Discrete differential geometry of surfaces. Variational principles, algorithms, and implementation

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2 Discrete conformal maps

Definition 1. *Two triangulations T and $\tilde{T}$ are* discrete conformal equivalent *if there is a map $u : V \to \mathbb{R}$ such that for any edge ij it is*

$$l_{ij} = e^{u_i + u_j} \tilde{l}_{ij}$$

Definition 2. *A* discrete flat metric *is a map $l : E \to \mathbb{R}_+$ such that triangle inequalities are satisfied and angle sums around each inner vertex are equal to 2π.*

2.1 Euclidean case

Construction of discrete flat metrics. A discrete Euclidean flat metric is the minimizer of a convex functional.

$$\begin{aligned} \lambda_{ij} &:= 2\log l_{ij} && (1)\\ \tilde{\lambda}_{ij} &:= \lambda_{ij} + u_i + u_j && (2)\\ f(u_i, u_j, u_k) &:= \alpha_i \tilde{\lambda}_{jk} + \alpha_j \tilde{\lambda}_{ki} + \alpha_k \tilde{\lambda}_{ij} + 2\left(Л(\alpha_i) + Л(\alpha_j) + Л(\alpha k)\right) && (3) \end{aligned}$$

Definition 3.

$$E_{Euc}(u) = \sum_{ijk \in F} f(u_i, u_j, u_k) - \frac{\pi}{2}\left(\tilde{\lambda}_{jk} + \tilde{\lambda}_{ki} + \tilde{\lambda}_{ij}\right) + \sum_{i \in V} 2\pi u_i \tag{4}$$

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