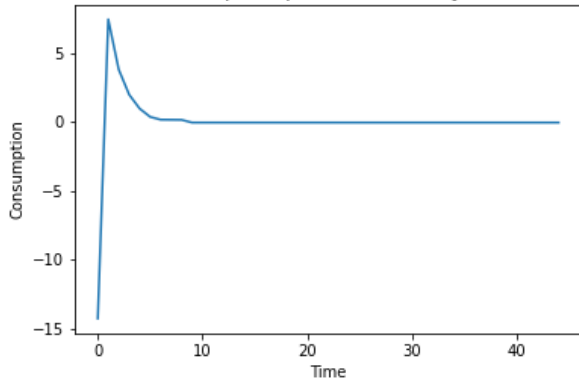
(b) Consumption policy function for quadratic case
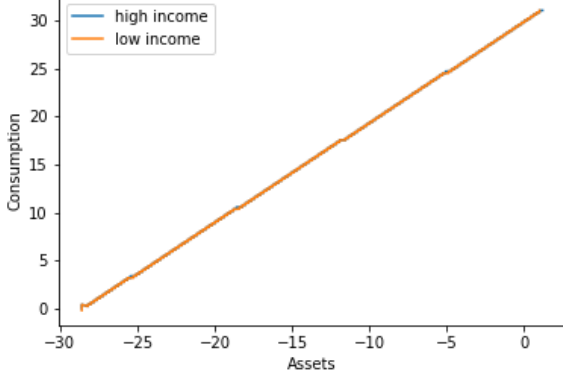

(c) Consumption path function for CRRA case
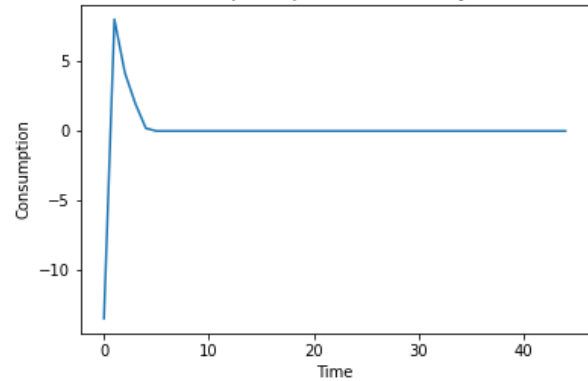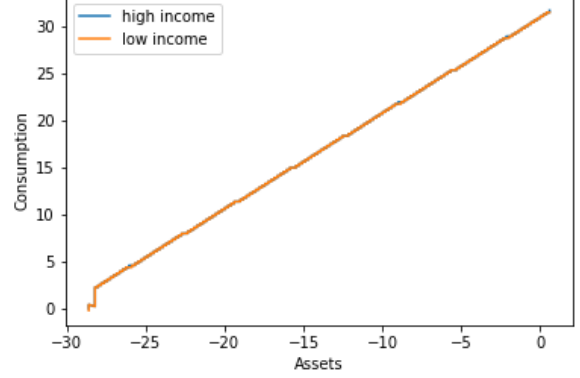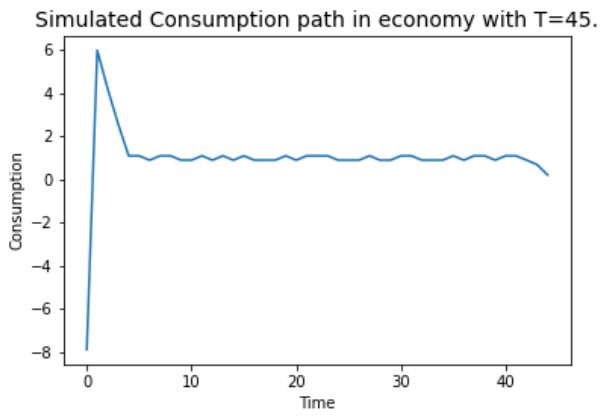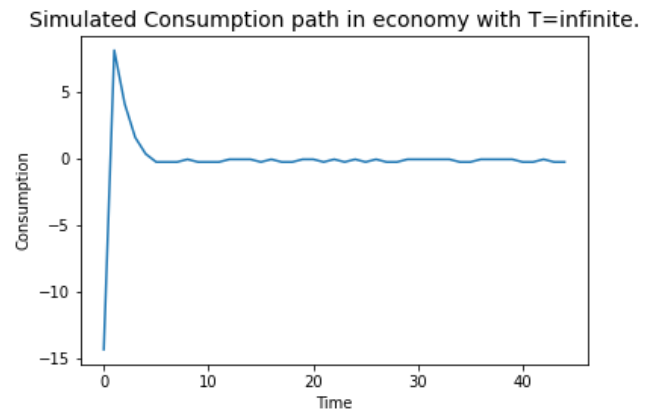

