Laboratory Session : April 8, 2025

Exercises due on : April 27, 2025

Exercise 1 - Discrete random variable

- Let $X \sim \text{ZTB}(n, p)$ be a random variable following a *Zero-Truncated Binomial Distribution* in which:
  - $n \in N$ is the number of Bernoulli trials.
  - $p \in (0, 1)$ is the probability of success on a single trial.

  Then, the probability mass function (PMF) is defined as:

$$P(X = k) = \frac{\binom{n}{k} p^k (1-p)^{n-k}}{1 - (1-p)^n}, \quad \text{for } k = 1, 2, \ldots, n$$

- Assuming $\mathbf{n = 14}$ and $\mathbf{p = 0.15}$,
  1) Write the R functions for the PMF and cumulative distribution (CDF), using the R naming convention.
  2) Produce two plots showing the PMF and CDF, separately.
  3) Compute the mean value and variance of the ZTB distribution using R. Compare the obtained values with the analytical statistical moments for the zero-truncated and the standard binomial distributions.
  4) Generate a sample of random numbers from this distribution, show them in an histogram, and evaluate the sample mean.

Exercise 2 - Continuous random variable

- The energy distribution of CR muons at sea level can be approximated as follows

$$p(E) = N \begin{cases} 1 & \text{for } \; E < E_0 \\ (E - E_0 + 1)^{-\gamma} & \text{for } \; E \geq E_0 \end{cases} \tag{1}$$

  where $\mathrm{E}_0 = 7.25$ GeV and $\gamma = 2.7$.

a) Compute the normalisation factor N using R.

b) Plot the probability density function in R.

c) Plot the cumulative density function in R.

d) Compute the mean value using R.

e) Generate 10^6 random numbers from this distribution, show them in an histogram and superimpose the pdf (with a line or with a sufficient number of points).

Exercise 3 Web server requests

- The average number of requests to a small web server is 7 per day.

a) Find a bound for the probability that at least 30 web server requests will occur tomorrow.

b) Under regular conditions, the number of web server requests can be modeled as Poisson process, calculate the probability that at least 30 requests will occur tomorrow. Compare this value with the bound obtained in the previous point a).

c) Let the variance of the number of requests be 5 per day, find a bound on the probability that tomorrow at least 30 requests will occur.

Exercise 4 Photon detector

- A photon detector measures an average of 350 light particles per second with a standard deviation of 75 particles.

  a) How many minutes an experimentalist must run the detector to be 95% confident of collecting at least 1 million signals?

Exercise 5 Rumor spreading process

- Let a small community be composed of 1000 persons.
- Initially, 5 people know a rumor, and 995 people don't.
- Every person who knows the rumor spreads it to one randomly chosen person, including those who already know it.

  a) Simulate how the rumor spreads over 15 interactions using R.

  b) Evaluate the mean number of people know the rumor after 15 interactions.

  c) Find an upper bound for the probability of at least 500 persons know the rumor after 15 interactions.

Exercise 6 Passenger arrivals

- In a stationary bus at the departure station, a passenger gets on the bus, on average every 30 seconds.

a) Compute the probability of getting more than 6 passenger after 2 minutes. Evaluate the probability of having less than 4 passenger after 3 minutes.

b) Simulate the distribution of the arrival time of the third passenger and superimpose the corresponding pdf.

c) Repeat the procedure of the point b) for the difference in arrival time between the fifth and the first passenger.