Reminder for mid-trimester test

2024 Trimester 2

Set's notation and limit

- If $g(x)$'s domain is $[0, x]$, $h(x) = g(\frac{x}{3})$'s domain is $[0, 3x]$
- Multipule sum:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} i = \sum_{i=1}^{m} n \cdot j \cdot i = n \cdot j \cdot (1 + 2 + \cdots + m)$$

- Convergent = 收敛 Divergent = 发散

Combination and permutation

- If you want to choose 7 pieces of fruit from 5 kinds of fruit, you should use stars and bars to calculate the combination:

$$\binom{7+5-1}{7} = 330$$
$$|*|*|*|**|*|$$
$$*:7$$
$$|:4$$

  the $7+5-1$ means there are 7 of your choice 5 of the kinds, so you would need to divide the whole stars into 12 sections which means you just need $7+5-1$ bars(the outside bars are not included). And 7 means you need to choose 7 pieces at last, which is 7 stars. You should calculate the combination of choose the stars among the stars and bars.

Probability

- Bayes theory `(Given B to calculate A's probability)` :

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- Total probability :

$$P(A) = \sum_{n} P(A|B_n)P(B_n)$$

- Expectation:

$$E(aX + bY) = aE(x) + bE(Y)$$

- Variance:

$$\begin{aligned} Var(X) &= E[(X - E[X])^2] \\ &= E[X^2 - 2XE[X] + E[X]^2] \\ &= E[X^2] - 2E[X]E[X] + E[X]^2 \\ &= E[X^2] - E[X]^2 \end{aligned}$$

Matrix

- The product of matrices:

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \cdot \begin{bmatrix} g & h & i \\ j & k & l \end{bmatrix} = \begin{bmatrix} ag + bj & ah + bk & ai + bl \\ cg + dj & ch + dk & ci + dl \\ eg + fj & eh + fk & ei + fl \end{bmatrix}$$

- Design matrix for linear regression:
  Consider there is 3 linear points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$

The design matrix would be like:

$$\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \end{bmatrix}$$

- The determinant of matrix:
  Consider there is a matrix A as below,

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$det(A) = aei + bfg + cdh - ceg - bdi - afh$$