Variational Methods for Discrete Surface Parameterization. Applications and Implementation.

vorgelegt von
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Discrete Differential Geometry - Software Packages

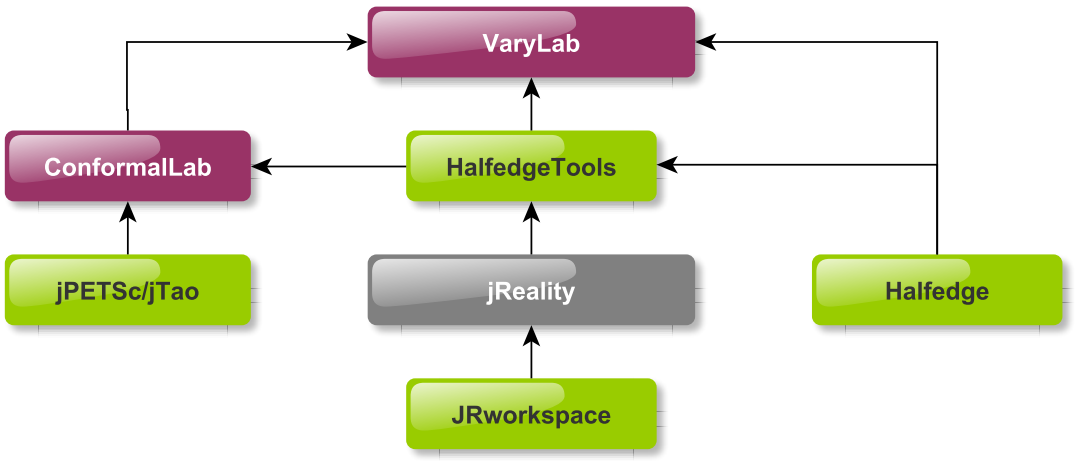

Figure 1.1: Software architecture and dependencies of the DDG Framework. JTEM library packages (green), mathematical software packages (red).

In the field of Discrete Differential Geometry (DDG) there is a special need for experiments conducted with the help of computer software. Especially if the methods of DDG are applied to problems in computer graphics, geometry processing, or architecture, algorithms have to be implemented and convincing examples have to be presented. Additionally a suitable visualization of the results has to be included in a state-of-the-art publication.

There is a growing knowledge of software development in the mathematical community. This is partly due to the curricula of universities which started to include programming courses for undergraduate students. This enables the students to extend their abilities of creating visualizations and mathematical software, where former generations of students solely used Mathematica and MatLab.

This Chapter is the description and getting-started manual of a set of software packages (DDG Framework) written in Java. They are specifically designed for the creation of custom interactive software for experiments with algorithms and geometries treated within DDG. Section 1.1 introduces the JRworkspace library of the Jtem project [jdt13b]. It is the foundation of any application created with the DDG Framework. It is also the user interface basis of Jreality, a mathematical visualization library that uses JRworkspace as plug-in and user interface tool [jdt13a]. Section 1.2 introduces the Halfedge and HalfedgeTools package. It implements a half-edge data structure and various user interface tools and algorithms for interaction and editing. In Section 1.3 we describe the software ConformalLab. This package implements the methods of the publications [BPS10, Sec12, SRB12, BSS]. Section 1.4 introduces VaryLab the software implementation of the methods described in the publications [LGSR11, LSRG12, SRB12]. This package is also released to partners of the development group as VaryLab[Gridshells], VaryLab[Ultimate], or even online as VaryLab[Service].

Figure 1.1 shows the dependencies of the packages. Every application depends on JRworkspace which implements plug-in functionality. It is the basis of the Jreality plug-in system. HalfedgeTools is using Jreality for visualization and is build on top of the Jtem project Halfedge. ConformalLab and VaryLab use Jpetsc/Jtao to perform numerical optimization. Their algorithms are implemented as JRworkspace plug-ins.

The development of the described software is joint work with Thilo Rörig (HalfedgeTools, VaryLab), the Jreality members [jdt13a], Hannes Sommer (Jpetsc/Jtao) [Som10], Paul Peters (JRworkspace), and Boris Springborn (Halfedge).

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