PHYC30019 Astrophysics

Project 3: Expansion of the Universe

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Motivation

During this workshop you will gain a clearer idea of how the expansion of the universe and its geometry affect the observation of very distant objects. In particular, we want to end by thinking about the sort of experiments you could perform to determine the type of expansion the universe is undergoing. In this workshop, assume that the current values of the energy densities are:

$$\Omega_{r,0} = 10^{-4}, \quad \Omega_{m,0} = 0.27, \quad \Omega_{\Lambda,0} = 0.73$$

You should use an $H_0 = 70\,\mathrm{km/sec/Mpc}$. For the order-of-magnitude problems this workshop, try to get to a solution with minimal effort - remember it is an order of magnitude solution you are looking for.

1 Order-of-magnitude estimates

1.1 Order-of-magnitude estimate 1

Suppose that you have been asked to find new extra-solar planets. You need to think about the techniques that you will use for these experiments. What accuracy is required to detect extra-solar planets by their astrometric wobble? By Doppler shift? Which would you expect to be easier to measure?

We assume that the plane of the planet's orbit around the observed star is perpendicular to our line-of-sight.

We take the line-of-sight distance from Earth to a star as d, and we assume the astrometric wobble of the star due to the extrasolar planet is

$$r = R_{\mathrm{star}}, \tag{1}$$

that is, its wobble is of order the star's radius. For small angles we can approximate $\tan\theta \approx \theta$, so that we have

$$\tan\theta \approx \theta = \frac{r}{d} \tag{2}$$

Taking the distance to the extrasolar planet as[1]

$$d \approx 20\,\mathrm{ly} \sim 10\,\mathrm{ly} \tag{3}$$

[1] *The Extrasolar Planet Encyclopaedia*, exoplanet.eu

and the radius of its star as[2]

$$\begin{aligned} r &= R_\odot && (4) \\ &\approx 7.36 \times 10^{-8}\,\text{ly} && (5) \\ &\sim 10^{-7}\,\text{ly} && (6) \end{aligned}$$

we can calculate θ as

$$\begin{aligned} \theta &= \frac{10^{-7}\,\text{ly}}{10\,\text{ly}}\ \text{radians} && (7) \\ &= 10^{-8}\,\text{radians} && (8) \\ &= 10^{-2}\,\text{arcsec} && (9) \end{aligned}$$

That is, we need an angular resolution of 10^{-2} arcsec to detect extrasolar planets by their astrometric wobble. The Hubble Space Telescope (HST) has an angular resolution of 0.05 arcsec = 5×10^{-2} arcsec, which we will use as a basis for our "most accurate" measurement [3]. We see that it would be quite difficult to measure this astrometric wobble!

Now we shall investigate the Doppler shift due to wobble. We start with an expression for Doppler shift caused by the radial velocity of a star wobbling around an extrasolar planet,

$$\frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} = 1 + \frac{v_s}{c} \tag{10}$$

where λ_{obs} is the observed wavelength, λ_{em} is the wavelength of the light emitted, and v_s is the (radial) velocity of the source. In this order-of-magnitude calculation we assume that the star is rotating in a plane parallel to our line-of-sight; we take the maximum radial velocity[4] as $v_s \sim 2.3 \times 10^5$ m/s. We can rewrite (10) in terms of the fractional change in λ, i.e. the accuracy to which we would have to be able to measure λ to determine radial velocity and hence detect extrasolar planets by Doppler wobble.

$$\begin{aligned} \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} &= 1 + \frac{v_s}{c} && (11) \\ \Rightarrow \lambda_{\text{obs}} &= \left(1 + \frac{v_s}{c}\right)\lambda_{\text{em}} && (12) \\ \Rightarrow \Delta\lambda &= \frac{v_s}{c} && (13) \end{aligned}$$

For radial velocity $v_s \sim 2.3 \times 10^5$ m/s,

$$\begin{aligned} \Delta\lambda &\sim \frac{2.3 \times 10^5}{3 \times 10^8} && (14) \\ &\sim 10^{-3} && (15) \end{aligned}$$

That is, we must be able to measure wavelength to an accuracy of 1 part in 10^3 in order to detect extrasolar planets! We expect that this is easier to measure than the astrometric wobble. This is backed up by the many extrasolar planets which have been detected by Doppler wobble.

