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# PHYC30019 Astrophysics

## Course Summary

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## 0 Introduction

### 0.1 Housekeeping

#### 0.1.1 Texts and References

- Maoz - Astrophysics in a Nutshell (free download apparently)

#### 0.1.2 Marking system

- $3 \times 10\%$  workshops Weeks 4, 8 and 11
- 70% exam

### 0.2 Definitions

#### 0.2.1 Astronomy

- Collects photons
- Some branches (small at current) investigate cosmic rays
- Some investigate gravitational waves
- Measured quantities for photons:
  - 2 angular dimensions
  - Spectrum
  - Luminosity (distance)
  - Flux (amount)
- Objects investigated:
  - Planets, asteroids, comets (not investigated in class)
  - stars, globular clusters, galaxies, gas clouds, clusters of galaxies, superclusters, active galactic nuclei

#### 0.2.2 Astrophysics

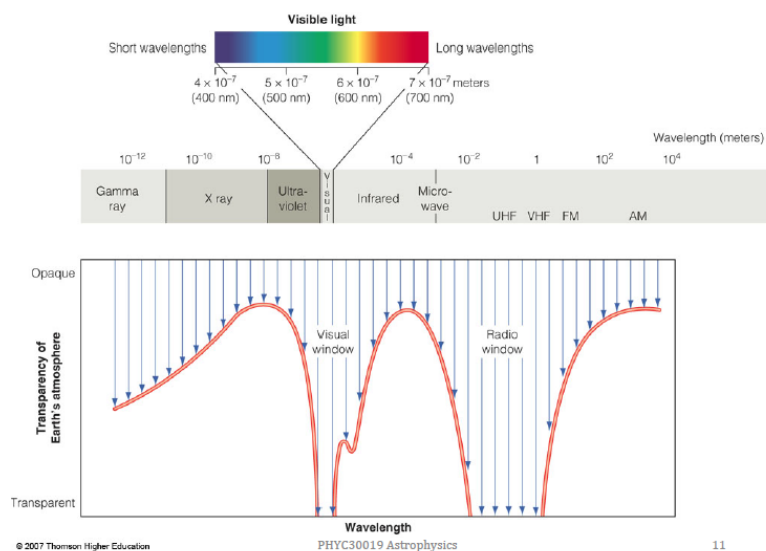
- Applies physics and mathematics to understand the universe and its components (formation, structure, evolution, distribution)
- Uses many branches of physics and maths
- Often things are measured imprecisely (order of magnitude)
- Computational processes are used (some high precision measurements are taken, e.g. for pulsars)
- Types of radiation processes:
  - thermal (emitting a blackbody), synchrotron, bremsstrahlung, Compton, inverse Compton

### 0.3 Photons

- Can be defined by frequency  $c = \lambda\nu$
- Or energy  $E = h\nu$
- Or temperature  $E = kT$

Radiation	Wavelength	Frequency (Hz)	Energy/ photon (J)	Temperature (K)
Gamma rays	<0.01nm	>3x10 <sup>19</sup>	>2x10 <sup>-14</sup>	<10 <sup>9</sup>
X-rays	0.01 -10nm	3x10 <sup>19</sup> -3x10 <sup>16</sup>	2x10 <sup>-14</sup> -2x10 <sup>-17</sup>	10 <sup>9</sup> -10 <sup>6</sup>
Ultraviolet	10-300nm	3x10 <sup>16</sup> -10 <sup>15</sup>	2x10 <sup>-17</sup> -7x10 <sup>-19</sup>	10 <sup>6</sup> -5x10 <sup>4</sup>
Optical	300-700nm	10 <sup>15</sup> -4x10 <sup>14</sup>	7x10 <sup>-19</sup> -3x10 <sup>-19</sup>	5x10 <sup>4</sup> -2x10 <sup>4</sup>
Infrared	700nm-1mm	4x10 <sup>14</sup> -3x10 <sup>11</sup>	3x10 <sup>-19</sup> -2x10 <sup>-22</sup>	2x10 <sup>4</sup> -10
Microwave	1mm-1cm	3x10 <sup>11</sup> -3x10 <sup>10</sup>	2x10 <sup>-22</sup> -2x10 <sup>-23</sup>	10-1
Radio	1cm-30m	3x10 <sup>10</sup> -10 <sup>7</sup>	2x10 <sup>-23</sup> -7x10 <sup>-27</sup>	1-5x10 <sup>-4</sup>