(d) Consumption path function for quadratic case

Figure 4: Plots for II.4.1 point 2
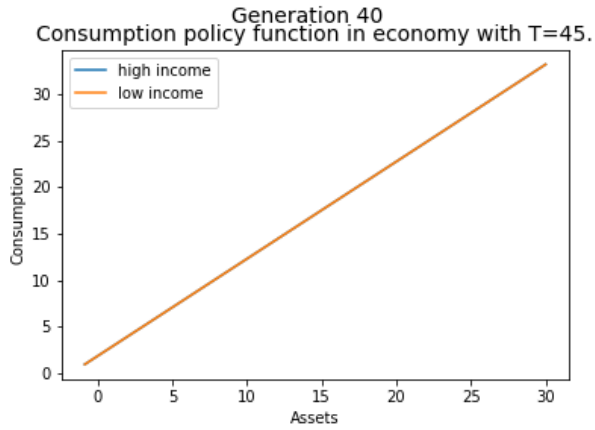
(a) Consumption policy function for CRRA case
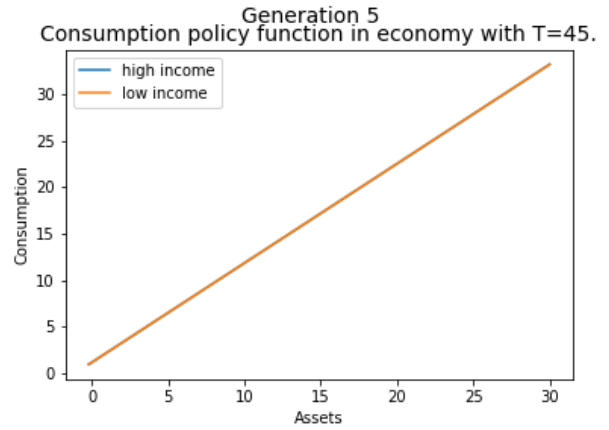
(b) Consumption policy function for quadratic case
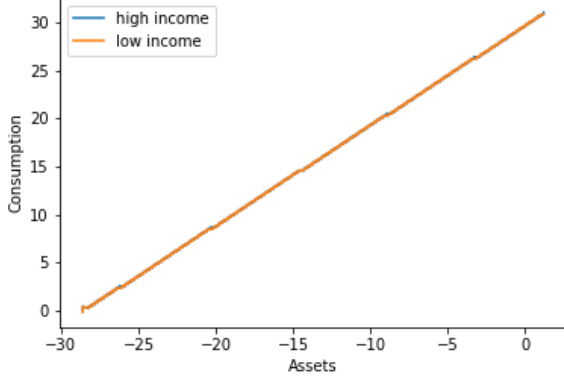

