

Linear shift-invariant filtering  
 Box filter  
 Separable filters  
 Conv and correlation  
 → better for image look

# image filtering

image transformation → filtering: changes pixel values  
 → warping: changes pixel locations

image filtering → point processing  
 { invert (255-x)  
 darken (x-128)  
 lower contrast (x/2)  
 non-linear rise contrast  $((\frac{x}{255})^2 \cdot 255)$

## Correlation

$$f \otimes h = \sum_k \sum_l \underbrace{f(k,l)}_{\text{image}} \underbrace{h(i+k, j+l)}_{\text{kernel}}$$

## Convolution

$$f * h = \sum_k \sum_l \underbrace{f(k,l)}_{\text{image}} \underbrace{h(i-k, j-l)}_{\text{kernel}}$$

we flip the kernel in x and y axes and then apply correlation.

Gaussian Filter → low-pass filtering.

$$g(x) = e^{-\frac{x^2}{2\sigma^2}}$$

$$g(x,y) = e^{-\frac{(x,y)^2}{2\sigma^2}}$$



mask values for gaussian filter

- most common natural model
- it has infinite number of derivatives
- Fourier of a Gaussian is Gaussian.  
 → if we want to look at image in frequency level.
- Convolution of Gaussian with itself is Gaussian.  
 → if both image and the kernel is gaussian for ex.

As  $\sigma$  increases more pixels are involved in avg.  
 As  $\sigma$  increases image is more blurred  
 As  $\sigma$  increases noise is more effectively suppressed.

$$\text{kernel} \Rightarrow \frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}_y = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}_x \otimes \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

separable

Laplace Filter → second derivative filter

$$1D \text{ derivative filter} \Rightarrow f'(x) = \lim_{h \rightarrow 0} \frac{f(x+0.5h) - f(x-0.5h)}{h}$$

$$\text{Laplace filter} \Rightarrow f''(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

$$\rightarrow \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$$

## Image Gradients

$$\text{magnitude} = \sqrt{(S_x^2 + S_y^2)}$$

$$\text{direction} = \theta = -\tan^{-1} \frac{S_y}{S_x}$$

- gradient is perpendicular to the edge

## Filtering

image  $\otimes$  kernel = filter output

Gaussian filtering: weighted averaging.  
 → you want more center pixel more important than others.

we use  
 Edge Detectors → derivative filters

## Prewitt and Sobel

- Compute derivatives in x and y direction
- Find gradient and magnitude
- Threshold gradient magnitude



$$\text{Sobel} \Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}_x \quad \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}_y$$

## Laplacian of Gaussian

Laplacian filters are derivative filters to find areas of edges. Since derivative filters are very sensitive to noise, it's common to smooth the image (eg. using a Gaussian) before applying the Laplacian.