# ELECTROWEAK RADIATIVE CORRECTIONS AND THE VALUE OF $\sin^2\theta_W$.

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We have calculated analytically the first-order electroweak radiative corrections to differential and total charged- and neutral-current cross sections and to the parity violating asymmetry observed in ed scattering at SLAC. These corrections reduce slightly the value of $\sin^2\theta$, defined in the $\overline{\mathrm{MS}}$ scheme with $\mu = M_W$. A similar result was previously obtained keeping only "leading logarithms" but this is fortuitous.

If the SU(2) × U(1) model of electroweak interactions is correct, different experiments must yield the same value for $\sin^2\theta_W$. The data have usually been analysed treating the electroweak interactions in Born approximation. Higher-order corrections, which differ between experiments, could easily change the effective values of $\sin^2\theta_W$ by ±0.02, which is bigger than the errors in some experiments and enough to change the predicted values of $M_{W,Z}$ by 4 GeV. These corrections must be included to test the theory accurately, including its renormalizability, and to obtain a precise value of $\sin^2\theta_W$ for comparison with the predictions of more comprehensive schemes which unify SU(2) and U(1).

We define

$$\delta(\mu) = \sin^2\theta(\mu) - \sin^2\theta^{\mathrm{expt}} ,$$

where $\sin^2\theta^{\mathrm{expt}}$ is the value obtained using the Born approximation and $\sin^2\theta(\mu)$ is the theoretical value defined in some renormalization scheme at a scale $\mu$. The radiative correction $\delta$ contains two parts. The first includes propagator and vertex corrections. The second, which is $\mu$ independent, includes the contributions of box diagrams and bremsstrahlung; these contributions are experiment dependent and *cannot* be absorbed in a universal running $\sin^2\theta$. To lowest order $\delta$ may contain terms

$$\mathrm{O}((\alpha/\pi)\ln M_W^2/\mu^2) + \mathrm{O}((\alpha/\pi)\ln M_W^2/q^2) + \mathrm{O}((\alpha/\pi)\ln m_f^2/q^2) + \mathrm{O}(\alpha/\pi \sin^2\theta) + \mathrm{O}(\alpha/\pi),$$

where $m_f$ is a fermion mass. With $\mu = M_W$, which simplifies the GUT predictions and the formulae for $\delta$, it is seen that $\delta$ is ±0.02 in order of magnitude.

Values of $\sin^2\theta$ have been obtained from measurements of:

(1) $\nu N \to \nu x/\nu N \to \mu x$.

(2) The difference in scattering of left- and right-handed electrons on deuterium observed at SLAC.

(3) $\nu_\mu e \to \nu_\mu e$.

(4) The forward backward asymmetry in $e^+e^- \to \mu^+\mu^-$.

(5) Parity violating optical effects in atoms.

We have calculated all the first-order electroweak radiative corrections for (1) and (2) in the minimal SU(2) × U(1) model with one Higgs doublet, up to but not including terms of $\mathrm{O}(m_f^2/q^2)$. We have analytic results for the corrections to $\sigma^\nu$, $d\sigma^\nu/dy$ and $d^2\sigma^\nu/dx\,dy$ for charged and neutral currents; only the integrals over parton distributions for the SLAC experiment must be done numerically.

Partial results for (1) and (2) have been reported previously [1], omitting non-logarithmic terms in $\delta$ and/or bremsstrahlung. The purely electromagnetic corrections to charged-current cross sections have been calculated numerically [2]. Recently we received a preprint by Marciano and Sirlin [3] who have carried out a complete calculation of the corrections to the ratio of the total neutral- and charged-current cross sections, allowing for the experimental cuts in an approximate way; we will compare our results with theirs.

 

In ref. [4] corrections to (2) are calculated but only the final answers can be compared since unitary gauge was used whereas we employ the renormalizable Feynman–'t Hooft gauge. Corrections to $\nu_\mu e$ scattering are given in ref. [5] and to $e^+e^-$ annihilation in ref. [6]. One of us (J.F.W.) has studied (5); the results are reported separately [7].

