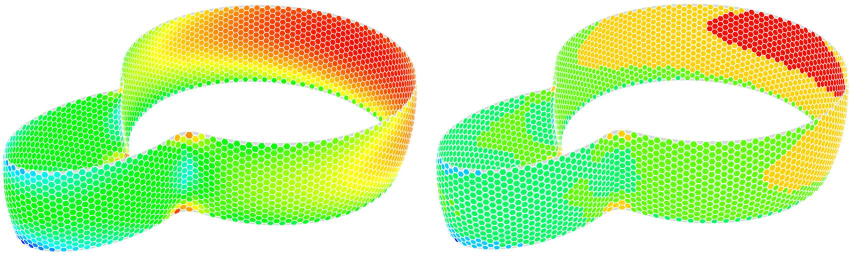

Figure 1: Panelization of a doubly curved variant. Left: Unquantized panelization. Right: Quantization to 11 panel sizes

{fig:quantization}

1 Rationalization: Hexagon optimization

{sec:regular_hexagons}

Starting from the conformal parameterization we optimize the obtained hex-mesh to have identical regular hexagons. We use a global optimization approach and define energies to achieve *planarity*, *regularity*, and *equality*.

Planarity. The planarity function is a simple adaptation of the usual energy used to planarize quad-meshes: A quadrilateral $\{A,B,C,D\}$ is planar if the volume of the tetrahedron $\{A,B,C,D\}$ is zero. So if we require the volume of all tetrahedra spanned by the vertices a polygon to be zero we obtain a planar polygon. This can easily be expressed in terms of determinants.

Regularity. A regular planar polygon is characterized by having equal edge lengths and equal angles at all vertices. As for planarity we define an energy that is minimized in case of regular polygons. The interior angle at a vertex of a regular p-gon is $\frac{p-2}{p}\pi$. So an energy E_{reg} that is minimized for a regular p-gon with vertices $\{v_1,\ldots,v_p\}$ and corresponding angles $\{\alpha_1,\ldots,\alpha_p\}$ is

$$E_{reg}(P) = \sum_{i=1}^{p}(\alpha_i - \frac{p-2}{p}\pi)^2 + \sum_{(v_i,v_{i+1})}(\|v_i - v_{i+1}\| - \ell_P)^2,$$

where ℓ_P is the desired target edge length for the polygon. In a first step, the target length can be chosen to be the average edge length of the polygon or the shortest edge length among the edges to avoid overlap. Note that, the normalization of the angles already implies planarity of the polygons. Nevertheless we consider the planarity energy since it increases the rate of convergence.

Starting from a cylindrical or conical periodic conformal parameterization we construct a hex-mesh that may or may not be aligned with the boundary. As a consequence of the conformality of the parameterization the angles of the hexagons are almost $\frac{2\pi}{3}$. In the following we do not work with a water tight mesh but split the surface into individual hexagonal panels. We optimize the edge lengths of the hexagons to be constant per face. To avoid overlap we choose the length of the shortest edge of each face as target length. Due to the conformal map used for parameterization, the hexagons are now already almost regular. We add the planarity and angle regularity constraint and obtain planar and regular hexagons. Each of the hexagons has its own constant edge length. We can rationalize the panelization further, by choosing a discrete set of edge lengths as target lengths for the polygons in the regularity functional. This quantization process is illustrated in Figure 1.

The range of lengths used depends on the initial hex-mesh constructed on the choosen target geometry. The effect of the different periodic conformal parametrizations on the quantization is shown in Figures **??** and **??**.