(c) Consumption policy function for quadratic case
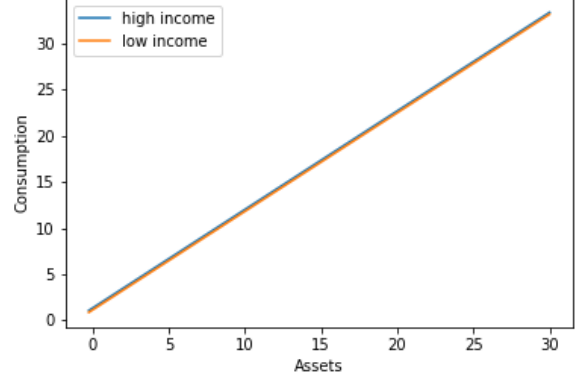

(d) Consumption policy function for quadratic case

(e) Consumption path function for CRRA case

(f) Consumption path function for quadratic case

Figure 5: Plots for II.4.2 point 1.

(a) Consumption policy function for CRRA case.

(b) Consumption policy function for quadratic case

(c) Consumption path function for CRRA case

(d) Consumption path function for quadratic case

(e) Consumption path function for CRRA case

(f) Consumption path function for quadratic case

6
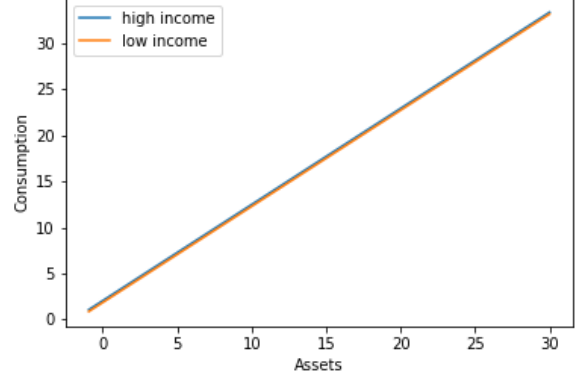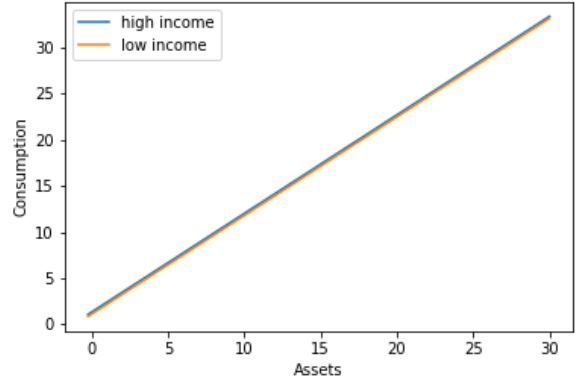
Figure 6: Plots for II.4.2 point 2.
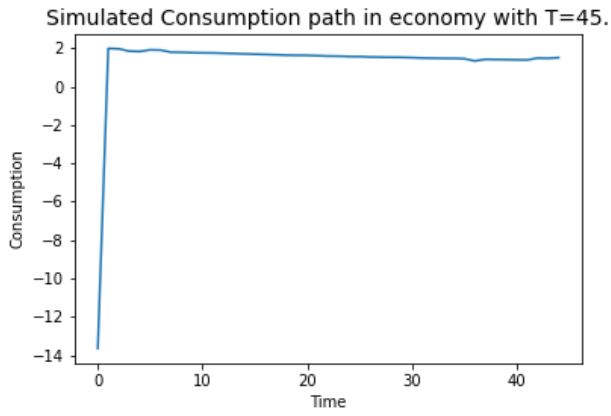
(a) Consumption path function for CRRA case.
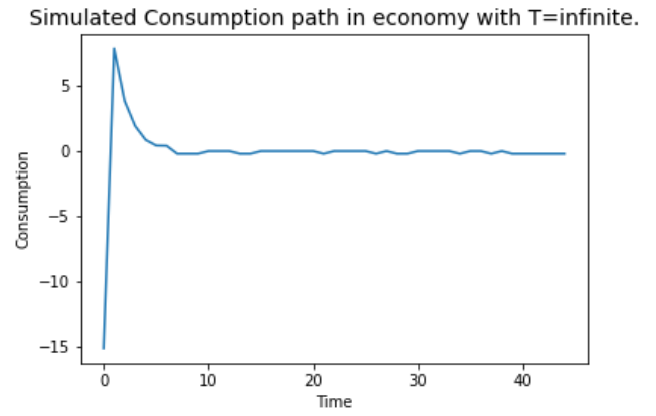
(b) Consumption policy function for CRRA case

(c) Consumption path function for quadratic case.

