

Mathematical Foundations of Data Science

Assignment 4

Trimester 2, 2024

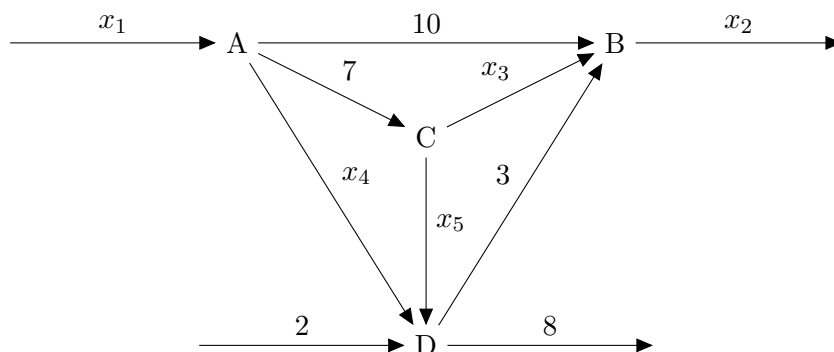
1. Find all matrices A such that $A^2 = \begin{bmatrix} 4 & -2 \\ 0 & 1 \end{bmatrix}$. Show all working to demonstrate your method for finding each solution.
2. Suppose that W , X , Y , and Z are matrices such that $W^{-1}(X + (YZ)^T)$ is defined. If X has 3 columns, Y has 4 columns, and Z has 5 columns, determine the sizes of all four matrices W , X , Y , and Z , showing all working.
3. Consider the following system of linear equations in the variables x_1 and x_2 , with some unknown parameter δ :

$$x_1 + \delta x_2 = 10$$

$$x_1 - 2x_2 = \delta$$

In this question, you must directly manipulate the equations to solve each part, showing all working. Do not use matrices or row-reduction.

- (a) What is the solution to this system if $\delta = 4$?
 - (b) Find any values of δ for which this system has no solutions.
 - (c) If there is a solution with $x_2 = 3$, find the corresponding values of x_1 and δ .
4. Consider the following section of a road network with traffic flows measured in thousands of cars per hour:



At each junction (A , B , C , D), the amount of traffic entering must be equal to the amount leaving. All traffic are one-way, with the directions of flow indicated by arrows. Set up a system of linear equations that describes the possible flows x_1 , x_2 , x_3 , x_4 , x_5 and then express the system in matrix form $A\mathbf{x} = \mathbf{b}$. Note, this question involves formulation only. You do NOT need to solve the system of equations.