Minimum and Maximum Masses for Stars

 Most main sequence stars have a mass in the range from about 0.1 to about 50 solar mass

Two questions arise:

- (1) What fundamental constants of nature determine the order of magnitude of the mass of a main sequence star?
- (2) Why is the range in mass so limited?

 The key ingredient in the calculation of minimum and maximum stellar masses is the condition for hydrostatic equilibrium. To do so, we use the following equation

$$P_c \approx (\frac{\pi}{36})^{1/3} G M^{2/3} \rho_c^{4/3}$$

Minimum mass of a main sequence star

 A contracting system must be sufficiently massive to generate a central temperature which is high enough for thermonuclear reactions To estimate the maximum temperature achievable at the centre of a contracting gas cloud, we shall assume that a stage is reached in which the electrons are fully degenerate and the ions are classical

$$P_c = K_{NR} \ n_e^{5/3} + n_i \, k \, T_c$$

In order to simplify the algebra, we shall assume the mass is entirely composed of hydrogen so that

$$n_e = n_i = \frac{\rho_c}{m_H}$$

$$P_c = K_{NR} \left[\frac{\rho_c}{m_H} \right]^{5/3} + \frac{\rho_c}{m_H} k T_c$$

But we also have

$$P_c \approx (\frac{\pi}{36})^{1/3} G M^{2/3} \rho_c^{4/3}$$

$$kT_c = \left[\frac{\pi}{36}\right]^{1/3} Gm_H M^{2/3} \rho_c^{1/3} - K_{NR} \left[\frac{\rho_c}{m_H}\right]^{2/3}$$

The maximum temperature reached at the centre of a contracting mass of hydrogen is

$$[kT_c]_{\text{max}} \approx \left[\frac{\pi}{36}\right]^{2/3} \frac{G^2 m_H^{8/3}}{4 K_{NR}} M^{4/3}$$

This condition is that the maximum central temperature reaches the ignition temperature for nuclear fusion of hydrogen. Thus,

$$M_{\min} \approx \left[\frac{36}{\pi}\right]^{1/2} \left[\frac{4 K_{NR}}{G^2 m_H^{8/3}}\right]^{3/4} \left[k T_{ign}\right]^{3/4}$$

If we take 1.5*10^6 K as the ignition temperature, then the minimum mass is 0.05 Solar Mass.

More accurate calculations give values close to 0.08 Solar Mass

Maximum mass of a main sequence star

 The central pressure P_c is the sum of a gas pressure P_g due to electrons and ions, and a radiation pressure P_r due to photons:

$$P_c = P_g + P_r$$

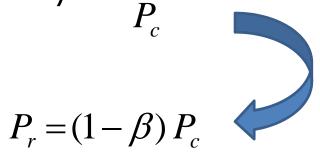
$$P_g = \frac{\rho_c k T_c}{\mu m_H}$$

$$P_r = \frac{1}{3} a T_c^4$$

We introduce

$$\beta = \frac{P_g}{P_c}$$

$$P_r = (1 - \beta) P_c$$



Eliminate T_c

$$P_{r} = \frac{1}{3} a T_{c}^{4}$$

$$T_{c} = \frac{\mu m_{H}}{\rho_{c} k} P_{g}$$

$$P_r = \frac{1}{3} a \left(\frac{\mu m_H}{\rho_c k} P_g \right)^4$$

$$P_c = \left[\frac{3}{a} \frac{(1-\beta)}{\beta^4}\right]^{1/3} \left[\frac{k \rho_c}{\mu m_H}\right]^{4/3}$$

Equating the above pressure to the pressure given before

$$P_c \approx (\frac{\pi}{36})^{1/3} G M^{2/3} \rho_c^{4/3}$$

$$\left[\frac{\pi}{36}\right]^{1/3} G M^{2/3} = \left[\frac{3}{a} \frac{(1-\beta)}{\beta^4}\right]^{1/3} \left[\frac{k}{\mu m_H}\right]^{4/3}$$

We assume that not more than 50 percent of the pressure at the centre of the star is due to radiation

$$1 - \beta < 0.5$$

Then, an estimate of about 100 Solar Mass for the maximum mass is obtained