(d) Consumption policy function for quadratic case

Figure 7: Plots for II.4.2 point 3, prudence = 5.
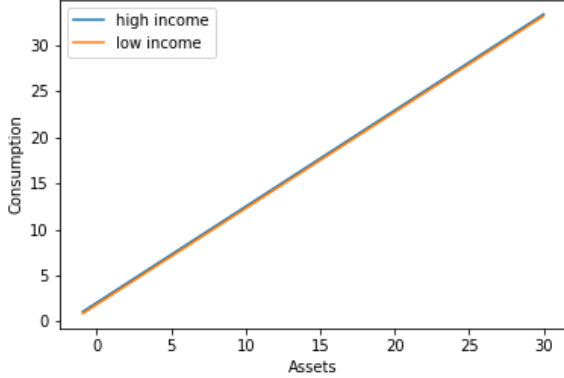
(a) Consumption path function for CRRA case.

(b) Consumption policy function for CRRA case
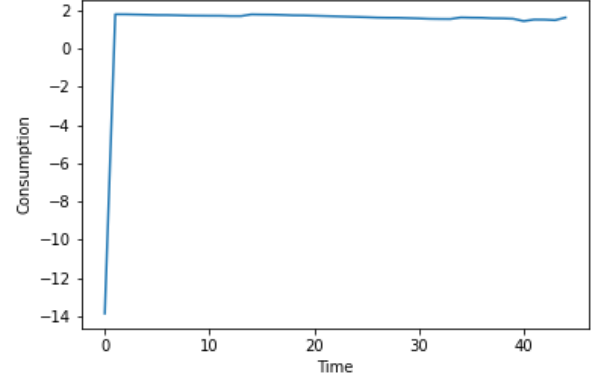
(c) Consumption path function for quadratic case.

(d) Consumption policy function for quadratic case
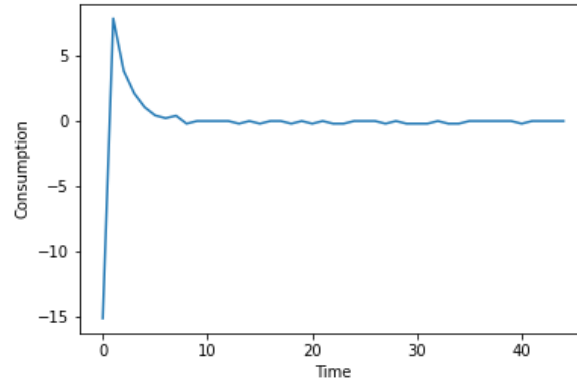
(e) Consumption policy function for quadratic case

