and,

$$\mathbf{T}_{\hat{7},7} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0.0 & 0.0 \\ \sin\theta_i & \cos\theta_i & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}. \tag{16}$$

One should note that numerical values for all unknown variables related to geometric dimensions, i.e., the Ls, are specified on Fig. 11.

3.2 Calibrated Homogeneous Transforms

The calibrated FK relationship can be expressed by the following equation:

$$\begin{aligned} \mathbf{T}_{0,IDM} = &(\mathbf{T}_{0,\hat{1}}\mathbf{T}_{\hat{1},cal}\mathbf{T}_{\hat{1},1})(\mathbf{T}_{1,\hat{2}}\mathbf{T}_{\hat{2},cal}\mathbf{T}_{\hat{2},2})(\mathbf{T}_{2,\hat{3}}\mathbf{T}_{\hat{3},cal}\mathbf{T}_{\hat{3},3})(\mathbf{T}_{3,\hat{4}}\mathbf{T}_{\hat{4},cal}\mathbf{T}_{\hat{4},4}) \\ &(\mathbf{T}_{4,\hat{5}}\mathbf{T}_{\hat{5},cal}\mathbf{T}_{\hat{5},5})(\mathbf{T}_{5,\hat{6}}\mathbf{T}_{\hat{6},cal}\mathbf{T}_{\hat{6},6})(\mathbf{T}_{6,\hat{7}}\mathbf{T}_{\hat{7},cal}\mathbf{T}_{\hat{7},7})(\mathbf{T}_{7,tool}\mathbf{T}_{tool,IDM}\mathbf{T}_{IDM,cal}), \end{aligned} \tag{17}$$

where

$$\mathbf{T}_{\hat{i},cal} = \mathbf{T}_{\hat{i},geometric}\mathbf{T}_{\hat{i},deflection}, \text{ for } \quad i \ = \ 1,2,3,4,5,6,7,IDM. \tag{18}$$

On one side, the geometric calibration transforms are defined as:

$$\mathbf{T}_{\hat{i},geometric} = Trans(\delta^g_{i,x}, \delta^g_{i,y}, \delta^g_{i,z})R_z(\epsilon^g_{i,z})R_y(\epsilon^g_{i,y})R_x(\epsilon^g_{i,x}), \tag{19}$$

which can be reformulated as follows:

$$\begin{aligned} \mathbf{T}_{\hat{i},geometric} = &\begin{bmatrix} 1.0 & 0.0 & 0.0 & \delta^g_{i,x} \\ 0.0 & 1.0 & 0.0 & \delta^g_{i,y} \\ 0.0 & 0.0 & 1.0 & \delta^g_{i,y} \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} \cos(\epsilon^g_{i,z}) & -\sin(\epsilon^g_{i,z}) & 0.0 & 0.0 \\ \sin(\epsilon^g_{i,z}) & \cos(\epsilon^g_{i,z}) & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \\ &\begin{bmatrix} \cos(\epsilon^g_{i,y}) & 0.0 & \sin(\epsilon^g_{i,y}) & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ -\sin(\epsilon^g_{i,y}) & 0.0 & \cos(\epsilon^g_{i,y}) & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & \cos(\epsilon^g_{i,x}) & -\sin(\epsilon^g_{i,x}) & 0.0 \\ 0.0 & \sin(\epsilon^g_{i,x}) & \cos(\epsilon^g_{i,x}) & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}. \end{aligned} \tag{20}$$

One should note that the ZYX Euler-angles convention is selected here in order to construct the rotation part of the transform matrices. Moreover, based on what was stated in Sub-section 2.1.4, $\{\delta^g_{i,x}$, $\delta^g_{i,y}$, $\delta^g_{i,z}$, $\epsilon^g_{i,x}$, $\epsilon^g_{i,y}$, $\epsilon^g_{i,z}\}$ are the six geometric correction parameters in translation and in rotation along the x, y, and z axes, respectively, for each rigid body. These six variables are the ones directly optimized during the calibration process in order to correct the geometric defects on each of the eight rigid bodies forming the robotic arm.

On the other side, the deflection calibration transforms are defined as:

$$\mathbf{T}_{\hat{i},deflection} = Trans(\delta^d_{i,x}, \delta^d_{i,y}, \delta^d_{i,z})R_x(\epsilon^d_{i,x})R_y(\epsilon^d_{i,y})R_z(\epsilon^d_{i,z}), \tag{21}$$

which can be reformulated as follows:

$$\mathbf{T}_{\hat{\imath},deflection} = \begin{bmatrix} 1.0 & 0.0 & 0.0 & \delta^d_{i,x} \\ 0.0 & 1.0 & 0.0 & \delta^d_{i,y} \\ 0.0 & 0.0 & 1.0 & \delta^d_{i,y} \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & \cos(\epsilon^d_{i,x}) & -\sin(\epsilon^d_{i,x}) & 0.0 \\ 0.0 & \sin(\epsilon^d_{i,x}) & \cos(\epsilon^d_{i,x}) & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$
$$\begin{bmatrix} \cos(\epsilon^d_{i,y}) & 0.0 & \sin(\epsilon^d_{i,y}) & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ -\sin(\epsilon^d_{i,y}) & 0.0 & \cos(\epsilon^d_{i,y}) & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} \cos(\epsilon^d_{i,z}) & -\sin(\epsilon^d_{i,z}) & 0.0 & 0.0 \\ \sin(\epsilon^d_{i,z}) & \cos(\epsilon^d_{i,z}) & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}. \quad (22)$$

One should note that the XYZ Euler-angles convention is selected here to construct the rotation part of the transform matrices. Moreover, based on what was stated in Sub-Section 2.1.4, $\{\delta^d_{i,x}, \delta^d_{i,y}, \delta^d_{i,z}, \epsilon^d_{i,x}, \epsilon^d_{i,y}, \epsilon^d_{i,z}\}$ are the six elasto-static correction parameters in translation and in rotation along the X, Y, and Z axes, respectively, for each rigid body.

As it was previously explained, however, these six variables are not the ones directly optimized during the calibration process but rather defined as follows:

$$\begin{bmatrix} \delta^d_{i,x} \\ \delta^d_{i,y} \\ \delta^d_{i,y} \\ \epsilon^d_{i,x} \\ \epsilon^d_{i,y} \\ \epsilon^d_{i,z} \end{bmatrix} = \begin{bmatrix} c_{i,x} & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & c_{i,y} & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & c_{i,z} & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & c_{i,rx} & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & c_{i,ry} & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & c_{i,rz} \end{bmatrix} \begin{bmatrix} F_{i,x} \\ F_{i,y} \\ F_{i,y} \\ \tau_{i,x} \\ \tau_{i,y} \\ \tau_{i,z} \end{bmatrix}. \quad (23)$$

As seen from the Eq. (23), in order to take into account the deflections induced by the gravitational effects applied on each of the eight rigid bodies forming the robotic arm, the compliance parameters $\{c_{i,x}, c_{i,y}, c_{i,z}, c_{i,rx}, c_{i,ry}, c_{i,rz}\}$ are the variables that are directly optimized during the calibration process, since they represent constant mechanical properties for each rigid bodies. One should note that the variables $\{F_{i,x}, F_{i,y}, F_{i,z}, \tau_{i,x}, \tau_{i,y}, \tau_{i,z}\}$ represent the six components of the Cartesian wrench applied on each rigid body. These Cartesian wrenches are calculated by the gravity model implemented in the robot base (for more details, see the document Gravity Model Report (MS7) - CTR_PR37_D02rev2.pdf).