[2] Haberreiter, M; Schmutz, W; Kosovichev, A.G., "Solving the Discrepancy between the Seismic and Photospheric Solar Radius", Astrophysical Journal 675 (1): L53-L56

[3] http://hubblesite.org/hubble_discoveries/hstexhibit/telescope/about.shtml

[4] csep10.phys.utk.edu/lightcone/demo/vlab/vlab3/dopwob.html

1.2 Order-of-magnitude estimate 2

Many galaxies rotate. Can you actually observe the rotation of nearby galaxies over a reasonable (i.e. measurable) timescale?

We will see if we can measure the angular displacement over a reasonable timeframe of an object in a rotating galaxy, and hence whether we can actually observe the rotation of that galaxy.

We take the average speed of an object in the Solar System[5] to be $\sim 200\,\mathrm{km/s}$ (note this will differ with increasing distance from the center of the galaxy). We take the radius of an arbitrary galaxy as the radius of the Milky Way[6] galaxy $= 7.7\,\mathrm{kpc} \sim 10\,\mathrm{kpc} \sim 3 \times 10^{17}\,\mathrm{km}$ (however we note that the Milky Way galaxy is not an averagely sized galaxy). We will call our "reasonable" timescale $50\,\mathrm{yr} \sim 10\,\mathrm{yr}$.

We will assume that in measuring the rotation of nearby galaxies, we are actually measuring the rotation of its components. Assuming that a star (or some other object rotating in a galaxy) travels only a negligible fraction of its orbit around the centre of the galaxy over 50 years, we approximate the distance travelled as

$$\text{Distance travelled} = 200\,\mathrm{km/s} \times 50\,\mathrm{yr} \times \frac{3 \times 10^{7}\,\mathrm{s}}{\mathrm{yr}} \tag{16}$$

$$= 3 \times 10^{11}\,\mathrm{km} \tag{17}$$

$$\sim 10^{11}\,\mathrm{km} \tag{18}$$

This leads to the fraction of the orbit travelled as

$$\frac{\Delta d}{d} = \frac{10^{11}}{2\pi \times 3 \times 10^{17}} \sim \frac{10^{11}}{10^{18}} = 10^{-7} \tag{19}$$

One simple way to measure the angular displacement is to assume that we are "at the center" of the galaxy (one radius away). Then,

$$\text{Angular displacement} = 10^{6}\,\mathrm{arcsec} \times 10^{-7} = 0.1\,\mathrm{arcsec} \tag{20}$$

where there are $\sim 10^{6}$ arcsec in a circle. Obviously, this number would be smaller if we were further from the object.

As discussed in order-of-magnitude 1, the HST could probe angular resolutions of this scale, for nearby galaxies; note the angular scale would decrease as we observe more distant galaxies.

[5] http://www.space.com/19915-milky-way-galaxy.html
[6] http://imagine.gsfc.nasa.gov/features/cosmic/milkyway_info.html

1.3 Order-of-magnitude estimate 3

Consider a galaxy acting as a gravitational lens. You know that the galaxy comprises, stars, globular clusters, dark matter, gas etc. Estimate the probability that a background quasar is gravitationally lensed by a globular cluster, as a function of the probability that is it lensed by the whole galaxy. Can you make a similar estimate for the probability of lensing by a star?

Make sure that you provide details of the criteria that you are using.

We start by assuming that the relation between $\mathcal{P}_{\text{cluster}}$ and $\mathcal{P}_{\text{galaxy}}$ (where $\mathcal{P}$ is the probability of lensing a background quasar) as

$$\frac{\mathcal{P}_{\text{cluster}}}{\mathcal{P}_{\text{galaxy}}} = \frac{\sigma_{\text{cluster}}}{\sigma_{\text{galaxy}}} \tag{21}$$

In equation (21), σ is the cross-sectional area of the object denoted by the subscript. The cross-section σ is proportional to r^2; instead of the physical radius we will instead use the Einstein radius [7]

$$r \equiv \theta_{\text{ER}} \sim 2'' \left(\frac{M}{10^{12}\ M_\odot}\right)^{1/2} \left(\frac{D}{0.3\,\text{Gpc}}\right)^{-1/2} \tag{22}$$

This radius takes the galaxy and globular clusters at cosmological distances as point-masses, and considers the effect of mass on lensing (which is more important than physical cross-section). D is a constant related to distances between the source and lens, and M is the mass of the lens.

