Quasiisothermic Mesh Layout

Stefan Sechelmann

Institut für Mathematik, TU-Berlin

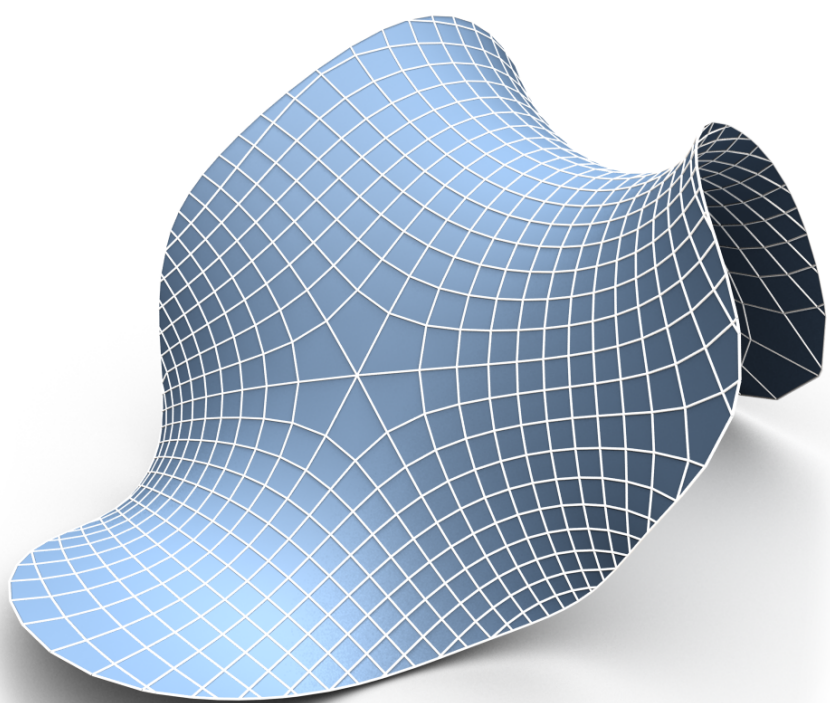
joint work with
Thilo Rörig and Alexander I. Bobenko

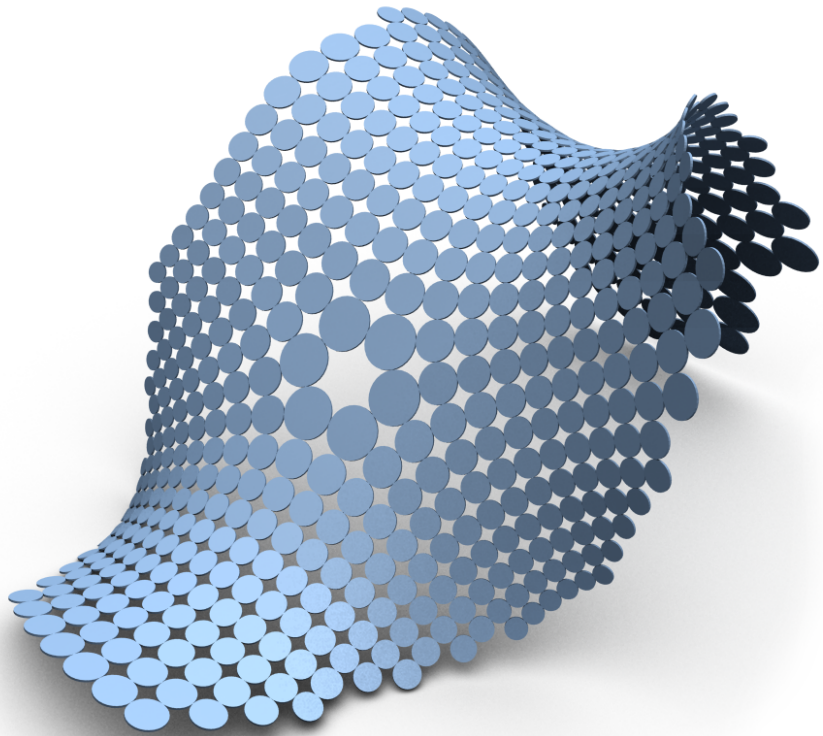
DFG Research Center MATHEON
SFB/TR 109: Discretization in Geometry and Dynamics

