



Figure 1: State of the art unroll methods can create patterns on a closed surface. The `ApplyCrv` command of Rhinoceros produces boundary aligned periodic patterns but introduces unacceptable non-isotropic stretch (left). The `SquishBack` method creates sufficiently regular elements but does not respect the periodicity of the surface (middle). Non-periodic conformal maps align with the boundary of a cut surface. Along the cut the map is not continuous (right).

1 Related Work

In this section we will review different methods used to unroll/parametrize surfaces that are accessible in the architectural design process. All available tools lack either one or the other key feature of the proposed method: Conformality of the mapping avoids non-uniformly stretched panels. Periodicity of the parameterization is needed to layout panels seamlessly on the surface.

Rhinoceros CreateUVCrv/ApplyCrv. A NURBS surface is naturally equipped with a parameterization, i.e., a map from a rectangle domain to the surface. For the surface of Figure ?? such a map can be used to project a pattern from this rectangle to the surface (`ApplyCrv` in Rhinoceros). The pattern can be constructed periodically as it is defined in the uv-rectangle of the surface. In general the uv-method however does not produce satisfactory results in terms of quality of elements as elaborated in the introduction. The UV-parameterization is not conformal and thus introduces non-isotropic stretch preventing the elements to appear regular on the surface, see Figure 1, left. This limitation exists even for developable surfaces.

Rhinoceros Squish/SquishBack. The `Squish/SquishBack` command of Rhino maps a surface onto the plane while minimizing the amount of stretch. While this is geometrically not a conformal map it produces acceptable patterns on the surface. It is however not capable of calculating periodic maps onto the surface thus not applicable in our situation, see Figure 1, middle.

PanelingTools for Rhino. The paneling tools of Rhinoceros use the UV-parameterization of the underlying surface to populate grid-points over the surface. With the help of such a grid, panels are placed onto the surface, see [McNeel]. The shape of the panels depend heavily on the behavior of the NURBS-parameterization. Results look similar to Figure 1, left.

Hexagonal tilings In the architectural context, hexagonal panelizations have been studied by [Zimmer et al. 2013, Troche 2008] and [Schiftner et al. 2009]. These

approaches however do not include periodicity or special boundary alignment as introduced in the current work. In [Schiftner et al. 2009] the result of the panelization depends on the choice on an initial triangle mesh that is optimized towards touching incircles. This allows for a torsion free support structure. Hexagonal tilings for triangulated surfaces have been studied by [Nieser et al. 2012]. They do not

Mesh based parameterization There are a vast number of parameterization schemes for meshes. To elaborate on all methods is beyond the scope of this section and we describe only the most important results here. General purpose parameterization methods for triangle meshes produce high quality quad or hex meshes for unstructured input data, see for instance [Bommes et al. 2009, Alexa et al. 2000, Springborn et al. 2008]. They have been used with success in the architectural context, e.g., by [Bo et al. 2011] and [Sechelmann et al. 2013].

The basis of our method are conformally equivalent triangle meshes as described by [Springborn et al. 2008]. The straight forward method to map a surface with this approach is to cut it open and map it to a rectangle domain. This method yields boundary aligned conformal maps that however do not match along the introduced cut, see Figure 1, right. How to generalize this method to overcome this limitation is the content of Section ??.

References

- ALEXA, M., COHEN-OR, D., AND LEVIN, D. 2000. As-rigid-as-possible shape interpolation. In *Proceedings of the 27th Annual Conference on Computer Graphics and Interactive Techniques*, ACM Press/Addison-Wesley Publishing Co., New York, NY, USA, SIGGRAPH '00, 157–164.
- BO, P., POTTSMANN, H., KILIAN, M., WANG, W., AND WALLNER, J. 2011. Circular arc structures. *ACM Trans. Graphics* 30, #101, 1–11. Proc. SIGGRAPH.
- BOMMES, D., ZIMMER, H., AND KOBELT, L. 2009. Mixed-integer quadrangulation. In *ACM SIGGRAPH 2009 Papers*, ACM, New York, NY, USA, SIGGRAPH '09, 77:1–77:10.
- MCNEEL. Paneling tool documentation. <http://wiki.mcneel.com/labs/panelingtools>.
- NIESER, M., PALACIOS, J., POLTHIER, K., AND ZHANG, E. 2012. Hexagonal global parameterization of arbitrary surfaces. *IEEE Trans. Vis. Comput. Graph.* 18, 6, 865–878.
- SCHIFTNER, A., HÖBINGER, M., WALLNER, J., AND POTTSMANN, H. 2009. Packing circles and spheres on surfaces. *ACM Trans. Graphics* 28, 5, #139, 1–8. Proc. SIGGRAPH Asia.
- SECHELMANN, S., RÖRIG, T., AND BOBENKO, A. I. 2013. Quasiisothermic mesh layout. In *Advances in Architectural Geometry 2012*, L. Hesselgren, S. Sharma, J. Wallner, N. Baldassini, P. Bompas, and J. Raynaud, Eds. Springer Vienna, 243–258.

- SPRINGBORN, B., SCHRÖDER, P., AND PINKALL, U. 2008. Conformal equivalence of triangle meshes. *ACM Trans. Graph.* 27, 3 (Aug.), 77:1–77:11.
- TROCHE, C. 2008. Planar hexagonal meshes by tangent plane intersection. In *Advances in Architectural Geometry 2008*, 57–64.
- ZIMMER, H., CAMPEN, M., HERKRATH, R., AND KOBBELT, L. 2013. Variational tangent plane intersection for planar polygonal meshing. In *Advances in Architectural Geometry 2012*, L. Hesselgren, S. Sharma, J. Wallner, N. Baldassini, P. Bompas, and J. Raynaud, Eds. Springer Vienna, 319–332.