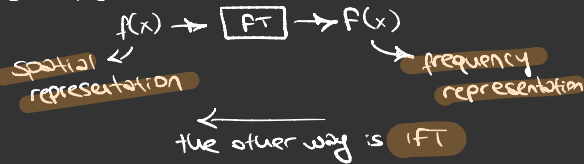


fourier transforms

Sinusoid $\Rightarrow f(x) = A \cdot \sin(\underbrace{2\pi kx}_{\text{amplitude}} + \phi)$

- If you have periodic function, you can describe it with sum of sinusoids, bassess.

- Represents a signal $f(x)$ in terms of Amplitudes and phases of its Constituent of Sinusoids.



infinite sum of sine waves $\Rightarrow A \sum_{k=1}^{\infty} \frac{1}{k} \cdot \sin(2\pi kx)$

discrete

FT $\Rightarrow f(k) = \frac{1}{N} \cdot \sum_{x=0}^{N-1} f(x) \cdot e^{-j2\pi kx/N}$

IFT $\Rightarrow f(x) = \sum_{k=0}^{N-1} F(k) \cdot e^{j2\pi kx/N}$

continuous

FT $\Rightarrow F(k) = \int_{-\infty}^{\infty} f(x) \cdot e^{j2\pi kx} dx$

we represent frequencies both negative and positive

IFT $\Rightarrow f(x) = \int_{-\infty}^{\infty} F(k) \cdot e^{j2\pi kx} dk$

$x \Rightarrow \text{space}$, $k \Rightarrow \text{frequency}$, $e^{\theta} = \cos \theta + i \sin \theta$

$e^{j\theta} = \cos \theta + j \sin \theta \rightarrow \text{Euler's formula}$

$f(x) = \sum_{k=0}^{N-1} F(k) \{ \underbrace{\cos(2\pi kx)}_{\text{scaling parameter}} + j \cdot \underbrace{\sin(2\pi kx)}_{\text{wave components}} \}$

DFT \rightarrow discrete Fourier transform

$$F = Wf$$

$$\begin{bmatrix} F(0) \\ F(1) \\ F(2) \\ \vdots \\ F(N-1) \end{bmatrix} = \begin{bmatrix} W^0 & W^0 & W^0 & W^0 & \dots & W^{N-1} \\ W^0 & W^1 & W^2 & W^3 & \dots & W^{N-1} \\ W^0 & W^2 & W^4 & W^6 & \dots & W^{N-2} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ W^0 & W^{N-1} & W^{N-2} & W^{N-3} & \dots & W^1 \end{bmatrix} \begin{bmatrix} f(0) \\ f(1) \\ f(2) \\ \vdots \\ f(N-1) \end{bmatrix}$$

$W = e^{-j2\pi/N}$

Frequency Domain Filtering

$F\{g * h\} = F\{g\} \cdot F\{h\}$

$F^{-1}\{g \cdot h\} = F^{-1}\{g\} * F^{-1}\{h\}$

Convolution for 1D continuous signals

Definition of linear shift-invariant filtering as convolution.

$(f * g)(x) = \int_{-\infty}^{\infty} \underbrace{f(y)}_{\text{filter}} \cdot \underbrace{g(x-y)}_{\text{input signal}} dy$

filtered signal

- we can interpret and implement all kinds of linear shift-invariant filtering as multiplication in frequency domain.

Properties of Fourier Transform

Property	Spatial Domain	Frequency Domain
Linearity	$\alpha f_1(x) + \beta f_2(x)$	$\alpha F_1(u) + \beta F_2(u)$
Scaling	$f(ax)$	$\frac{1}{ a } F\left(\frac{u}{a}\right)$
Shifting	$f(x-a)$	$e^{-i2\pi ua} F(u)$
Differentiation	$\frac{d^n}{dx^n} f(x)$	$(i2\pi u)^n F(u)$