Figure 8: Plots for II.4.2 point 3, prudence = 20.
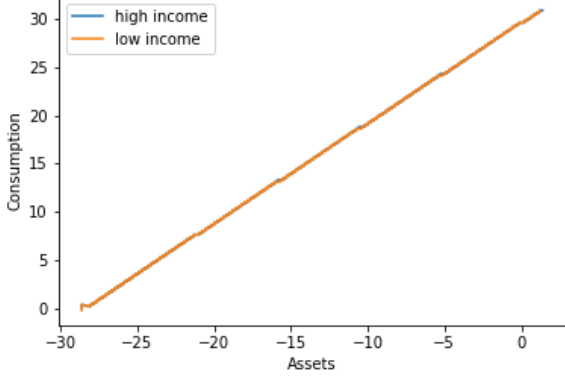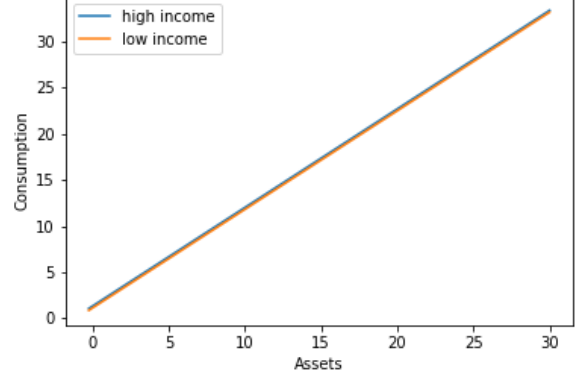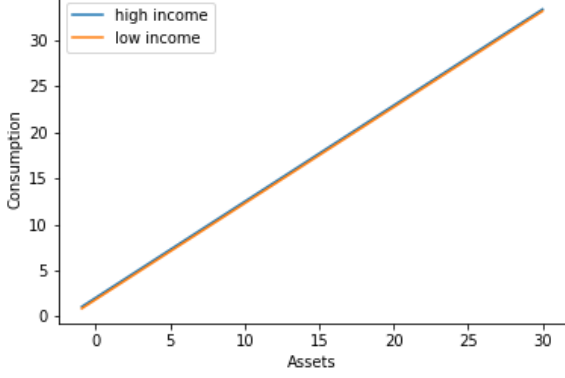
(a) Consumption path function for CRRA case.

(b) Consumption policy function for CRRA case
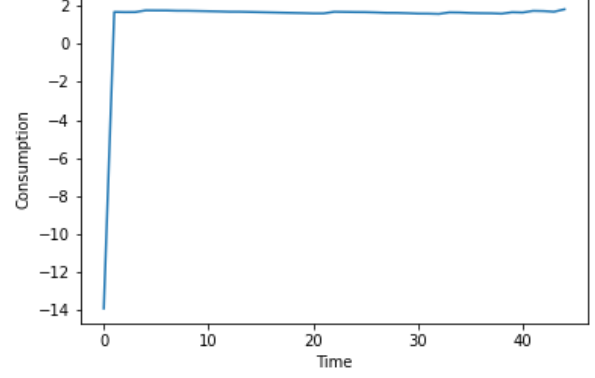
(c) Consumption path function for quadratic case.