Quasiisothermic Mesh Layout

Triangulated Surface

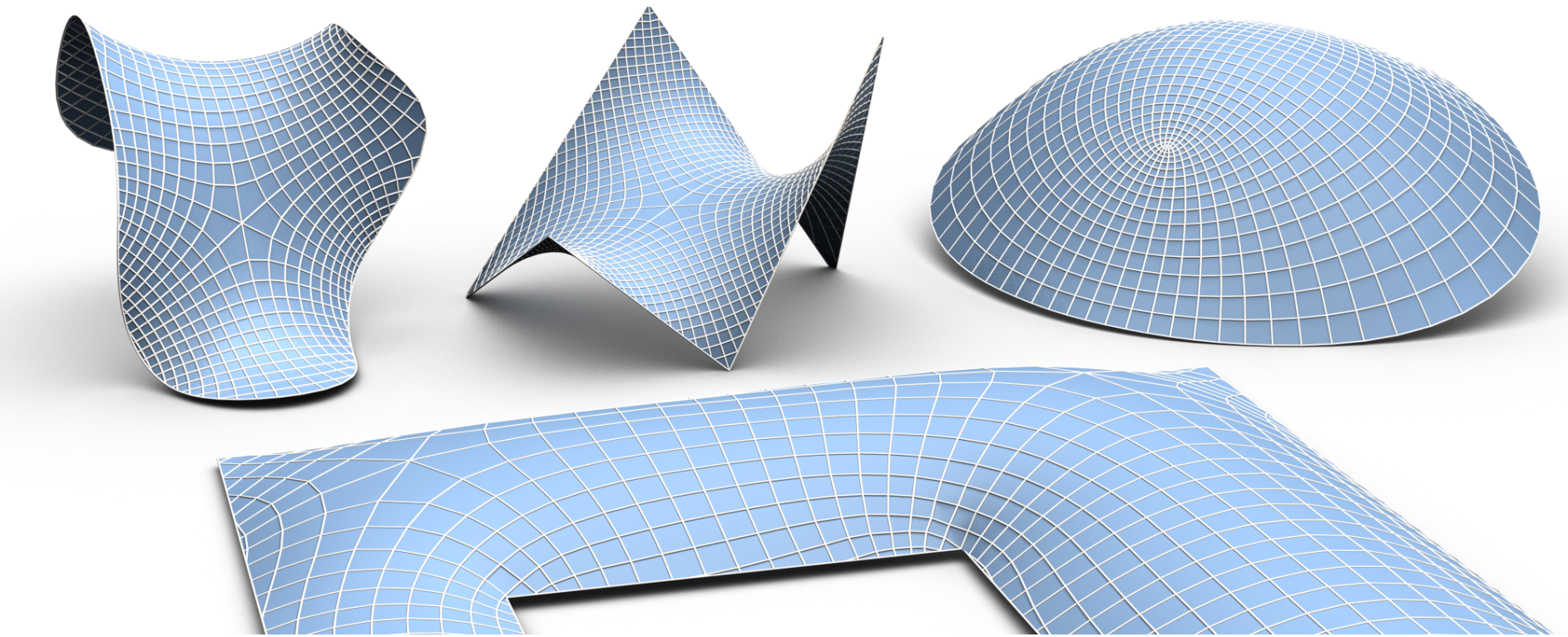
Quasiisothermic Mesh Layout



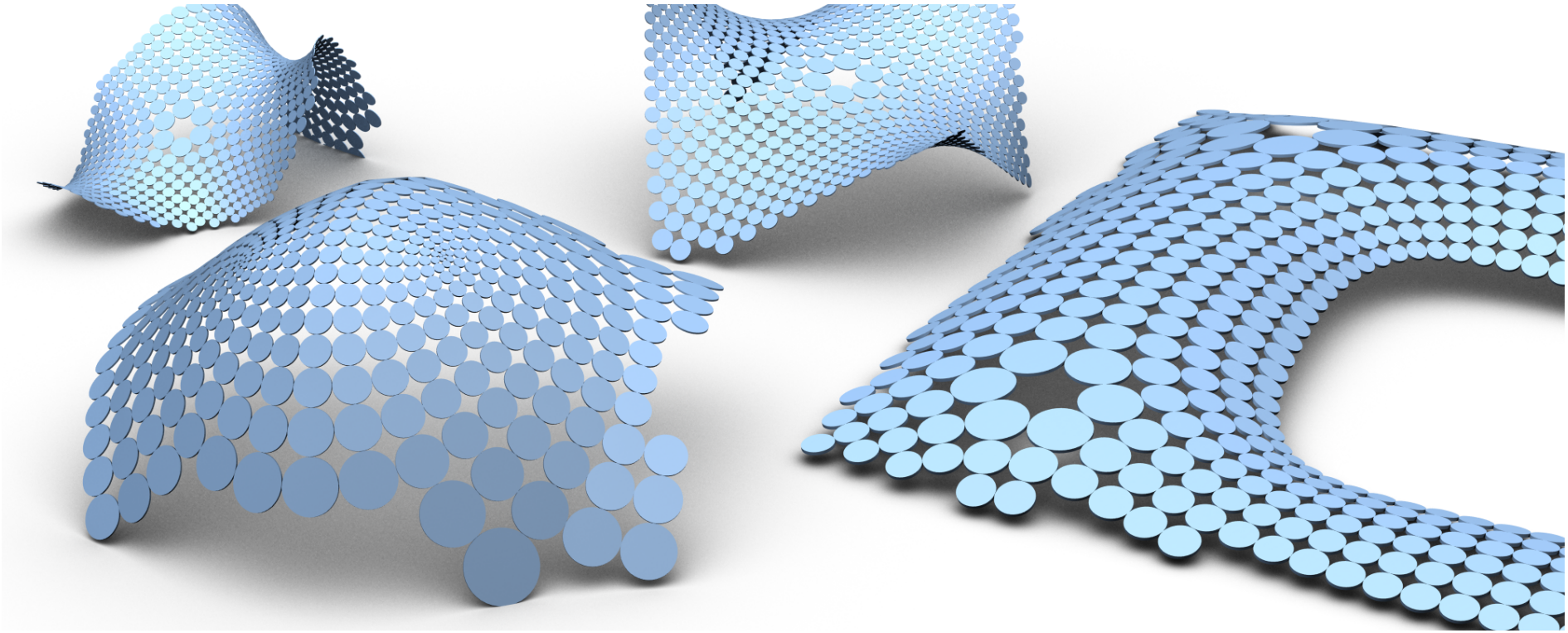


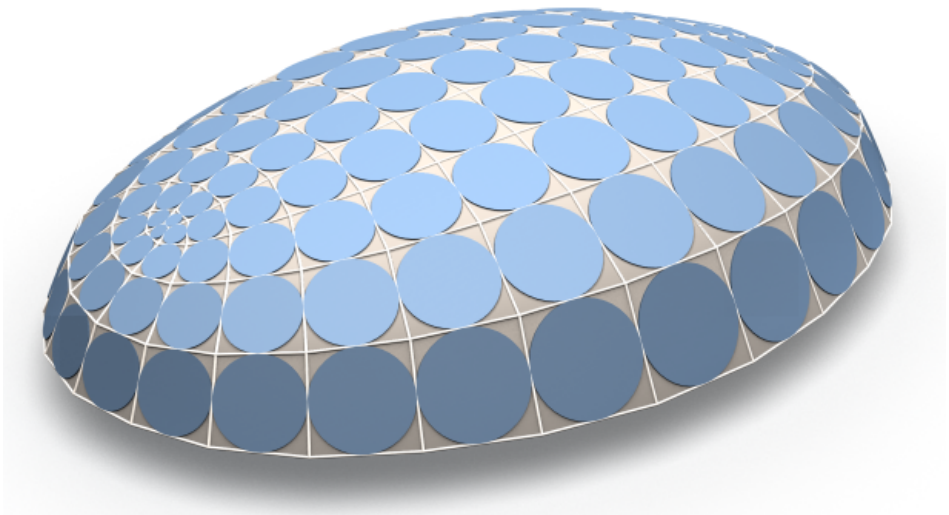
Optimized PQ-Mesh with touching incircles

