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# PHYC90011 Particle Physics

## Assignment 5

By


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# 1 Drag racing in space in the Greco-Roman era

Buoyed by the success of their intrepid interstellar experiment on the twin paradox (the legend is recounted in Section 1.13 of the 1st edition of A First Course in General Relativity, by B. F. Schutz), twin sisters Artemis and Diana decide to conduct a follow-up investigation into the physics of accelerated reference frames in flat space as follows. At the same instant, 1 the sisters jump into two rocket ships, which are initially at rest in a global inertial frame, 2 and race off in the x-direction with constant but unequal proper accelerations. In their space packs they carry a laser pointer (Artemis), a mirror (Diana), and identically manufactured clocks and rulers (Artemis and Diana). 3 Their departure points and trajectories are not the same. In the global inertial frame in which the rocket ships are initially at rest, Artemis starts out from the event $(t, x, y, z) = (0, g^{-1}, 0, 0)$, where $(t, x, y, z)$ are standard Minkowski coordinates, and g denotes her proper acceleration. As derived in lectures, her trajectory on a spacetime diagram is a hyperbola in the x-t plane defined parametrically by

$$
\begin{aligned}
t_A(\tau A) &= g^{-1} \sinh g\tau_A, &(1)\\
x_A(\tau A) &= g^{-1} \cosh g\tau_A, &(2)\\
y_A(\tau A) &= 0, &(3)\\
z_A(\tau A) &= 0 &(4)
\end{aligned}
$$

where $\tau_A$ is her proper time. Diana's departure point, trajectory, and proper acceleration remain unspecified for now.

**(a)** Before attempting any experiments, Artemis constructs a new coordinate system $(t, x, y, z)$ for making measurements in the neighbourhood of her rocket ship. The construction proceeds in three steps, which we copy here.

**i.** Fermi-Walker transport the Minkowski basis vectors $\vec{e}_0$, $\vec{e}_1$, $\vec{e}_2$, and $\vec{e}_3$ along Artemis's world line. Show that this procedure yields a new basis

$$
\begin{aligned}
\vec{e}_0(\tau_A) &= \vec{e}_0 \cosh g\tau_A + \vec{e}_1 \sinh g\tau_A, &(5)\\
\vec{e}_1(\tau_A) &= \vec{e}_0 \sinh g\tau_A + \vec{e}_1 \cosh g\tau_A, &(6)\\
\vec{e}_2(\tau_A) &= \vec{e}_2, &(7)\\
\vec{e}_3(\tau_A) &= \vec{e}_3 &(8)
\end{aligned}
$$

at the point along the world line labelled by $\tau_A$.

**ii.** Verify that one can obtain (5)-(8) by Lorentz boosting the Minkowski basis vectors into the rocket ship's momentarily comoving reference frame.

**iii.** Let the vector $x'\vec{e}_{1'} + y'\vec{e}_{2'} + z'\vec{e}_{3'}$ be the displacement of an event $P$ from the spacetime location of the centre of mass of Artemis's rocket ship. Let $t'$ be the proper time measured by Artemis at the space point $(x, y, z) = (0, 0, 0)$. Show that these definitions lead to the transformation

$$t = (g^{-1} + x') \sinh gt', \tag{9}$$

$$x = (g^{-1} + x') \cosh gt', \tag{10}$$

$$y = y' \tag{11}$$

$$z = z', \tag{12}$$

if $P$ occurs at $(t, x, y, z)$ in Minkowski coordinates.