- Some wavelengths have more attenuation (decay) than others (see figure below)



#### 0.3.1 Characteristic scales

- Cosmological
  - Scale of observable universe (light travelling over finite universe lifetime)  $c/H_0$
  - $H_0$  is Hubble constant
- Parsec
  - $1\text{pc} = 3.1 \times 10^{18}\text{cm} = 3.3\text{ly}$
  - Defined based on universe size
  - Distance at which 1AU (distance to Sun) subtends by an angle of one arcsecond

### 0.3.2 Copernican Principle

- We are not in a favoured position in the universe.
- Local physics is the same as distant physics
- Thus we rarely use "new" physics; exceptions are dark matter and dark energy

### 0.3.3 Cosmological Principle

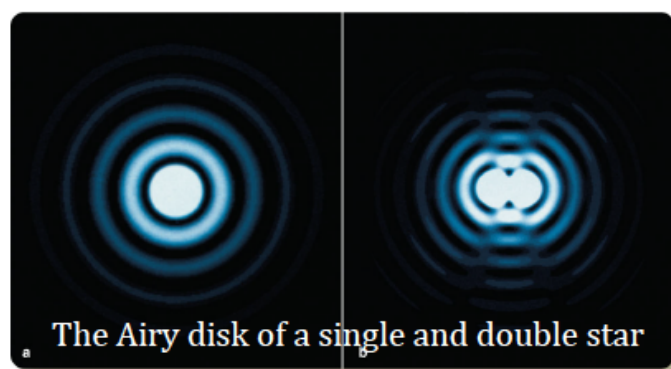
- On average, the universe is homogeneous and isotropic at any time in its evolution
- Local measurements of isotropy and homogeneity + Copernican principle imply the Cosmological principle
- Universe is homogeneous to about 1% on a scale of 100Mpc
- We need to average over a large area to get this homogeneity
- Isotropic = rotational invariant

## 0.4 Astronomical Observing

Four important dimensions to measure:

### 0.4.1 Angular resolution

- Smallest angle on the sky between two sources that telescopes can separate
- Point source produces diffraction pattern with central spot of radius  $\theta = 1.22\lambda/D$
- $D$  = diameter of aperture (known as Airy disk)



### 0.4.2 Light-gathering power

- Larger area = more photons collected
- Limited by ability to make large glass that doesn't deform

## 0.5 Integration time

- More sensitive if more photons are collected
- Hence, observe for longer
- Try to have minimal noise

### 0.5.1 Wavelength range

- Different telescopes observe different wavelengths
- Not all photons reach the Earth, so we have outer space telescopes

## 0.6 The Big Questions

- How did the universe begin and how will it evolve? – the behaviour of matter, energy, space and time
- What is the nature of the stuff the universe is made of?
- Physics in extreme physical environments: how does matter behave?
- How did the universe we inhabit form - stars, galaxies and planets?
- Are we alone: what do other planetary systems look like? i what is life?

# 1 Blackbody Emission

- We approximate a lot of things as blackbodies
- We assume we can model things by blackbody spectrums

$$u_\nu = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1} \quad (1)$$

- The blackbody spectrum is the energy density  $u_\nu$
- Units:  $\text{erg cm}^{-3} \text{ Hz}^{-1}$ .
- Flow of energy  $I_\nu$ ; derivative of the energy density w.r.t solid angle and multiplying by  $c$
- = energy passing through unit area per unit time

$$I_\nu = c \frac{du_\nu}{d\Omega} = c \frac{u_\nu}{4\pi} \quad (2)$$

$$= \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \quad (3)$$

$$= B_\nu \quad (4)$$

- Called  $B_\nu$  because blackbody
- Flow of energy in a particular direction inside a BB is the intensity  $I_\nu$
- Related to outgoing flux by  $df_\nu = I_\nu \cos \theta d\Omega$

Okay, I'll just try to summarize stuff as I go along. Very little on written on the board, at the moment.