More surfaces



More surfaces

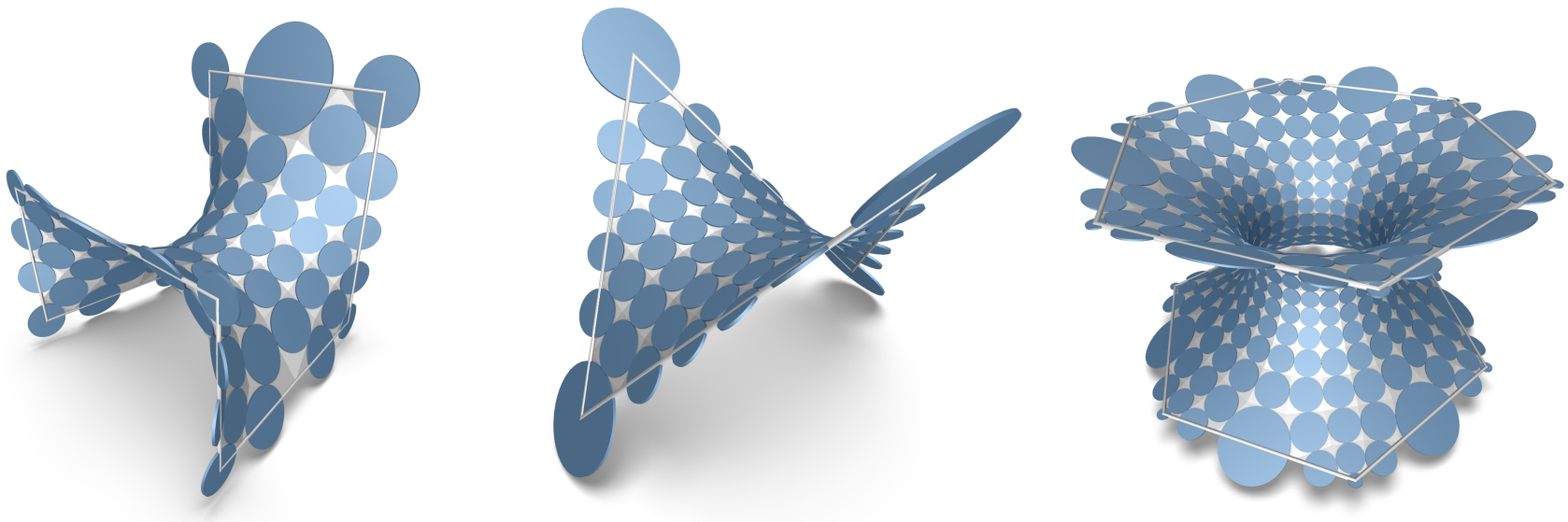




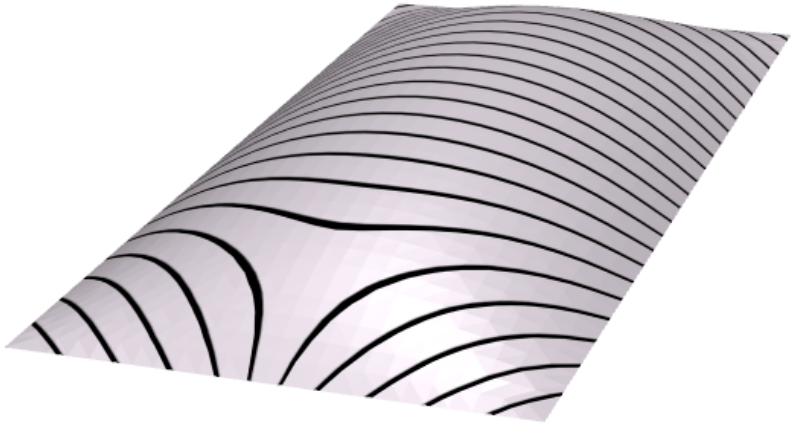
From Discrete Differential Geometry

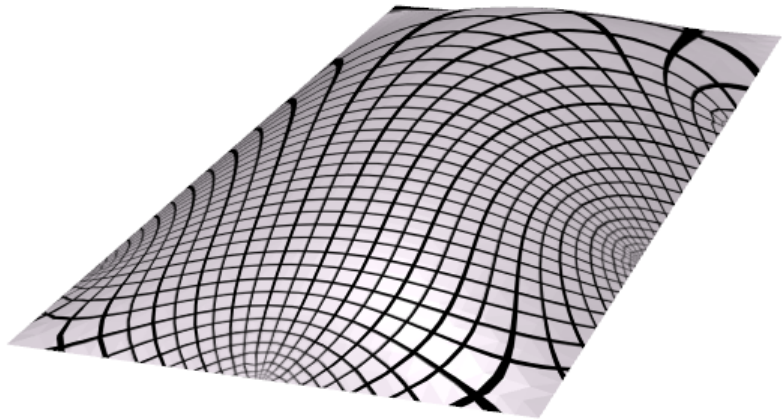
Planar quadrilaterals with touching incircles approximate conformal curvature line (*isothermically*) parameterized surfaces.

Was used before to create some minimal surfaces

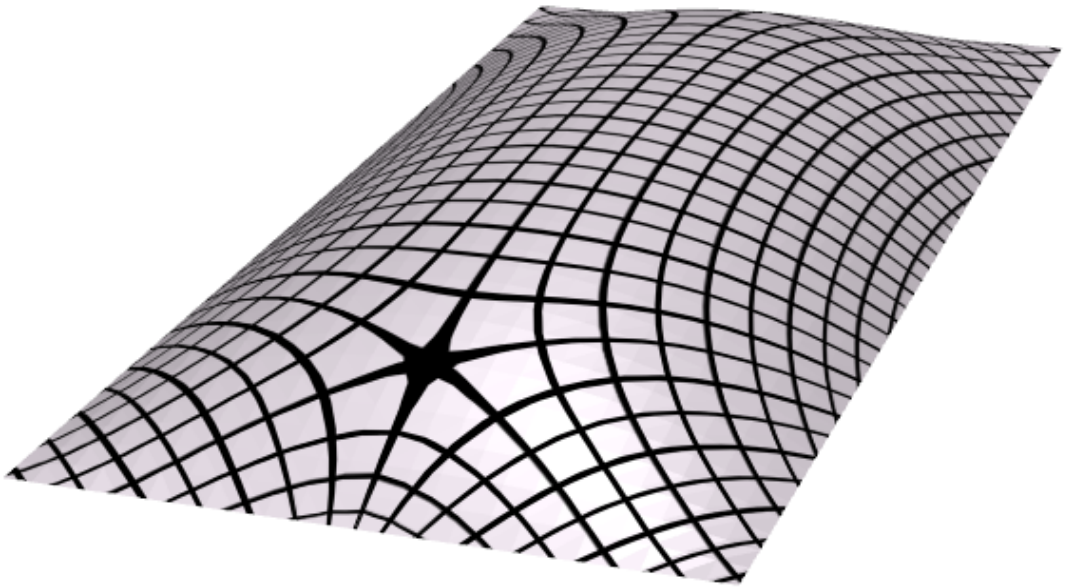


Isothermic Parameterization





Curvature Lines and Conformal $\longrightarrow$ Isothermic.



Algorithm Outline

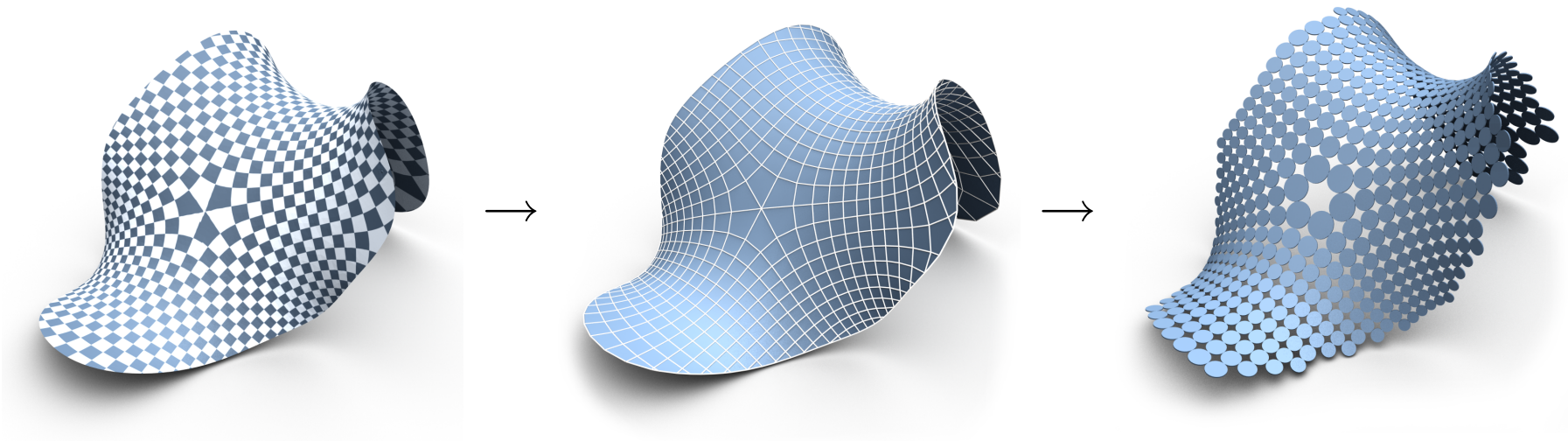
Isothermic Parameterization of a Triangle Mesh