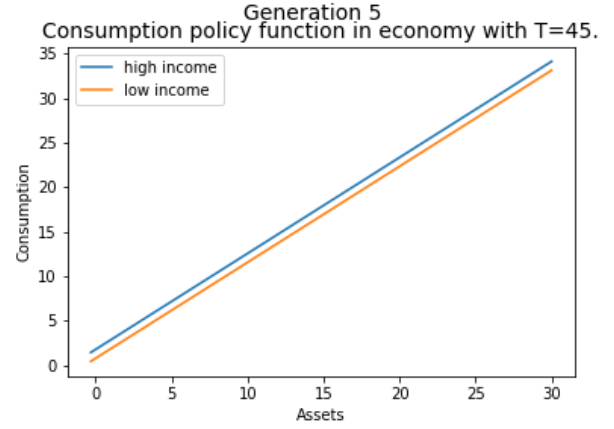(d) Consumption policy function for quadratic case
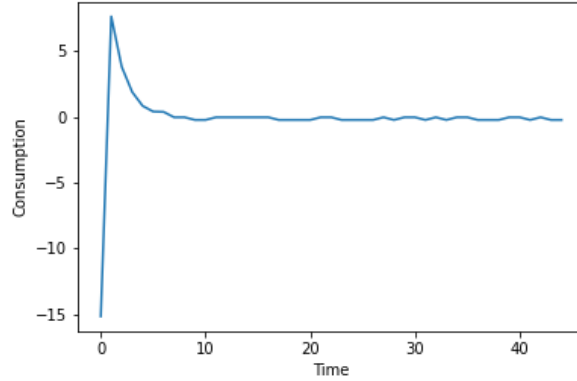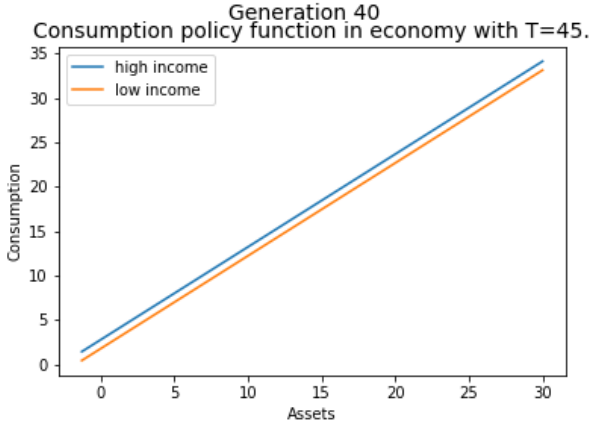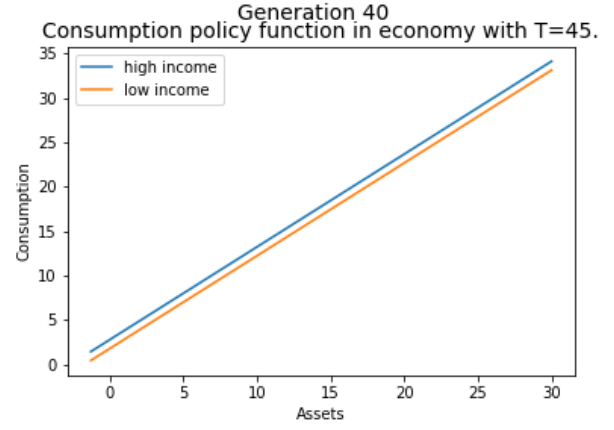
(e) Consumption policy function for quadratic case

Figure 9: Plots for II.4.2 point 4, Variance = 0.5.
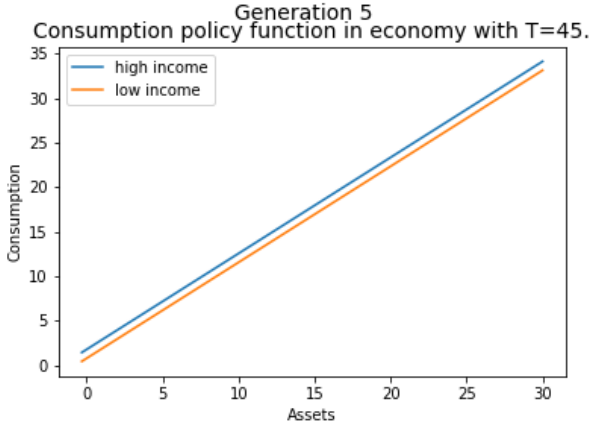
(a) Consumption path function for CRRA case.
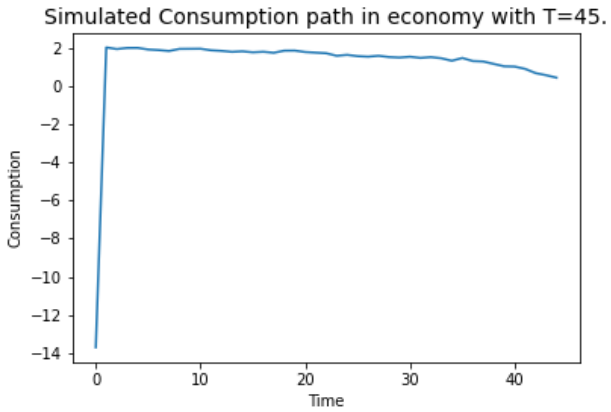