- Flux we observe depends on luminosity and distance from star ( $f_\nu = \frac{L_\nu}{4\pi^2}$ )
- Luminosity given by  $L_\lambda d\lambda = 4\pi^2 R^2 B_\lambda d\lambda$
- Inverse square law:

$$\frac{f_1}{f_2} = \frac{d_2^2}{d_1^2} \quad (5)$$

## 1.1 Limiting Blackbody Spectra

At low frequencies,

$$h\nu \approx kT$$

$$B_\nu \sim \frac{2\nu^2}{c^2} kT$$

At high frequencies,

$$h\nu \approx kT$$

$$B_\nu \sim e^{-(h\nu/kT)}$$

## 2 Stars: Observations

### 2.1 Stellar Attributes

- Distance
- Temperatures
- Luminosity
- Radius
- Mass

### 2.2 Distance

Parallax, arcsecond, parsec.

The Moon is 30 arcminutes across.

Distance from Earth to Sun = 1AU.

$$d = \frac{1AU}{\tan(\alpha)} \approx \alpha^{-1} AU \quad (6)$$

For small angles (and most stars have very small angles to us) we can use the small angle approximation

$$\tan \theta = \theta + O(\theta^3) \quad (7)$$

### 2.3 Temperature

Photon random walks its way out of a star (interacting with electrons). Could take  $\sim 3$  seconds to get out straight, but actually takes thousands of years.

The photosphere is the point where star density is low so a photon can escape without interacting. This is the “surface”.

#### 2.3.1 Brightness temperature

Temperature measured at a particular frequency (equivalent temperature for a BB to generate that frequency)

#### 2.3.2 Colour temperature

Planck spectrum is fitted to spectrum.

The colour of a star is the ratio of two different wavebands.

### 2.3.3 Effective temperature

By using

$$L = 4\pi r^2 \sigma T^4 \quad (8)$$

We can measure luminosity, if we know the radius we can find temperature.

## 2.4 Magnitudes

Definition of apparent magnitude:

$$m_\lambda = -2.5 \log_{10} f_\lambda + C \quad (9)$$

Mimics the human eye (which views things logarithmically). Colours are defined as the difference between magnitudes. UBV system = Ultraviolet, Blue, Visual.

$$B - V = m_B - m_V \quad (10)$$

More B = bluer. NOTE: independent of distances (they cancel out b/c logarithmic difference).

## 2.5 Luminosity

Determined after finding distance  $d$  and flux  $f$ . Usually defined in a waveband.

$$L = 4\pi d^2 f \quad (11)$$

Integrating over all wavelengths gives bolometric luminosity.

## 2.6 Radius

Difficult to measure stellar radii (because they're too far away); often use indirect measurements. Can also use occultation, interferometry and eclipsing binaries. Indirect way:

$$L = 4\pi r^2 \sigma T^4 \quad (12)$$

Only  $\sim 100$  stars with known radii. Betelgeuse is the only star (apart from the Sun) with a directly measured radius; 0.125 arcsec. Recent data gives mass-radius relationship for low-mass stars as basically linear.