So, we find

$$\begin{aligned} \frac{\mathcal{P}_{\text{cluster}}}{\mathcal{P}_{\text{galaxy}}} &= \frac{(M_{\text{cluster}}^{1/2})^2}{(M_{\text{galaxy}}^{1/2})^2} && (23)\\ &= \frac{M_{\text{cluster}}}{M_{\text{galaxy}}} && (24)\\ &= \frac{10^6\ M_\odot}{10^{12}\ M_\odot} && (25)\\ &= 10^{-6} && (26) \end{aligned}$$

where we have used[8] [9]

$$\begin{aligned} M_{\text{cluster}} &= 10^6\ M_\odot && (27)\\ M_{\text{galaxy}} = M_{\text{Milky Way}} &= 10^{12}\ M_\odot && (28) \end{aligned}$$

[7] Wyithe, S and Pindor B. 2015, "640-612: Physical Cosmology", *PHYC90009: Physical Cosmology*, University of Melbourne, Melbourne.

[8] M_{cluster}: `http://relativity.livingreviews.org/Articles/lrr-2013-4/articlese2.html`

[9] M_{galaxy}: Kafle, P.R.; Sharma, S.; Lewis, G.F.; Bland-Hawthorn, J. (2014). "On the Shoulders of Giants: Properties of the Stellar Halo and the Milky Way Mass Distribution". The Astrophysical Journal 794 (1): 17. arXiv:1408.1787

For the probability of lensing by a star of mass $M_\odot$, it is easy to see that

$$\frac{\mathcal{P}_{\mathrm{star}}}{\mathcal{P}_{\mathrm{galaxy}}} = 10^{-12} \tag{29}$$

2 Research Tasks

2.1 Research Task 1

When we consider the fluctuations in the CMB, we usually plot a power spectrum to describe the scales where there is most power. Describe the physical quantities that are measured on the two axes of the typical plot of the fluctuations - a full explanation is expected. Then describe the physical interpretation of the two main peaks in the plots.

The x-axis of the plot of fluctuations is measured in multipole moments. The multipole moment is inversely proportional to the angular scale, meaning that as the multipole moment increases, the angular scale decreases. The angular scale refers to the size of the angle on the sky ($\theta \sim \frac{1}{l}$). The y-axis of the CMB power spectrum measures the fluctuation in temperature for a specific angular scale. The fluctuation in temperature (ΔT) is measured in microKelvin (μK) .

The first peak in the CMB power spectrum is due to the compression of a large region that reaches its maximum compression at the time of decoupling. It also shows the size of hot and cold spots due to this. The most important feature, the angular location of the first peak, is important in determing the geometry of the universe (indicated to be spatially flat).

The ratio of the second peak amplitude to the first peak amplitude tells us about the ratio of baryon density to the critical density for baryons (Ω_b)[10 11 12].

[10]www.background.uchicago.edu/ whu/intermediate/summary.html
[11]`www.astro.umd.edu/~miller/teaching/astr422/lecture21.pdf`
[12]Wyithe, S and Pindor B. 2015, "640-612: Physical Cosmology", *PHYC90009: Physical Cosmology*, University of Melbourne, Melbourne.

3 Calculations

3.1 Calculation 1

Using the final form of the differential equation that describes the behaviour of the scale factor with time, work out the redshift when dark energy first dominated the expansion of the universe. The equation is given by:

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{R^4} + \frac{\Omega_{m,0}}{R^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{R^2}$$

where the subscript 0 refers to the present value and the subscripts r, m and Λ refer to radiation, matter and the cosmological constant respectively.

