PHYC30019 Astrophysics

Project 1: Something

By

Braden Moore
Jude Prezens

The University of Melbourne

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1 Order-of-magnitude estimate 1

Are there more grains of sand on the beaches of Earth or more stars in the Milky Way?

1.1 Grains of sand

We begin by estimating the number of beaches on Earth.

$$\begin{aligned}\text{no. of continents} &= 7 \sim 10\\ \text{no. of beaches/continent} &\sim 10,000\\ &\Rightarrow 100,000 \text{ beaches on Earth}\end{aligned}$$

We then make an estimate of the average volume of a beach covered by sand.

$$\begin{aligned}\text{average length of beach} &= 1000\,\text{m}\\ \text{average width}^1 \text{ of beach} &= 10\,\text{m}\\ \text{average depth}^2 \text{ of beach} &= 2\,\text{m}\\ &\Rightarrow = 20.000\,\text{m}^3 \text{ of sand per beach}\end{aligned}$$

Next we estimate the number of grains of sand in a $1\,\text{cm}^3$ box to be ~ 1000.

$$\begin{aligned}&\Rightarrow 1000 \text{ grains of sand/cm}^3\\ &\Rightarrow 10^3 \times 10^6 \text{ grains of sand/m}^3\end{aligned}$$

We can now estimate the number of grains of sand on the beaches of Earth as

$$\begin{aligned}\text{number of grains of sand} &\sim 100,000 \text{ beaches} \times 20.000\,\text{m}^3/\text{beach} \times 10^9 \text{ grains/m}^3\\ &\sim 2 \times 10^5 \times 10^5 \times 10^9 \text{ grains}\\ &\sim 2 \times 10^{19} \text{ grains}\end{aligned}$$

1.2 Stars

We begin by determining a rough measure of volume for the Milky Way.

$$\begin{aligned}\text{diameter of Milky Way} &\sim 100.000\,\text{ly}\\ \text{thickness of Milky Way} &\sim 2.000\,\text{ly}\\ \Rightarrow \text{volume} &= \pi \left(\frac{100.000\,\text{ly}}{2}\right)^2 \times 2.000\,\text{ly}\\ &\sim \frac{6}{4} \times 10^3 \left(10^5\right)^2 (\text{ly})^3\\ &\sim 1.5 \times 10^{13}\,(\text{ly})^3\end{aligned}$$

We then estimate the average distance between stars. To do this, we take the distance from the Sun to its nearest neighbour as $\sim 4\,\mathrm{ly}$. Using this distance we then suppose that each star exists within a "starbox" of dimensions $2 \times 2 \times 2\,(\mathrm{ly})^3$.

$$\text{Volume per star} \sim 8\,(\mathrm{ly})^3/\text{star}$$

Now we can estimate the number of stars as:

$$\begin{aligned}\text{Number of stars in the Milky Way} &= \frac{\text{volume of Milky Way}}{\text{volume/star}} \\ &\sim \frac{1.5 \times 10^{13}}{8} \\ &\sim \frac{10^{13}}{10} \\ &= 10^{12}\,\text{stars}\end{aligned}$$

1.3 Conclusion

From the above, we claim that

$$\begin{aligned}\text{Number of grains of sand on the beaches of Earth} &\sim 10^{19} \\ \text{Number of stars in the Milky Way} &\sim 10^{12}\end{aligned}$$

From the rough order-of-magnitude calculations performed, we deduce *there are more grains of sand on the beachs of Earth than there are stars in the Milky Way.* Even though we have made many assumptions as well as liberal rounding of numbers, it is unlikely that these values are more than an order of magnitude greater or smaller than quoted.

It is validating to note that the common answer for number of stars in the Milky Way[3] is quoted as $\sim 10^{11}$.

2 Research task 1

What is the lowest mass for a star?

THe lowest mass for a star is the mass of a brown dwarf, with mass

$$M_{\mathrm{BD}} \sim (13 \to 65) M_{\mathrm{Jup}} \tag{1}$$

where M_{Jup} is the mass of Jupiter. In terms of solar mass, this gives

$$M_{\mathrm{BD}} < 0.07 M_{\odot} \tag{2}$$

[3] See http://asd.gsfc.nasa.gov/blueshift/index.php/2015/07/22/how-many-stars-in-the-milky-way/

What Physics determines this mass?

Minimum mass is determined by the Jeans mass

$$M_J = \frac{3kT}{G\mu m} r \tag{3}$$

Will compact objects form below this mass - and if so what are they?

Can we observe them?

3 Calculation 1

What is the most massive star that can form? State the physics that you have used the derive this estimate. How could a star form that might avoid this limit?

Assuming the central pressure ρ_c of a star is the sum of the pressure due to gravity ρ_g and the pressure due to radiation ρ_r (where this radiation is emission of photons), then

$$\rho_c = \rho_g + \rho_r \tag{4}$$

where

$$\rho_g = \frac{\rho_c k T_c}{\mu m_H} \tag{5}$$

$$\rho_c = \text{central density} \tag{6}$$

$$k = \text{Boltmann's constant} \tag{7}$$

$$T_c = \text{central temperature} \tag{8}$$

$$\mu m_H = \text{reduced mass of hydrogen} \tag{9}$$

and

$$\rho_r = \frac{1}{3} u \tag{10}$$

$$= \frac{1}{3} a T_c^4 \tag{11}$$

where $u = aT_c^4$ is the Stefan-Boltzmann law relating total energy density of a black body to its temperature.

We then introduce $\beta = \dfrac{\rho_g}{\rho_c}$, and it follows that

$$\rho_r = (1 - \beta)\rho_c \tag{12}$$

We can eliminate T_c by rearranging (5):

$$T_c = \frac{\mu m_H}{\rho_c k}\rho_g \tag{13}$$

then recalling (11), we find

$$\rho_r = \frac{1}{3}a\left(\frac{\mu m_H}{\rho_c k}\rho_g\right)^4 \tag{14}$$

$$\therefore \rho_c = \left[\frac{3}{a}\frac{(1-\beta)}{\beta^4}\right]^{1/3}\left[\frac{k\rho_c}{\mu m_H}\right]^{4/3} \tag{15}$$

From hydrostatic equilibrium we know

$$\rho_c \approx \left(\frac{\pi}{36}\right)^{1/3} GM^{2/3}\rho_c^{4/3} \tag{16}$$

Equating these equations we find

$$\left(\frac{\pi}{36}\right)^{1/3} GM^{2/3} = \left[\frac{3}{a}\frac{(1-\beta)}{\beta}\right]^{1/3}\left[\frac{k}{\mu m_H}\right]^{4/3} \tag{17}$$

Assuming that $(1-\beta) < 0.5^4$, we find the theoretically maximum mass of a star to be about $100M_\odot$. Stars cannot be more massive that this because if radiation pressure exceeds gravitational pressure, the star will blow itself apart. For a star to form and avoid this limit, there must be some mechanism by which this imbalance is avoided.

However, it is known that the star Eta Carinae A has mass between $100 - 150M_\odot$ (and even heavier stars exist).

4 Calculation 2

Suppose the gas cloud of $10^3 M_\odot$ collapses under gravity and forms stars. Under simple assumptions, what is the largest star that will form? You may assume the Salpeter IMF which has the form: $dN/dm \propto m^{-2.35}$.

5 Calculation 3

Once a gas cloud reaches an overdensity of ~ 1, it will colla[se in free fall. Determine the time scale of collapse of a gas cloud of mass M. Work out how this scales. What will stop our gas cloud collapsing into a black hole?

[4] That is, $\rho_c < \rho_g$. For $\rho_c > \rho_g$, ...