# 3 Stars: Observation II

## 3.1 Mass

To first order, the mass of a star tells you what sort of star you are looking at. Mass measurements are often used by binary systems



- Visual binary: two stars resolved implies long orbital timescales, so it takes a long time to get any information
- Astrometric binary: cannot resolve smaller object, so we observe orbital motion of brighter star; a “wobbling” orbit of a bright object implies there is another massive object there (this is how some exoplanets have been discovered)
- Eclipsing binary: orbital plane of binary is in line of sight; we measure parameters of an eclipse (most planets have been discovered using this)
- Spectroscopic binary: absorption or emission lines of both objects are observed in spectrum. Absorption lines will move with respect to each other (cannot resolve both stars, but can deduce the existence of a binary system)

We can investigate the curvature of an object in spacetime to determine mass, however this is a small perturbation so we often use other ways to determine mass.

### 3.1.1 Key attributes of a binary

The two stars orbit a common centre of mass; bigger object will be closer to the centre of mass.

$$r_1 M_1 = r_2 M_2 \quad (13)$$

Kepler’s 3 laws:

1. Both orbits are similar ellipses
2. Orbiting system conserves angular momentum; travels slower when further away etc.
3. When  $\tau$  = period,  $\omega$  = angular frequency,  $a$  = semi-major axis,  $M$  = masses, we have the equation:

$$\tau^2 = \frac{(2\pi)^2}{\omega^2} = \frac{a^3}{G(M_1 + M_2)} \quad (14)$$

### 3.1.2 Mass measurement

Assuming circular orbits (not elliptical),

$$\begin{aligned} \text{Centre of mass: } r_1 M_1 &= r_2 M_2 \\ a &= r_1 + r_2 \\ \text{Equation of motion: } M_1 \omega^2 r_1 &= \frac{G M_1 M_2}{a^2} \\ v_{1obs} &= \langle v_1 \rangle \sin(i) \\ \text{Projected velocities: } v_{2obs} &= \langle v_2 \rangle \sin(i) \\ \text{Final equation: } (M_1 + M_2) \sin^3(i) &= \frac{\tau (\langle v_{1obs} \rangle + \langle v_{2obs} \rangle)^3}{2\pi G} \end{aligned}$$

Note:  $i$  = angle between plane of motion and line-of-sight. We get the final equation using Kepler 3.

In a visual binary, if the period is 'short', then the angular radii can be directly measured  $\Rightarrow$  the ratio of masses. Kepler's 3rd Law then gives the individual masses.

In a spectroscopic binary, we only measure the projected velocities, and so can only determine the sum of the masses up to a factor of  $\sin^3(i)$ .

When one of the stars has a much smaller mass, the equations can simplify.

Can use Doppler velocity of star with orbiting planet to measure planet mass (sensitive to a couple of metres, can determine mass of at lightest  $M_{\text{Jupiter}}$ ).

### 3.1.3 Two monotonic empirical relationships

1. Mass-luminosity relationship:

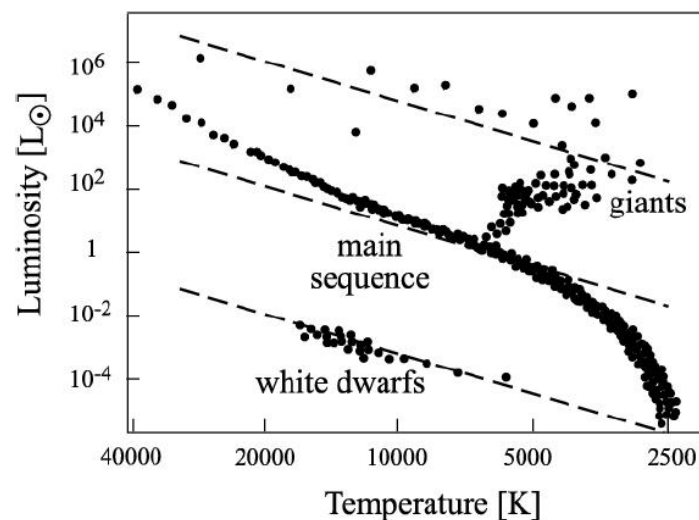
$$\frac{L}{L_{\odot}} = 0.35 \left( \frac{M}{M_{\odot}} \right)^{2.62} \quad M < 0.7M_{\odot}$$

$$\frac{L}{L_{\odot}} = 1.02 \left( \frac{M}{M_{\odot}} \right)^{3.92} \quad M > 0.7M_{\odot}$$

This implies more massive stars are much more luminous and 'burn' faster.