We recall that the Hubble constant H is related to the scale factor $R(t)$ as

$$H(t) = \frac{\dot{R}(t)}{R(t)} \tag{30}$$

where the dot represents a derivative with respect to time. So, we can write the Friedman equation in terms of R by

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{R^4} + \frac{\Omega_{m,0}}{R^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{R^2} \tag{31}$$

$$\Rightarrow \dot{R}^2 = H_0^2 R^2 \left[\frac{\Omega_{r,0}}{R^4} + \frac{\Omega_{m,0}}{R^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{R^2}\right] \tag{32}$$

$$\Rightarrow \dot{R} = H_0 \left[\frac{\Omega_{r,0}}{R^2} + \frac{\Omega_{m,0}}{R} + \Omega_{\Lambda,0}R^2 + (1-\Omega_0)\right]^{1/2} \tag{33}$$

We will assume that the universe is flat;

$$\Rightarrow \Omega_0 = 1 \tag{34}$$

This is justifiable since $\Omega_0 = \Omega_{r,0} + \Omega_{m,0} + \Omega_{\Lambda,0} \approx 1$ from the values given for this assignment.

Substituting in the energy densities into (33) we see

$$\dot{R} = H_0 \left[\frac{10^{-4}}{R^2} + \frac{0.27}{R} + 0.73R^2\right]^{1/2} \tag{35}$$

From this, we can determine when dark energy begins to dominate the expansion of the universe. The era before dark energy-domination is the matter-dominated period. In this period, the effects of radiation on the expansion of the universe are negligible. The redshift at which dark energy and matter both contribute equally to the expansion of the universe is given by

$$\Omega_{\Lambda,0}R^2 = \frac{\Omega_{m,0}}{R} \tag{36}$$

$$\Rightarrow 0.73R^2 = \frac{0.27}{R} \tag{37}$$

$$\Rightarrow R = 0.718 \tag{38}$$

We can write the redshift z in terms of the scale factor as

$$\frac{1}{1+z} = a \tag{39}$$

$$\Rightarrow z = \frac{1}{a} - 1 \tag{40}$$

$$\Rightarrow z_{\text{matter-dark energy equality}} = 0.393 \tag{41}$$

For all smaller redshifts, dark energy should be the dominant factor in the expansion of the universe (though not exactly, since we have neglected the radiation component, even though it is very small).

Taking the end of the matter-dominated era as 9.8×10^9 yr[13], we can compare redshifts to see whether our answer is feasible. In the dark matter-dominated era,

$$R \propto e^{H_0 t} \tag{42}$$

Today we have $R = 1$ and $t = t_0$; recalling $t_0 = \frac{1}{H_0}$ we have

$$1 = Ae^{H_0 t_0} = Ae \tag{43}$$

$$\Rightarrow A = e^{-1} \tag{44}$$

$$\Rightarrow R_{\text{matter-dark matter equality}} = e^{H_0 t - 1} \tag{45}$$

$$= e^{\frac{9.8}{13.8} - 1} \tag{46}$$

$$= 0.74 \tag{47}$$

Where we have converted $H_0 = 70\,\text{km/sec/Mpc}$ into units of yr,

$$\Rightarrow z_{\text{matter-dark energy equality}} = \frac{1}{0.74} - 1 \tag{48}$$

$$= 0.35 \tag{49}$$

This is indeed close to the redshift we found in (41), so we can justify our approximate value of z calculated early to being around the redshift of the beginning of the dark energy-dominated era.

[13] Ryden, Barbara, "Introduction to Cosmology", 2006

3.2 Calculation 2

For the cosmology given above, use the calculator from the previous workshop to determine the redshift where a galaxy of a standard size has the smallest angular extent or size.

`http://www.ph.unimelb.edu.au/cosmocalc/session.php`

Angular size is given by[14]

$$\Delta\theta = \frac{D(1+z)^2}{d_L(z)} \tag{50}$$

where D is the proper length of an object, and d_L is the luminosity distance. The proper distance is constant over all redshifts. Using the calculator provided[15] and using the given cosmology, we run over redshift $0.05 \to 5$ in intervals of 0.05 and use the data to determine the minimum angular size.