↓

Quad-Panel Generation

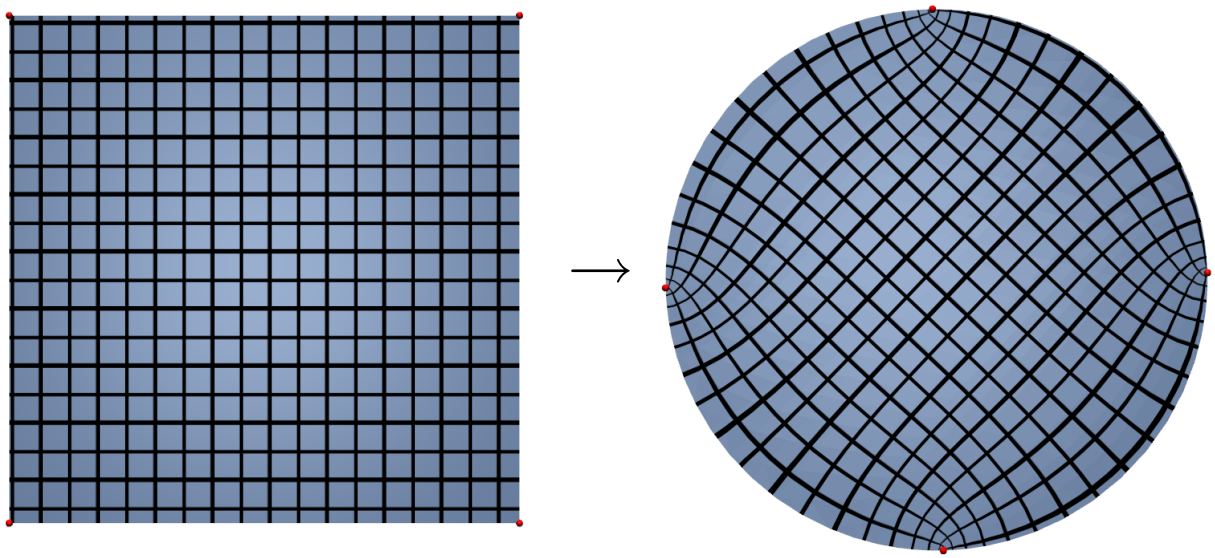
↓

PQ/Incircle Optimization



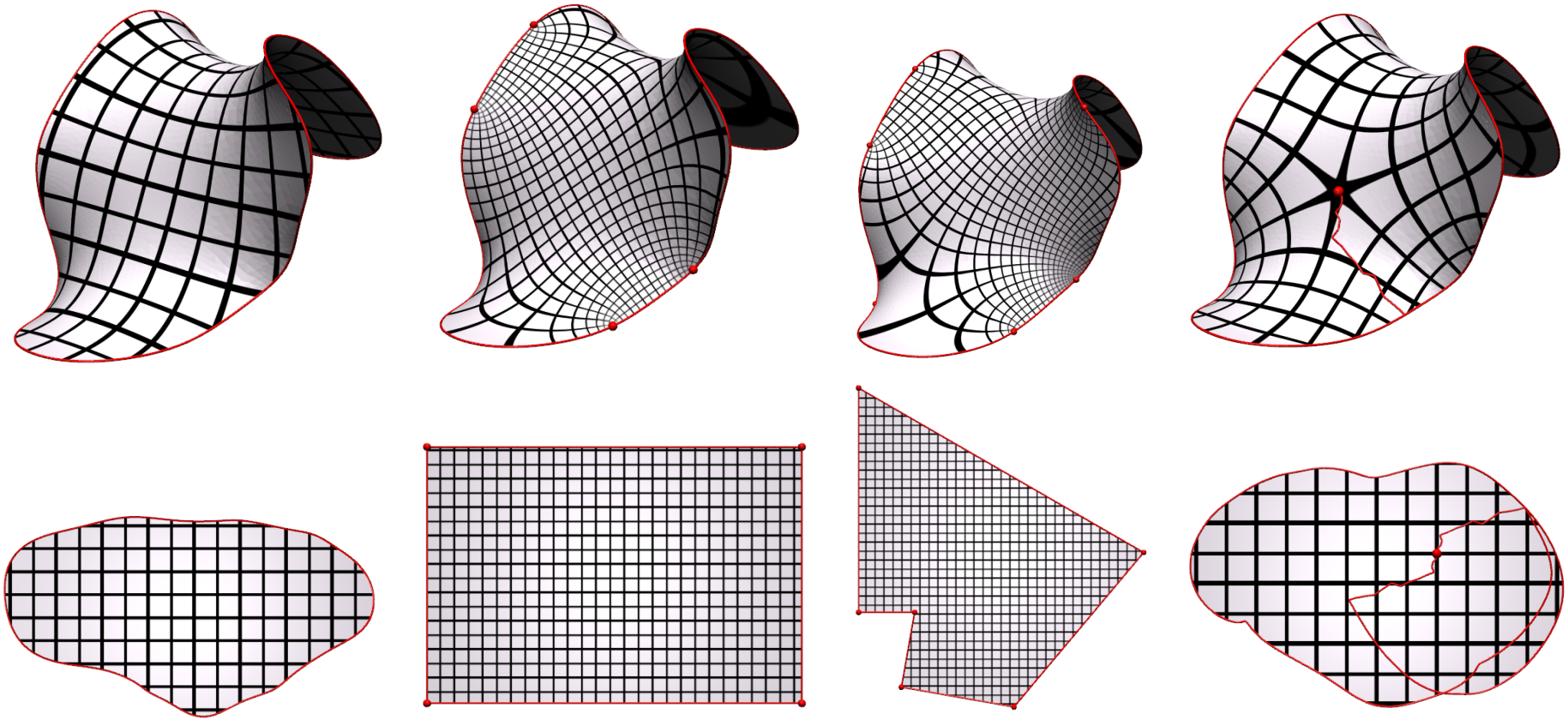
Discrete Conformal Parameterization

A map to the plane that mimmics the angle preserving property of smooth conformal maps.