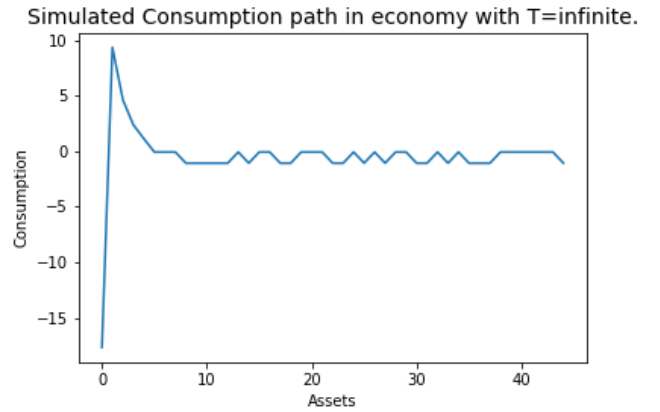(b) Consumption path function for quadratic case.

(c) Consumption path function for CRRA case.

(d) Consumption path function for quadratic case.
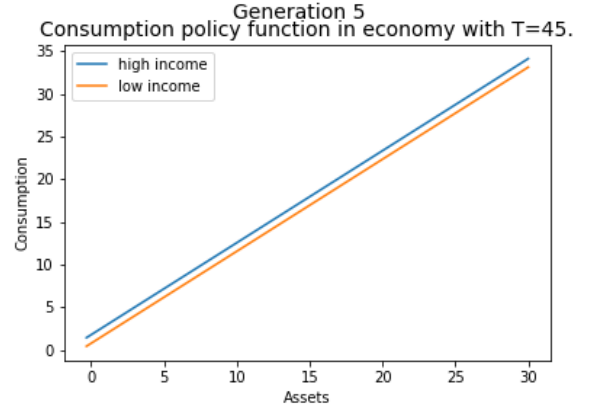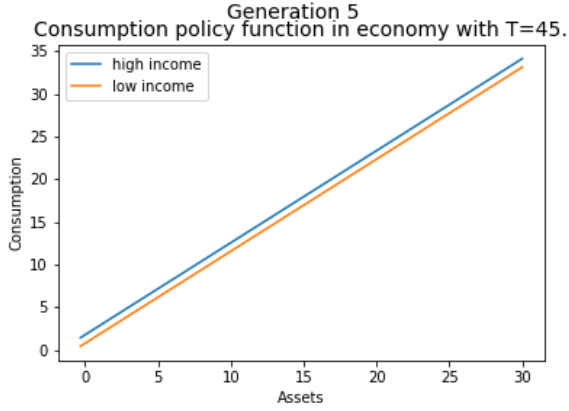

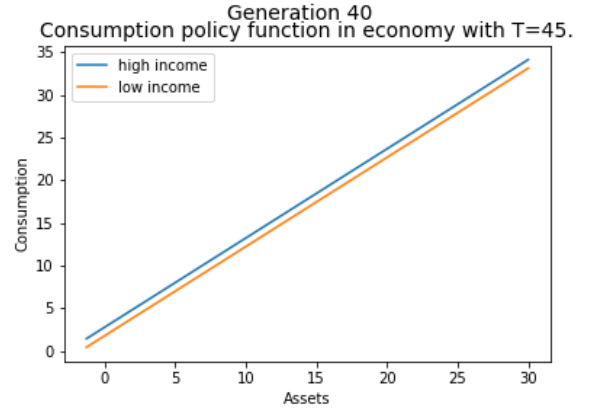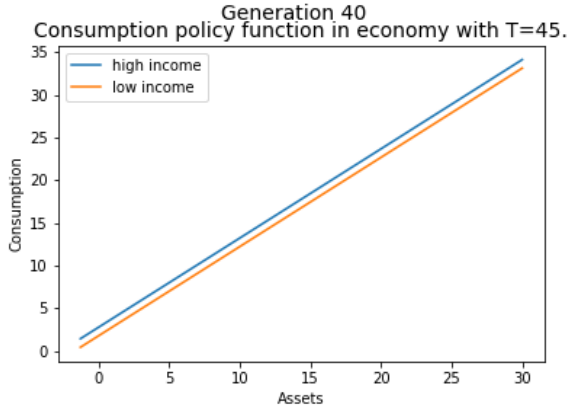(e) Consumption path function for CRRA case.