A minimum in angular size occurs at $z = 0.75$. In Figure 1 below we have plotted angular size against distance over our range of values (up to ~ 3.75, for illustrative purposes). We see that after reaching the minimum, angular size continues to increase.

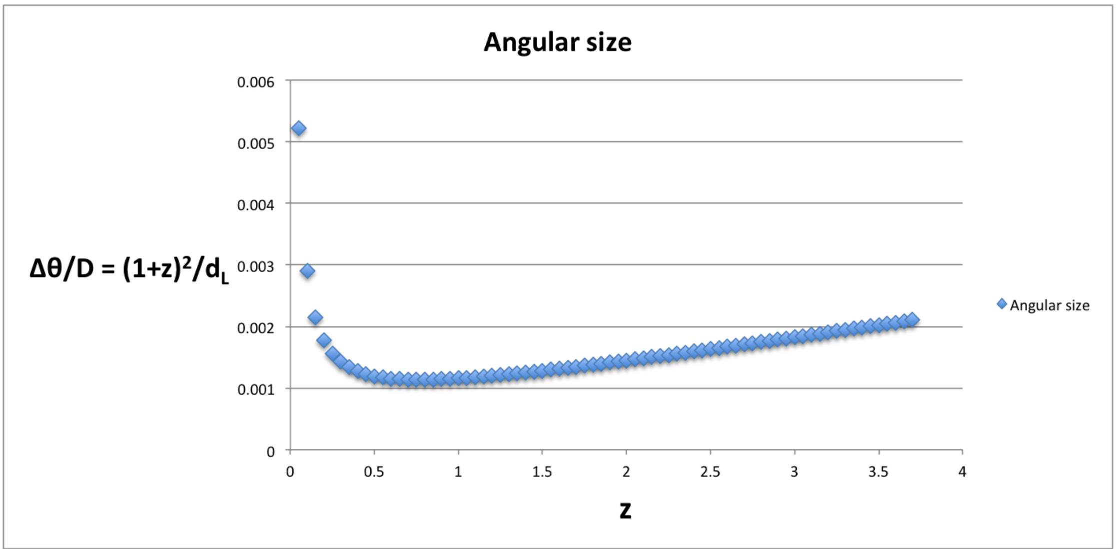

Figure 1: Angular size

[14] `https://ned.ipac.caltech.edu/level5/March02/Sahni/Sahni4_5.html`
[15] `http://www.ph.unimelb.edu.au/cosmocalc/session.php`

3.3 Calculation 3

Consider the surface brightness of a galaxy. How is it defined? Now explain how surface brightness changes as a function of distance in a Euclidean, non-expanding universe. How does surface brightness change in an expanding FRW universe?

What conclusions can you draw about the observations of very distant galaxies?

The surface brightness is the measured brightness for a specific area on an extended object (e.g. a galaxy). In a Euclidean, non-expanding universe, surface brightness is constant with distance. As distance increases and the brightness fades, the area begin observed also decreases at the same rate.

Flux decreases with the square of the distance, but so does area. So, these values cancel out and the surface brightness is not affected. However, flux is reduced by 2 factors of $(1 + z)$, one factor due to time dilation in the photon arrival time, and the other factor due to redshift.

In a Euclidean, non-expanding universe,[16]

$$\text{Surface brightness} \equiv \text{SB} = m + 2.5\log_{10}(A) \tag{51}$$

$$m = -2.5\log_{10}(f_E) \tag{52}$$

where $f_E = \frac{\text{flux}}{\text{reference flux}}$. In an expanding universe,

$$f = \frac{f_E}{(1+z)^2} \tag{53}$$

from lecture notes. Hence,

$$\text{SB} = -2.5\log_{10}\left(\frac{f_E}{(1+z)^2}\right) + 2.5\log_{10}(A) \tag{54}$$

$$= \underbrace{[-2.5\log_{10}(f_E) + 2.5\log_{10}(A)]}_{\text{independent of distance}} + 2.5\log_{10}\left(\frac{1}{(1+z)^2}\right) \tag{55}$$

$$\Rightarrow \Delta\text{SB} = 2.5\log_{10}\left(\frac{1}{(1+z)^2}\right) \tag{56}$$

$$= 2.5\log_{10}(R^2) \tag{57}$$

$$= 5\log_{10} R \tag{58}$$

[16]Sparke, L.; Gallagher, J. (2000). *Galaxies in the Universe: An Introduction (1st ed.).* Cambridge University Press.