Classical Riemann-Map to the Circle

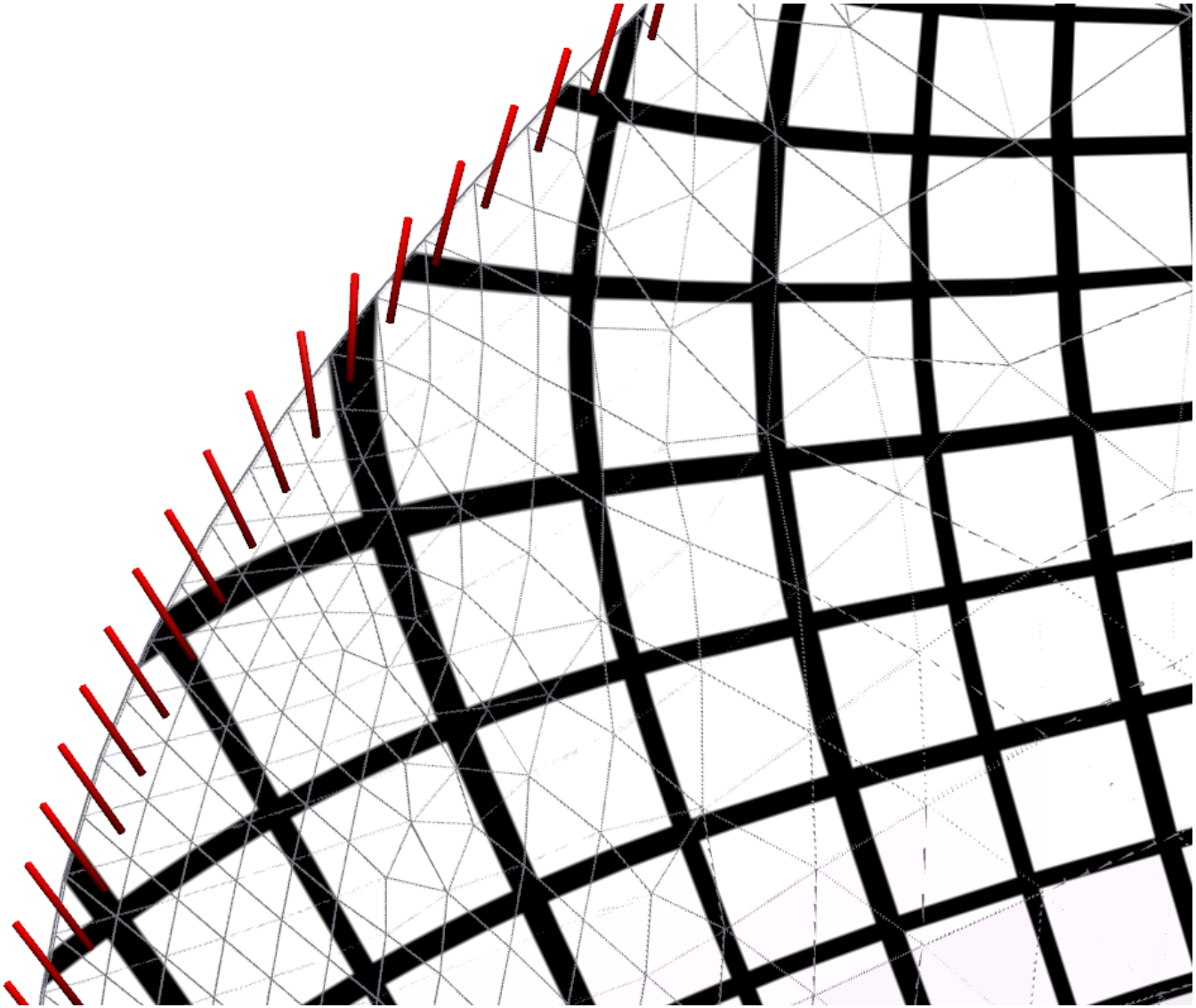
Conformal maps with different boundary conditions.



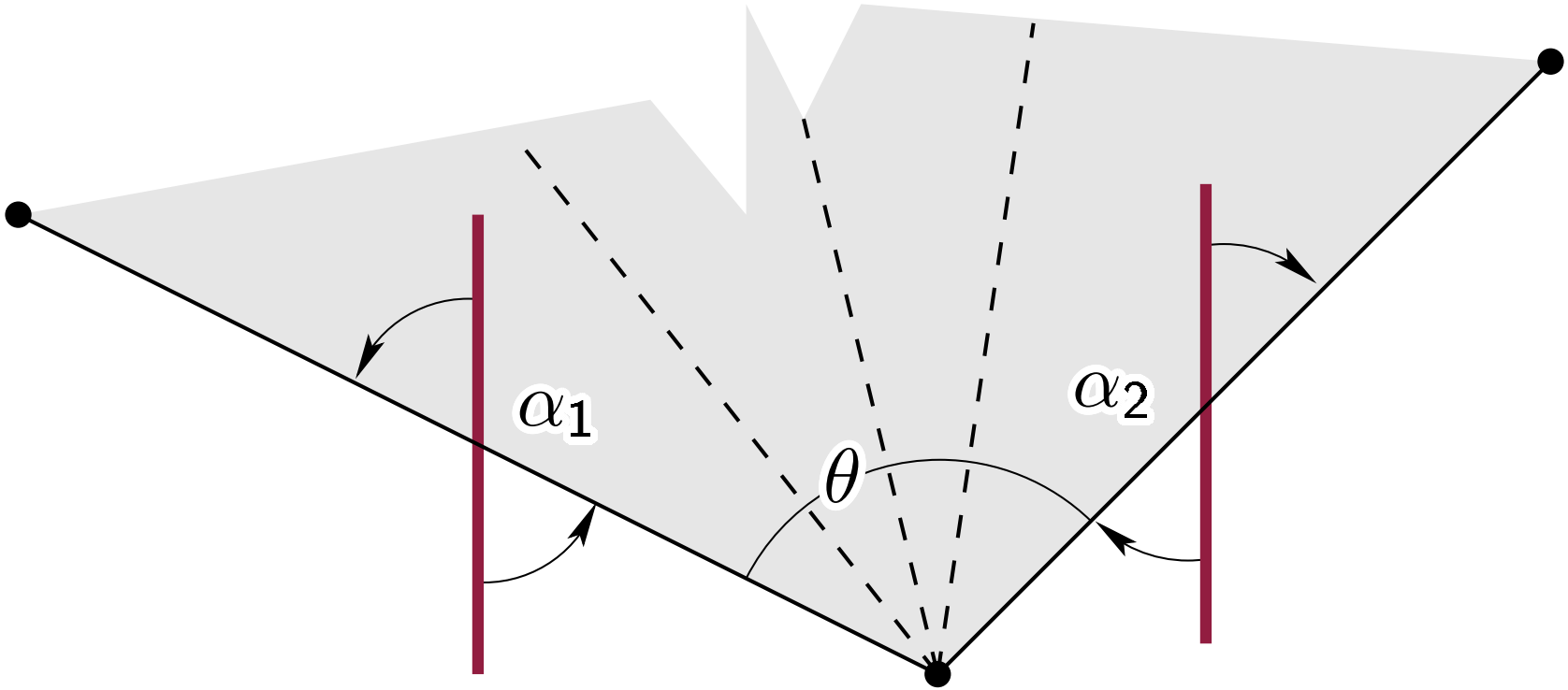
Algorithm: B. Springborn, P. Schröder, U. Pinkall. Conformal Equivalence of Triangle Meshes.

ACM Transactions on Graphics 27:3 [Proceedings of ACM SIGGRAPH 2008]

