Minimum and Maximum Masses for Stars

- Most main sequence stars have a mass in the range from about 0.1 to about 50 solar mass

Two questions arise:

(1) What fundamental constants of nature determine the order of magnitude of the mass of a main sequence star?

(2) Why is the range in mass so limited?

- The key ingredient in the calculation of minimum and maximum stellar masses is the condition for hydrostatic equilibrium. To do so, we use the following equation

$$P_c \approx (\frac{\pi}{36})^{1/3} \, G M^{2/3} \, \rho_c^{4/3}$$

Minimum mass of a main sequence star

- A contracting system must be sufficiently massive to generate a central temperature which is high enough for thermonuclear reactions

- To estimate the maximum temperature achievable at the centre of a contracting gas cloud, we shall assume that a stage is reached in which the electrons are fully degenerate and the ions are classical

$$P_c = K_{NR}\; n_e^{5/3} + n_i\, k\, T_c$$

In order to simplify the algebra, we shall assume the mass is entirely composed of hydrogen so that

$$n_e = n_i = \frac{\rho_c}{m_H}$$

$$P_c = K_{NR}\,[\frac{\rho_c}{m_H}]^{5/3} + \frac{\rho_c}{m_H}\,k\,T_c$$

But we also have

$$P_c \approx (\frac{\pi}{36})^{1/3}\,G\,M^{2/3}\,\rho_c^{\;4/3}$$

$$k\,T_c = [\frac{\pi}{36}]^{1/3}\,Gm_H\,M^{2/3}\,\rho_c^{1/3} - K_{NR}\,[\frac{\rho_c}{m_H}]^{2/3}$$

The maximum temperature reached at the centre of a contracting mass of hydrogen is

$$[k\,T_c\,]_{\max} \approx [\frac{\pi}{36}]^{2/3}\ \frac{G^2\, m_H^{8/3}}{4\,K_{NR}}\ M^{4/3}$$

This condition is that the maximum central temperature reaches the ignition temperature for nuclear fusion of hydrogen. Thus,

$$\boxed{M_{\min} \approx [\frac{36}{\pi}]^{1/2}\ [\frac{4\,K_{NR}}{G^2\, m_H^{8/3}}]^{3/4}\ [k\,T_{ign}\,]^{3/4}}$$

If we take 1.5*10^6 K as the ignition temperature, then the minimum mass is 0.05 Solar Mass.

More accurate calculations give values close to 0.08 Solar Mass

Maximum mass of a main sequence star

- The central pressure P_c is the sum of a gas pressure P_g due to electrons and ions, and a radiation pressure P_r due to photons:

$$P_c = P_g + P_r$$

$$P_g = \frac{\rho_c \, k \, T_c}{\mu \, m_H}$$

$$P_r = \frac{1}{3} a \, T_c^4$$

We introduce

$$\beta = \frac{P_g}{P_c}$$

$$P_r = (1 - \beta) \, P_c$$

Eliminate T_c

$$P_r = \frac{1}{3} a T_c^4$$

$$T_c = \frac{\mu m_H}{\rho_c k} P_g$$

$$P_r = \frac{1}{3} a \left(\frac{\mu m_H}{\rho_c k} P_g\right)^4$$

$$P_c = \left[\frac{3}{a} \frac{(1-\beta)}{\beta^4}\right]^{1/3} \left[\frac{k \rho_c}{\mu m_H}\right]^{4/3}$$

Equating the above pressure to the pressure given before

$$P_c \approx \left(\frac{\pi}{36}\right)^{1/3} G M^{2/3} \rho_c^{4/3}$$

$$\boxed{[\frac{\pi}{36}]^{1/3} G M^{2/3} = [\frac{3}{a}\frac{(1-\beta)}{\beta^4}]^{1/3}[\frac{k}{\mu m_H}]^{4/3}}$$

We assume that not more than 50 percent of the pressure at the centre of the star is due to radiation

$$1-\beta<0.5$$

Then, an estimate of about 100 Solar Mass for the maximum mass is obtained