We assume that $\sin^2\theta$ has already been extracted from experiment using the Born approximation for the electroweak interactions but taking QCD scaling violations and the quark–antiquark sea into account, as discussed for example in ref. [8]. The value of $\delta$ is calculated using a valence parton model, with exact scaling; the effects of the neglected scaling violations and the $q\bar{q}$ sea are small corrections to a correction. Infrared divergences in intermediate stages of the calculation are dealt with using dimensional regularization and we keep $m_f \neq 0$ since the quantities calculated are not mass finite in general. We use the $\overline{\mathrm{MS}}$ scheme, in which all Ward identities are automatically respected. The only free parameter is $\sin^2\theta(\mu)$; $\alpha^{\mathrm{em}}$ and $G_F$ are taken from experiment, taking into account electroweak radiative corrections to muon decay.

The full results, especially for $d\sigma/dy$ and $d^2\sigma/dx dy$, are lengthy and will be given elsewhere [9] together with details of the calculations. Most of the virtual contributions were compared to published results and we checked $\sigma^\nu$ by integrating using two very different choices of variables.

The corrections to $\sin^2\theta_w$ measured in various neutrino experiments are given in table 1. When the different renormalization schemes are taken into account we agree completely with Marciano and Sirlin [3] in the case of the CHARM experiment. In other experiments with higher hadronic energy cuts we differ slightly; this may be because they used leading logarithmic results to correct for these cuts whereas we used the exact result. The principal contribution to $\delta$ comes from the electromagnetic corrections to charged-current scattering; the contribution of virtual weak graphs is small provided $M_{\mathrm{top}} \lesssim M_w$ and $M_{\mathrm{Higgs}} < O(1\ \mathrm{TeV})$ (again in agreement with ref. [3]). The fact that the leading logarithmic contributions to $\delta$ are similar to the exact result is fortuitous. The results are insensitive to the choice of parton distribution and we expect the error in $\delta$ due to the neglect of the $q\bar{q}$ sea and scaling violations to be O(10%).

In the case of the parity violating asymmetry in electron–deuteron scattering, observed at SLAC, $\delta = -0.007$ in the central kinematical region. The data [10,8] therefore imply $\sin^2\theta(M_w) = 0.216 \pm 0.015$. Although the sign and magnitude of $\delta$ turn out to be the same as in neutrino experiments, so that the resulting values of $\sin^2\theta(M_w)$ are consistent, the virtual weak contributions account for about half of $\delta$ in this case. Complete results as a function of the kinematic variables will be given in ref. [9]. Our value of $\delta$ is about 20% less than the value found in ref. [4]. This may be due to the neglect of corrections associated with the quark legs in the final results given in ref. [4] and the different choice of parton distributions.

In minimal SU(2) × U(1):

$$M_w = \frac{38.51\,[37.28]}{\sin\theta(M_w)}\ \mathrm{GeV}, \quad M_z = \frac{77.09\,[74.56]}{\sin 2\theta(M_w)}\ \mathrm{GeV},$$

where the first numbers include all first-order electroweak radiative corrections [11,3] and the bracketed numbers include only the purely electromagnetic corrections to muon decay, traditionally taken into account in defining $G_F$. Thus, for example, the CHARM experiment implies $M_w = 84.0 \pm 2.8$ GeV and $M_z = 94.6 \pm 2.3$ GeV taking all first-order corrections into account, whereas in the Born approximation the predictions are $79.5 \pm 2.6$ GeV and $90.0 \pm 2.1$ GeV, respectively. Finally we note that our analysis [1] of SU(5), which agrees with that of other authors, predicts

$$\sin^2\theta(M_w) = 0.206\,^{+0.016}_{-0.004},$$

which is clearly compatible with the data.

Table 1
The values of $\sin^2\theta^{\mathrm{exp}}$, which are taken from ref. [3], were obtained using the procedures described in ref. [8]; $\sin^2\theta(M_w)$ is the central value obtained including electroweak corrections in the $\overline{\mathrm{MS}}$ scheme.

| Experiment | $\sin^2\theta^{\mathrm{exp}}$ | Hadronic energy cut (GeV) | $\sin^2\theta(M_w)$ |
|---|---|---|---|
| CHARM | 0.220 ± 0.015 | 2 | 0.210 |
| BEBC | 0.217 ± 0.045 | 15 | 0.206 |
| CDHS | 0.230 ± 0.013 | 10 | 0.219 |
| CITF | 0.272 ± 0.055 | 12 | 0.263 |
| HPWF | 0.274 ± 0.075 | 4 | 0.266 |

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