

Mathematical Foundations of Data Science

Assignment 3

Trimester 2, 2024

1. An IT helpdesk receives, on average, 1 support ticket every 12 minutes. Let X be the number of tickets received by the helpdesk in an hour, and suppose that X has a Poisson distribution, i.e., $X \sim \text{Pois}(\lambda)$.
 - (a) Find the rate, λ .
 - (b) What is the probability that the helpdesk receives between 3 and 6 support tickets (inclusive) in the next hour? Please show all working and round your answer to the nearest 4 decimal places.
 - (c) How many minutes are required for the probability of the desk receiving at least one ticket to exceed $1/4$.
2. Consider the following distribution for a discrete random variable:

x	-4	-2	-1	0	1
$P(X = x)$	0.05	0.1	0.15	0.2	0.5

- (a) Calculate the expected value, $E[X]$.
- (b) Calculate the variance, $\text{var}(X)$, using $E[(X - E[X])^2]$.
- (c) Confirm your answer to part (b) by calculating $E[X^2] - E[X]^2$. Since you are expecting to get the same answer, you will not receive marks for just stating the answer. Your working must clearly show that you have independently calculated the variance using both methods.

Please show all working out. Do not round your answers to this question.

3. A social media platform is trialling a new tool designed to detect if accounts are controlled by humans or bots. The tool returns a positive test if it determines that an account is controlled by a bot. In the trial, it is discovered that:
 - the probability that the test is positive given that the account is controlled by a bot is 0.77; and
 - the probability that the test is positive given that the account is controlled by a human is 0.24.
 - (a) Suppose the tool is implemented in a setting where it is estimated that 68% of accounts are controlled by bots. What is the probability of a positive test result?

- (b) In the same setting as above, what is the probability that an account is controlled by a human if the test result is negative?

4. Consider the following matrices:

$$F = \begin{bmatrix} 0 & -1 & 1 \\ w & -1 & 0 \end{bmatrix}, G = \begin{bmatrix} 3 & 0 & 1 \\ 0 & x & 0 \\ 0 & 0 & -1 \end{bmatrix}, H = \begin{bmatrix} 1 & 0 \\ -2 & y \\ -1 & z \end{bmatrix}, \text{ where } w, x, y,$$

and z are unknown real-valued parameters. Calculate $FGH - H^T F^T$, showing all working.