Curvature Boundary Conditions

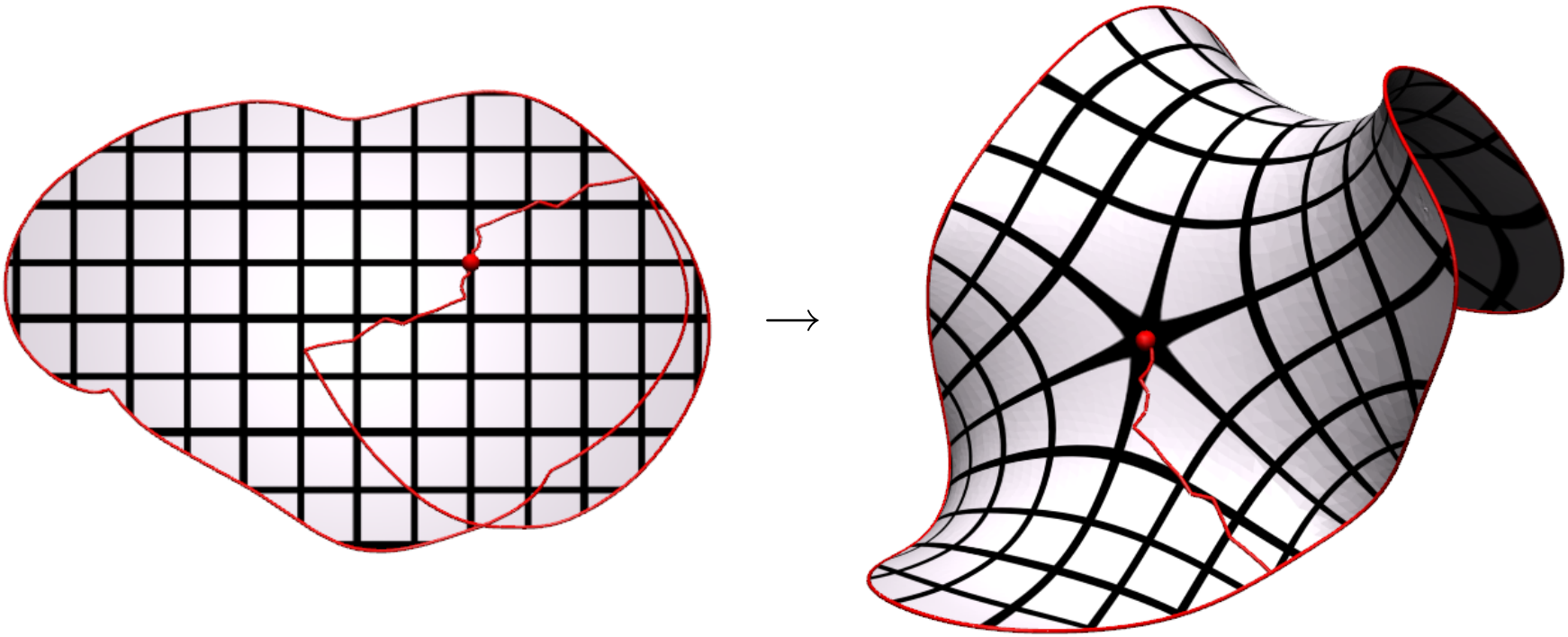


Curvature Boundary Conditions



Boundary angle sum θ at a boundary vertex

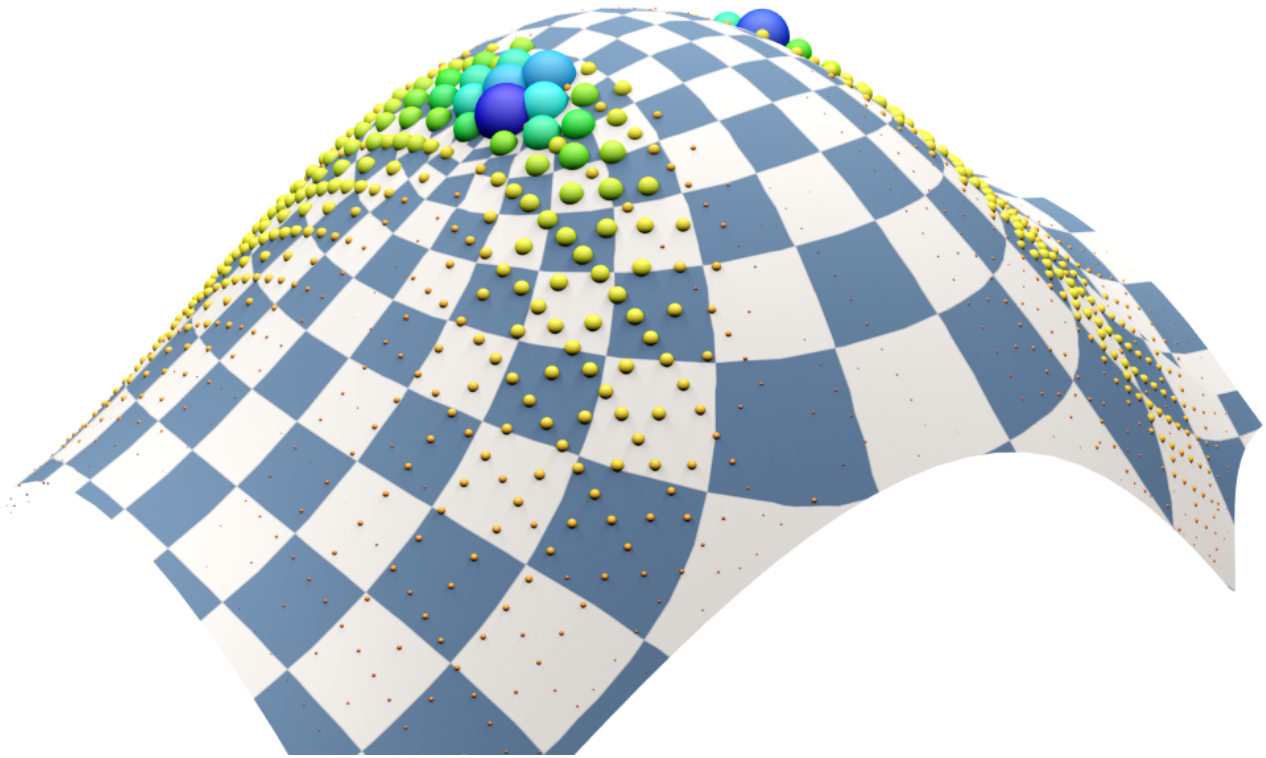
Curvature Boundary Conditions



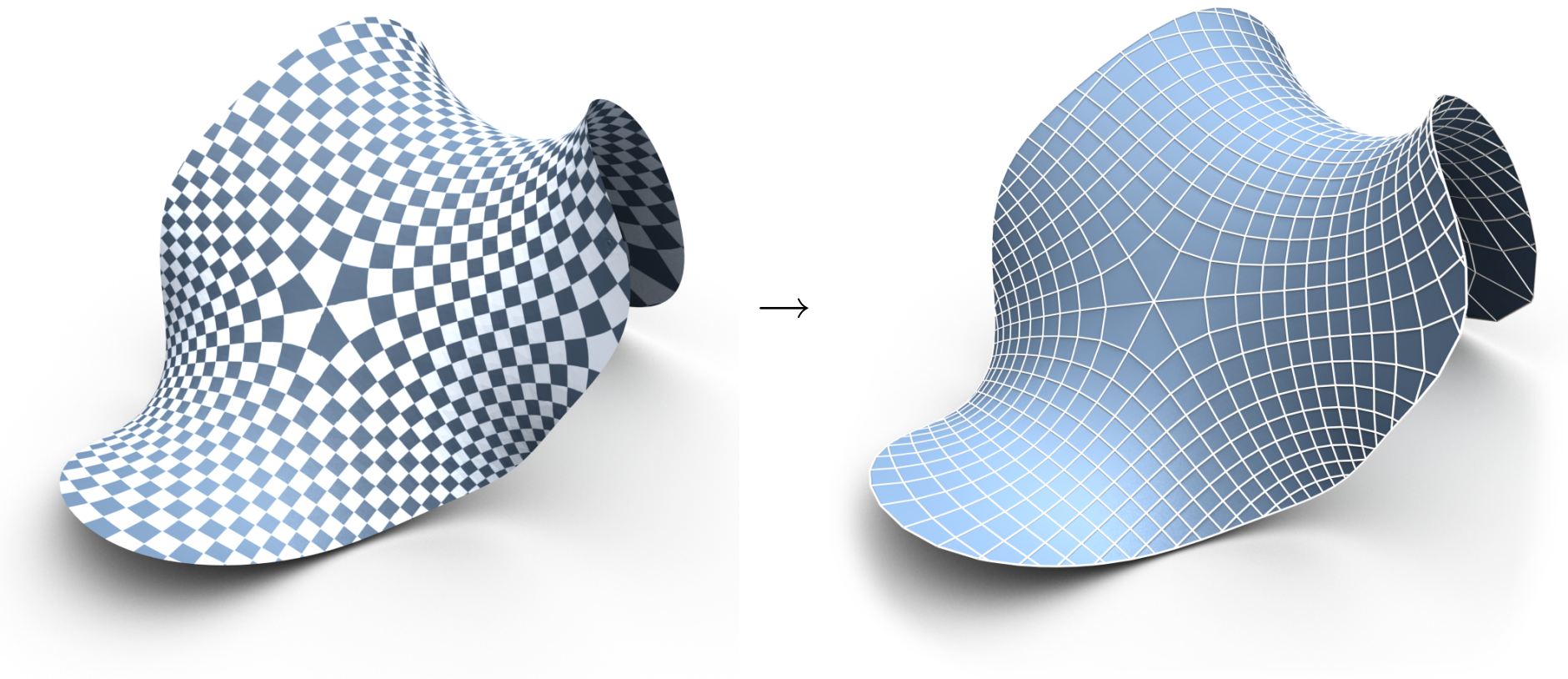
Curvature boundary conditions and a singularity with angle 540°

