

MATHS7027 Mathematical Foundations of Data Science

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Assignment 4 - Question 1

Consider $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, to make $A^2 = \begin{bmatrix} 4 & -2 \\ 0 & 1 \end{bmatrix}$, we could write an expression below:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 0 & 1 \end{bmatrix}$$

And then we could get the relationships of a, b, c and d :

$$\left\{ \begin{array}{l} a^2 + bc = 4 \\ ab + bd = -2 \\ ac + cd = 0 \\ bc + d^2 = 1 \end{array} \right. \begin{array}{l} (1) \\ (2) \\ (3) \\ (4) \end{array}$$

from (3), we could learn that: $ac = -cd$, if $c \neq 0$, so $a = -d$.

So we can change (2) to $ab - ab = -2$ which is obviously incorrect.

Then $c = 0$, which means a and d could be any value based on (3).

But still with (1) and (4) we could obtain $a^2 = 4, d^2 = 1$, So a could be either 2 or -2 while d could be either 1 or -1. And calculate (2) with all the cases, we could get:

$$\begin{array}{l} 2b + b = -2, a = 2, d = 1 \\ 2b - b = -2, a = 2, d = -1 \\ -2b + b = -2, a = -2, d = 1 \\ -2b - b = -2, a = -2, d = -1 \end{array}$$

Consider a vector of (a, d, b) , which could be:

$$(2, 1, -\frac{2}{3}), (2, -1, -2), (-2, 1, 2) \text{ or } (-2, -1, \frac{2}{3})$$

So the matrix A could be:

$$\begin{bmatrix} 2 & -\frac{2}{3} \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -2 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -2 & 2 \\ 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} -2 & \frac{2}{3} \\ 0 & -1 \end{bmatrix}$$

Assignment 4 - Question 2