Mathematical Foundations of Data Science
Assignment 5

Trimester 2, 2024

1. Consider the matrix
$$A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 3 & 0 \\ -2 & 0 & 1 \end{bmatrix}.$$

   Find the eigenvalues and eigenspaces of A, showing all working.

2. Consider the matrix
$$B = \begin{bmatrix} 2 & a & -1 \\ 0 & 2 & b \\ -1 & -1 & 3 \end{bmatrix}.$$

   (a) Suppose that $v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$ are both eigenvectors of B. Determine the values a and b.

   (b) Use the trace of B to find the remaining eigenvalue of B.

3. Let C be the matrix that is diagonalised by $P = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ 2 & 0 & 1 \end{bmatrix}$ with

   diagonal matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c \end{bmatrix}$ for some $c \in \mathbb{R}$.

   (a) Use row reduction to find P^{-1}, showing all working. For full marks, you must include details of all row operations you use.

   (b) Use matrix diagonalisaton to find an expression for C^n for any positive integer n, showing all working.

4. Consider the following network, comprising nodes A, B, C, and D:
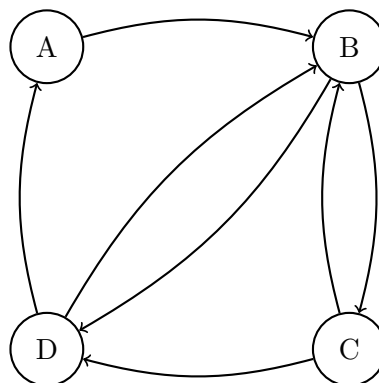
Using the PageRank algorithm, this network can be represented by

the matrix $M = \begin{bmatrix} 0 & 0 & 0 & 1/2 \\ 1 & 0 & 1/2 & 1/2 \\ 0 & 1/2 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \end{bmatrix}$ and vector $\boldsymbol{x} = \begin{bmatrix} x_A \\ x_B \\ x_C \\ x_D \end{bmatrix}$.

This question is contained in the Jupyter notebook `MFDS_A5_Q4.ipynb`. The notebook asks you to analyse the above network using the PageRank algorithm. You should follow the instructions in the notebook, filling in code and answers where required.

Please submit your answers to this question by converting your completed Jupyter Notebook to a PDF.

Please note: You must include the code you used to find each answer. Each answer submitted without code will receive a mark of 0.

Hint for submitting: You can "Download As PDF" in Jupyter, but this may not work on your computer. If it doesn't, you can download as HTML, then convert the HTML file to a PDF. It is best to submit all of your answers as a single file, so make sure you join your PDF to the rest of your answers before submitting. If you are unsure, the Python Computer Exercises Module on MyUni contains a video demonstrating how to save a Jupyter Notebook as a PDF.