Quasiisothermic Parameterization

We call a conformal parameterization with curvature boundary conditions a *quasiisothermic parameterization*



Mesh Generation



Quad-Mesh Generation

Variational Principle

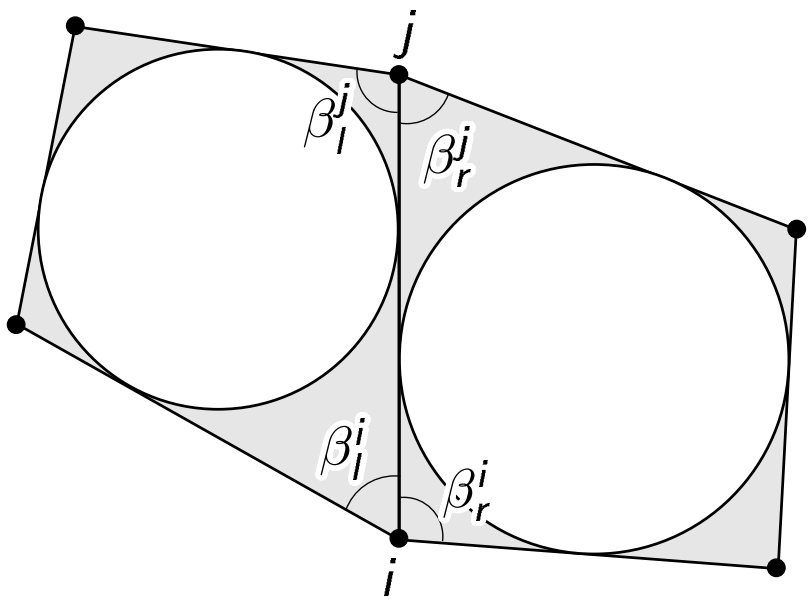
Linear combination of energies

$$E := \lambda_1 E_{\text{planar}} + \lambda_2 E_{\text{incircle}} + \lambda_3 E_{\text{touch}}$$

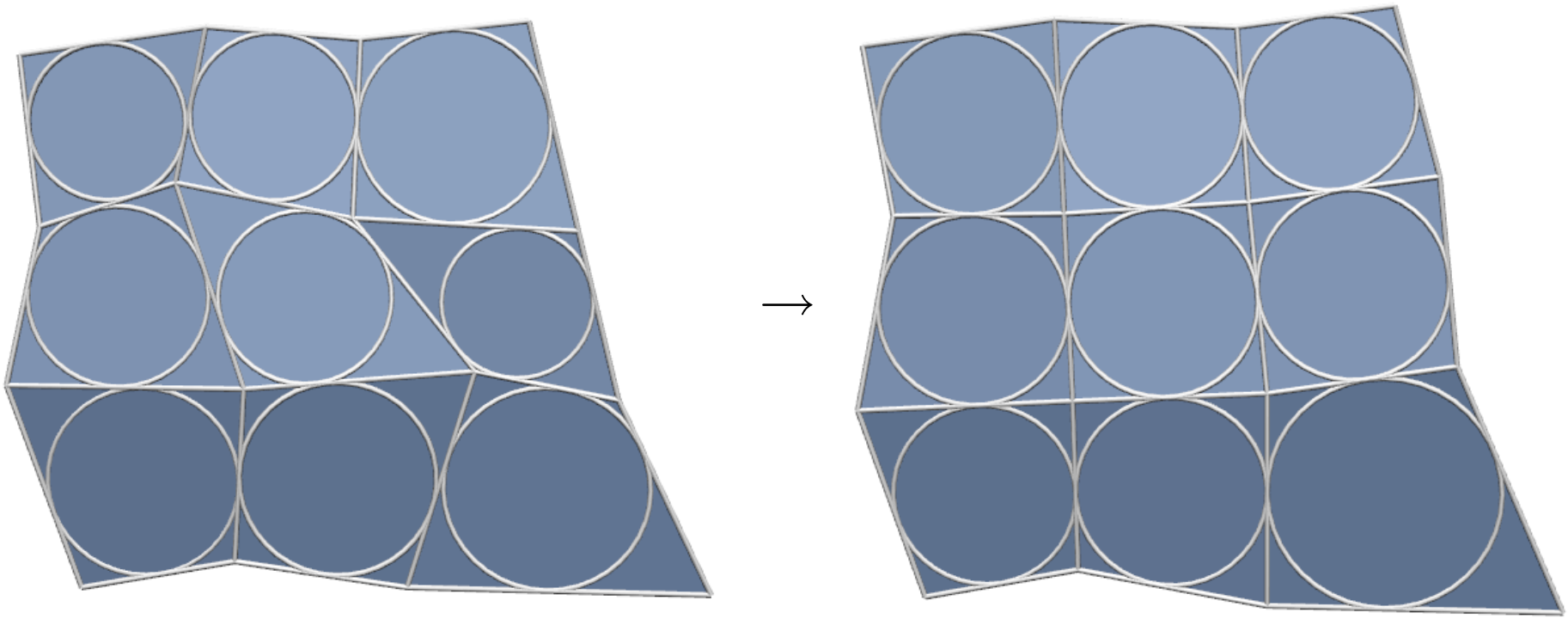
- E_{planar} is some planarity term
- E_{incircle} is due to A. Schiftner, M. Höbinger, J. Wallner, and H. Pottmann. 2009. *Packing circles and spheres on surfaces.* ACM Trans. Graph. 28
- E_{touch} critical for touching incircles

Touching Circles Energy

$$E_{\text{touch}}(ij) = \left(\cot \frac{\beta_l^j}{2} \cot \frac{\beta_r^i}{2} - \cot \frac{\beta_r^j}{2} \cot \frac{\beta_l^i}{2} \right)^2 .$$

