

# Mathematical Foundations of Data Science Assignment 1

Trimester 2, 2024

## Question 1

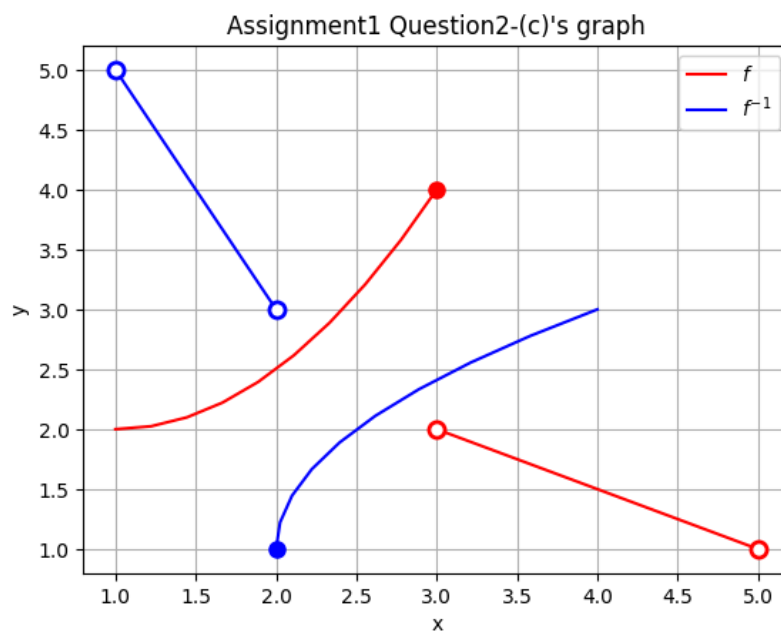
1. The statement in set notation means for any  $y$  belongs to rational numbers, there is an absolute value of  $y$  smaller than  $y$ .
2. The statement is false. For example there is a positive number like 5 belongs to rational numbers but its absolute value is 5 itself which equals 5 but not smaller.

## Question 2

- (a) The domain of  $g$  is  $(-1, 5)$ .
- (b) It is. As the graph shows the function has 2 dots at 3 but only the upper dot is solid which means the value of  $g$  when  $x = 3$  is 4.
- (c) The expression of  $g$  on domain  $[1, 5)$  is

$$f = \begin{cases} \frac{1}{2}(x-1)^2 + 2, & x \in [1, 3] \\ \frac{1}{2}x + \frac{7}{2}, & x \in (3, 5) \end{cases}$$

Then we change the places of  $x$  and  $f$ , then we can plot the graph of  $f^{-1}$  in Jupyterbook:



## Question 3

- (a)  $A \setminus \mathbb{Z} = \{\frac{1}{4}, \frac{3}{2}, \pi\}$
- (b)  $A \cap B = \{0, \frac{1}{4}, \frac{3}{2}\}$

## Question 4

- To find  $h^{-1}(x)$ , we just switch all instances of  $x$  and  $h(x)$ , which is

$$x = \frac{h^{-1}(x)+3}{2}$$

$$h^{-1}(x) = 2x - 3$$

- The domain of  $h(x)$  is  $[1, +\infty)$ , which indicates the range of  $h(x)$  is  $[2, +\infty)$ . As the concept of inverse functions we can learn the domain of  $h^{-1}(x)$  is the range of  $h(x)$ , which is  $[2, +\infty)$ .