4 Conclusion

> You have considered some of the changes in observables in different FRW cosmologies. Which observations do you think might be easiest to undertake to measure the curvature of the universe accurately?

Measuring the CMB power spectrum gives us a very powerful insight into the universe, its properties and its evolution. By measuring the locations of the peaks (even just the first peak) we can compare prediction with experiment. Below are several figures from *Dodelson* (2000)[17]

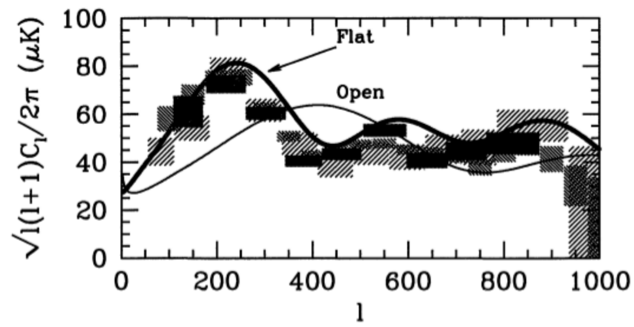

Figure 8.16. The anisotropy spectrum in flat versus open universe. Also shown are data from three small-scale experiments: DASI (darkest; Halverson *et al.*, 2002), Boomerang (medium; Netterfield *et al.*, 2002), and Maxima (lightest; Lee *et al.*, 2001). The pattern of peaks and troughs persists in the open universe but is shifted to smaller scales. The data clearly favor the flat case. Both curves have identical parameters $n = 1, \Omega_m h^2 = 0.15, \Omega_b h^2 = 0.02$ with no reionization, tensors, or cosmological constant. *Open* curve has $\Omega_k = 1 - \Omega_m = 0.7$; *flat* has the same parameters except $\Omega_k = 0$.

The figures below show how measured angular scale depends on the curvature of the universe. Therefore if we know the expected angle between two points, we can infer curvature from the actual measurement.

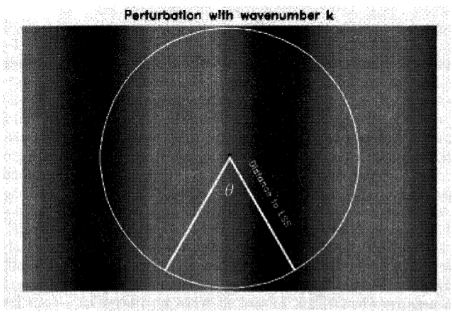

Figure 8.4. Free-streaming. Perturbations in the temperature at recombination from one plave wave with wavenumber k. Hot and cold spots are shaded light and dark. After recombination, photons from the hot and cold spots travel freely to us, here denoted by the star at the center. This mode contributes anisotropy on a scale $\theta \sim k^{-1}$/(Distance to last scattering surface).

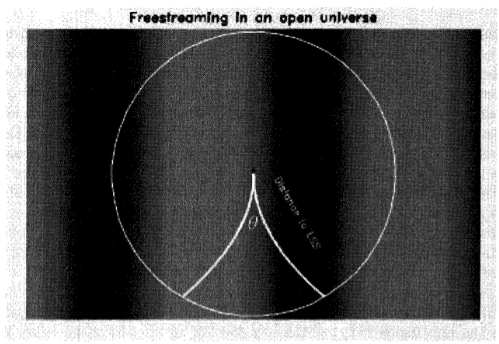

Figure 8.15. Photon trajectories in an open universe diverge. Perturbations at last scattering turn up on smaller scales in an open universe than they do in a flat universe.

[17] Dodelson, S. (2003) *Modern Cosmology*. San Diego, California: Academic Press.