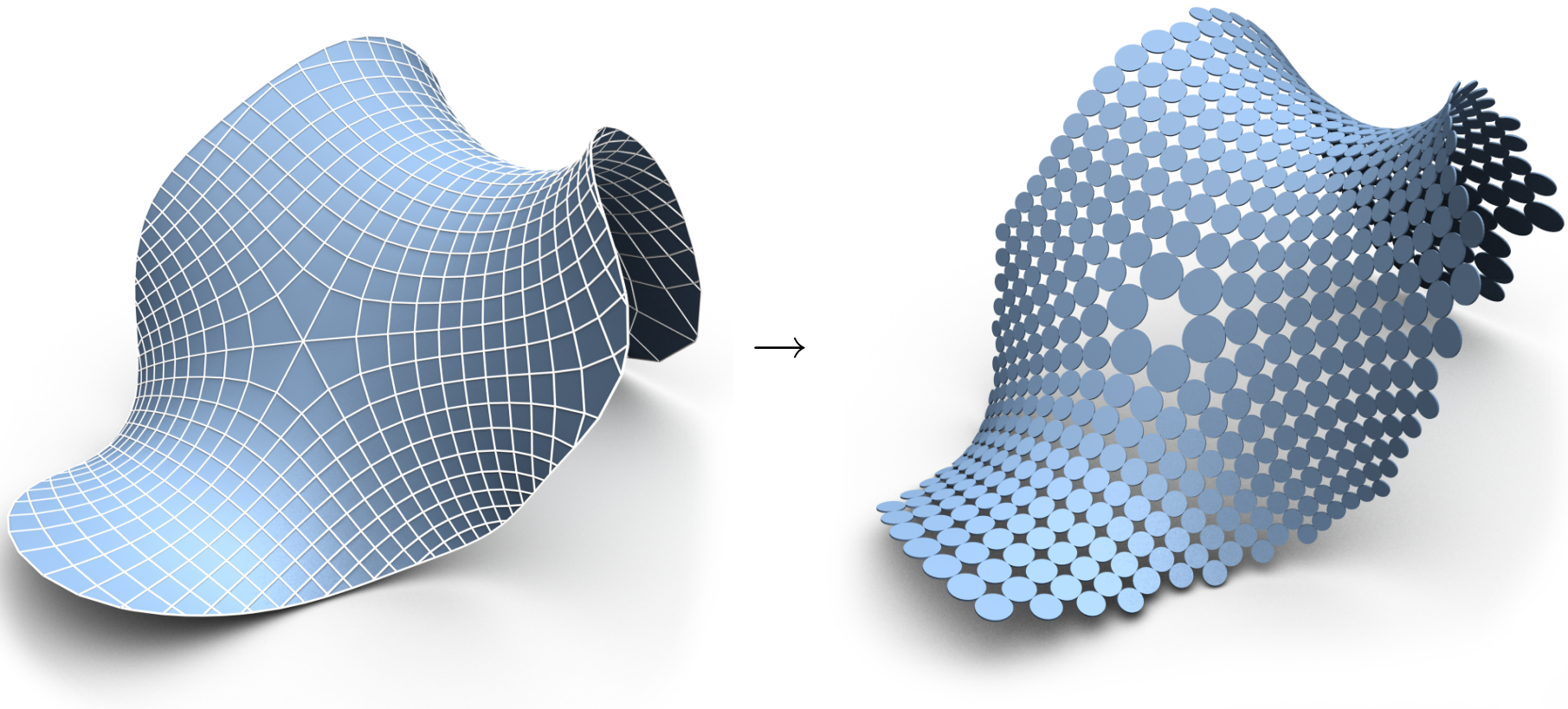
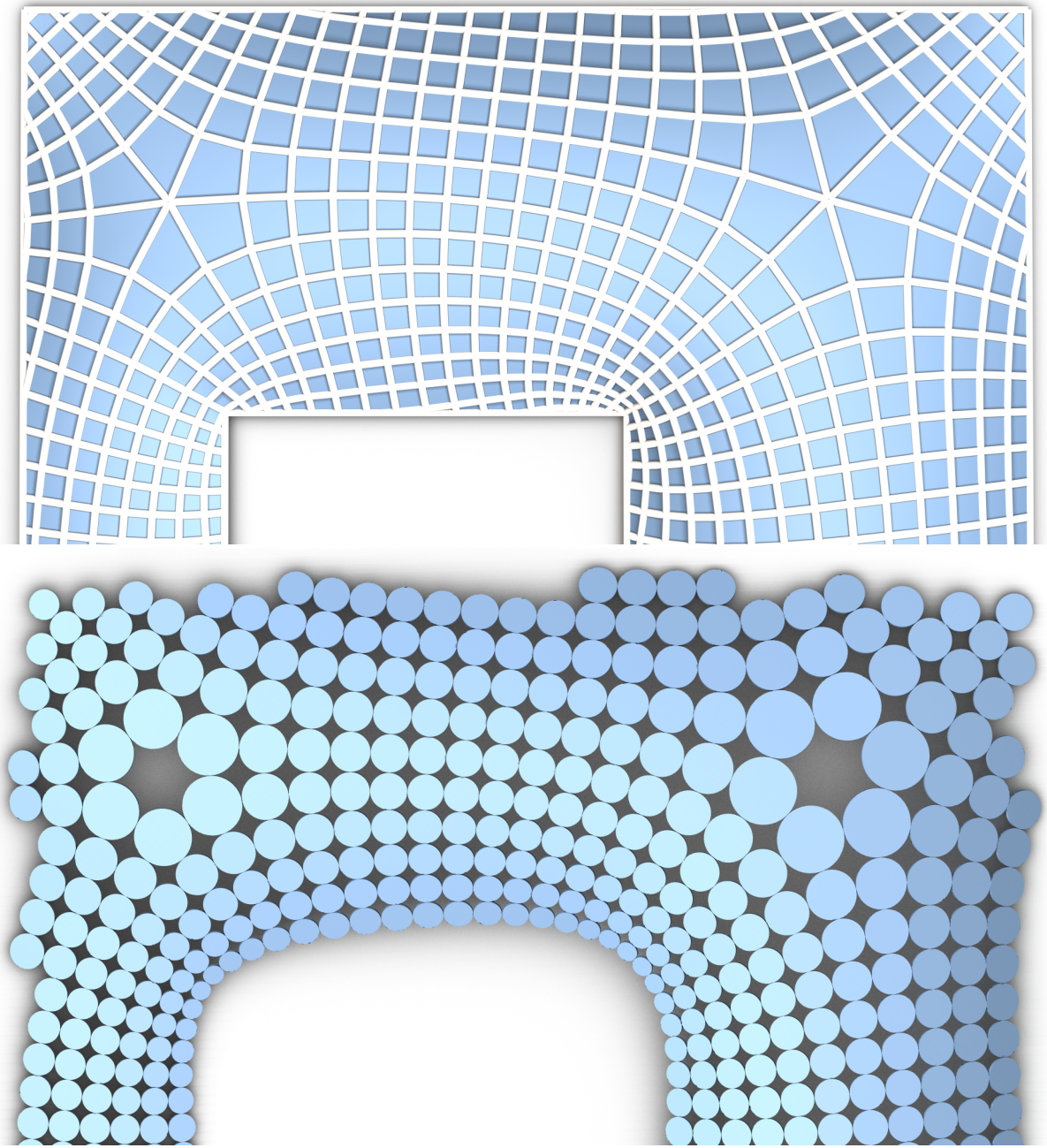


Curvature Boundary Conditions



Optimization of the energy

Optimization



Optimization of big meshes needs a good guess → Quasiisothermic parameterizations