Hough Transform

Line fitting

minimize average square distance

$$E = \sum \frac{(y_i - m x_i - c)^2}{N}$$

using $\rightarrow \frac{dE}{dm} = 0$ and $\frac{dE}{dc} = 0$

$$c = \bar{y} - m\bar{x} \qquad m = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}$$

Hough Transform → solves fitting and segmentation problem simultaneously

- a technique for having edges vote for plausible line locations.

→ original formulation

- each edge point votes for all possible lines passing through, and lines corresponding to high accumulator or bin values are examined for potential line fits.
- in hybrid strategy, each edgel votes for a number of possible orientation or location pairs centered around the estimate orie...
- Before voting the lines we must represent the lines by polar coordinates.
  → or other representations.

we convert the normal vector into an angle ← $\theta = \tan^{-1} n_y / n_x$

!
- Edges don't have to be connected.
- Lines can be occluded.
- Edges vote for possible models.

$y = mx + c$

image	parameter
line →	point
point →	line
2 points →	2 lines

→ those are actually maxima. (votes)

Line Detection by Hough Transform

1 – Quantize parameter space → (m, c)
2 – Create accumulator array. → $A(m, c)$
3 – Set $A(m,c) = 0 \quad \forall m, c$
4 – For each image edge (x_i, y_i)
  For each element in $A(m,c)$
    If (m,c) lies on the line → $c = -x_i m + y_i$
    Increment $A(m,c) = A(m,c) + 1$
  Find local minima in $A(m,c)$

⇒ In $y = mx + c$ parameterization, the space of m would be huge.
For better parameterization, we can use normal form:

$$\boxed{x.\cos\theta + y \sin\theta = p}$$

image ($y = mx + c$)	parameter ($x.\cos\theta + y.\sin\theta = p$)
point →	wave
line →	point

→ here, the rotation angle of the line is important!
→ positive rho ⇒ $x.\cos\theta + y.\sin\theta = p$
negative rho ⇒ $x.\cos(\theta + \pi) + y.\sin(\theta + \pi) = -p$

Implementation

1 – Initialize accumulator H to all zeros
2 – For each edge in the image
  For $\theta = 0$ to 180
    $p = x.\cos\theta + y.\sin\theta$
    $H(\theta, p) = H(\theta, p) + 1$
3 – Find the values of (θ, p) where $H(\theta, p)$ is a local maximum.
4 – The detected line in the image is given by $p = x.\cos\theta + y.\sin\theta$

Note: More noise yields fewer votes.

Hough Circles → if radius is not known 3D Hough space

$$(x-a)^2 + (y-b)^2 = r^2$$

with gradient we know → edge direction (x_i, y_i)
→ edge location ϕ_i

$a = x - r.\cos\phi \qquad b = y - \sin\phi$

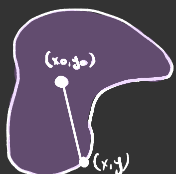
1 – Quantize parameter space.
2 – For each edge point (x,y)
  for $r = r_{min}$; $r \leq r_{max}$; $r{+}{+}$
    $x_0 = x - r.\cos\phi$
    $y_0 = y - r.\sin\phi$
    $P[x_0, y_0, r] = P[x_0, y_0, r] + 1$
3 – Find the lokal maxima

! Each point on a circle will map to a circle in the parameter space.

Final notes:

✓ Deals with occlusion well
✓ Detects multiple instances
✗ It's not robust to noise
✗ Does not have good computational complexity.
✗ It's not easy to set parameters.

Example



To find this shape in image
1 – Compute centroid
2 – For each edge compute its distance to centroid

$$r = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix}$$

3 – Find edge orientation
  gradient angle
4 – Construct a table of angles and r values