(f) Consumption path function for quadratic case.

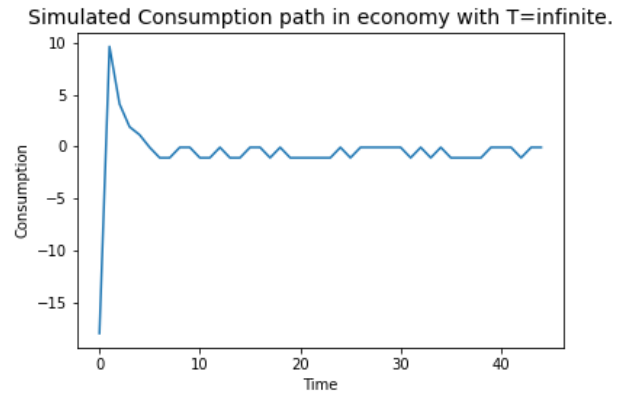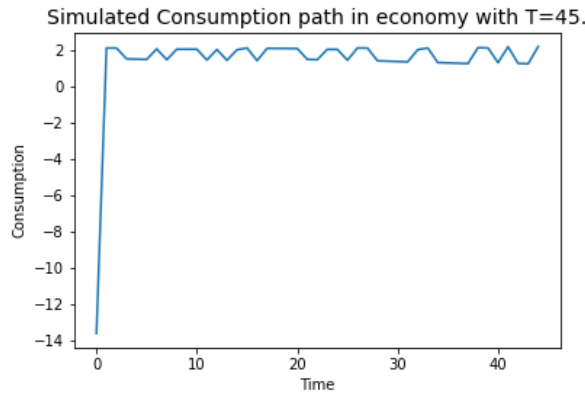Figure 10: Plots for II.4.2 point 5, persistence = 0.95.

(a) Consumption path function for CRRA case. (b) Consumption path function for quadratic case.
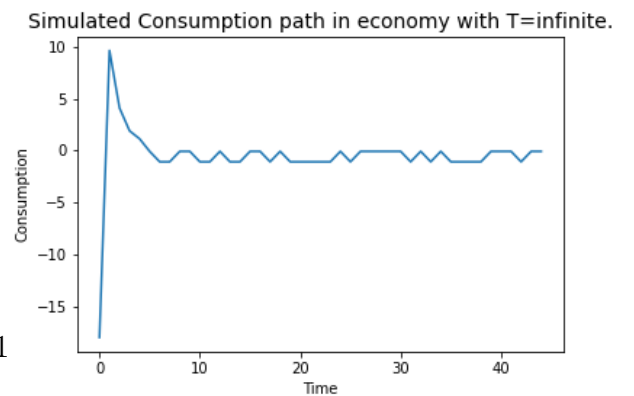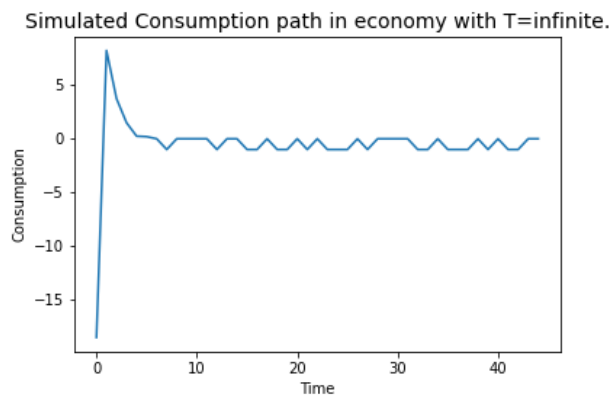
(c) Consumption path function for CRRA case. (d) Consumption path function for quadratic case.

(e) Consumption path function for CRRA case. (f) Consumption path function for CRRA case.

11

(g) Consumption path function for CRRA case. (h) Consumption path function for CRRA case.