2. Hertzsprung-Russel Diagram

This is the diagram that plots luminosity against temperature; see figure below.



### 3.1.4 Stellar Spectral Type

Stars can be divided into classes based on characteristics.

Spectral Class	Colour	B-V index	Temperature (K)		Examples
O	Blue-violet	-.035	28-50,000	ionised He	Naos, Mitaka
B	Blue-white	-0.16	10-28,000	neutral He, some H	Spica, Rigel
A	White	+0.13	7.5-10,000	strong H, some ionised metals	Sirius, Vega
F	Yellow-white	+0.42	6-7,500	H and ionised Ca & Fe	Canopus
G	Yellow	+0.70	5-6,000	ionised Ca, neutral metals	Sun
K	Orange	+1.2	3.5-5,000	neutral metals	Arcturus
M	Red-orange	+1.2	2.5-3,500	strong TiO bands	Betelguese

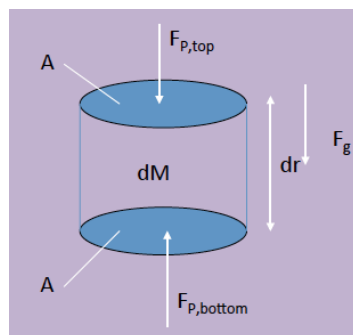
## 3.2 Stars: Physics

We can write down a set of equations which describe the structure of a star.

### 3.2.1 Hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} = g\rho \quad (15)$$

Mass  $M_r$  is mass enclosed in radius  $r$ .



$$\begin{aligned}
 F_g &= -\frac{GM_r dM}{r^2} \\
 dF_p &= AdP \\
 dm &= A\rho dr \\
 \Rightarrow \frac{dP}{dr} &= -\frac{GM_r\rho}{r^2} = g\rho
 \end{aligned}$$

### 3.2.2 Mass continuity

$$\frac{dM}{dr} = 4\pi r^2 \rho \quad (16)$$

### 3.2.3 Radiative energy transport

Energy can be transported via radiation, convection or conduction. Conduction is only important in white dwarfs and neutron stars. Convection is important in normal stars, but we will only consider radiative transport.

$$\frac{dT}{dr} = -\frac{3L\kappa\rho}{4\pi r^2 4acT^3} \quad (17)$$

### 3.2.4 Energy conservation

$\epsilon(r)$  defined as power produced by unit mass of stellar material. Luminosity produced within a thin shell:

$$\begin{aligned} dL &= \epsilon dM = \epsilon \rho 4\pi r^2 dr \\ \Rightarrow \frac{dL}{dr} &= 4\pi r^2 \rho \epsilon \end{aligned}$$

So we have 4 equations and 7 unknowns. Need 4 more equations. We will need 3 further functions -  $\epsilon$ ,  $\kappa$ , and an equation of state.

### 3.2.5 Assumptions

1. If gas is fully ionised, Thomson scattering is most important interaction;  $\sigma = 6.7 \times 10^{-25} \text{ cm}^2$
2. Can estimate number density. See example below.

#### Example 1:

$$\begin{aligned} \frac{M}{V} &= \langle \rho \rangle \\ V &= \frac{4}{3}\pi r^3 \sim 1.4 \times 10^{33} \text{ cm}^3 \\ n &= \frac{\langle \rho \rangle}{m_H} = \frac{2 \times 10^{33} \text{ g}}{1.4 \times 10^{33} \text{ cm}^3} \frac{1}{1.7 \times 10^{-24} \text{ g}} \\ &\sim 8.2 \times 10^{23} \text{ cm}^{-3} \\ l &= \frac{1}{n\sigma} = 1.8 \text{ cm} \end{aligned}$$