

SN: 587623

PHYC90012 General Relativity

Course Summary

By

Braden MOORE

Master of Science
The University of Melbourne

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1 Syllabus

1.1 Part I

1. Introduction to gravity
 - Order of magnitude estimates
 - Small amount of quantum gravity
2. Equivalence principle + experimental foundations
3. Geometric objects
 - Need to understand geometric components of GR
 - Vectors, metric, etc. that live on manifolds
 - Laws of nature do not depend on coordinates chosen
 - Hence can write laws of nature in terms of geometric objects w/o reference to coordinates
4. Kinematics
 - Time dilation, length contraction in GR framework
5. Calculus in curvilinear coordinates
 - Mass and energy curve spacetime
 - Hence geometric objects moved on curved manifolds
 - Distances are not only spatial but temporal; need to use mathematics of small change = calculus
 - Uses the covariant derivative (a geometric object; independent of basis/coordinate independent)
 - This point of the course we will not be considering curved space, but instead only curvilinear coords
 - A flat space can be covered (represented?) by curved coordinates, but an intrinsically curved surface cannot be covered by flat coordinates
6. Curved spaces
 - Manifolds
 - How to calculate lengths, volumes, angles in curved spaces
 - Introduces the idea of parallel transport \Rightarrow leads to curvature
 - Define the Riemann tensor, and its children etc. Ricci tensor, ...; these satisfy the Bianchi identities
7. Einstein's field equations
 - Stress-energy tensor
8. Weak-field limit
 - Gauge transformations

1.2 Part II - Applications

9. GR phenomena revisited

- GPS, Mercury's orbit, gravitational lensing, gravitational redshift, ...

10. Gravitational waves

- Propagation (phase speed, polarisation, ...)
- Generation*
- Detection*
 - * = “antenna problem”

11. Relativistic stars

- neutron stars
- equation of state (cannot study on Earth because largest nuclei only have 200 elements or so; need more density)

12. Black holes

- Event horizons, singularities, ...

13. Cosmology

- Friedman-Robertson-Walker (FRW) metric - describes a homogeneous, isotropic universe
 - We will derive this and the Friedman equations

2 Introduction to gravity

2.1 Strength of gravity

- Weak! Weakest of all fundamental forces
- Long-ranged force (like EM)
- Weakness determined by coupling constant
- Coupling constant = Newton's gravitational constant

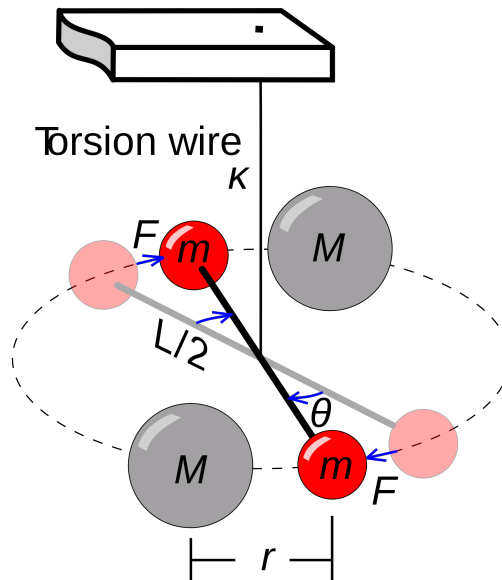
$$\vec{F} = \frac{Gm_1m_2}{r_{12}^2}\hat{r} \quad (1)$$

- G is hard to measure; least well known of coupling constants

In 1797-98, Cavendish used torsion balls (1.8m torsion balance) with rod of big masses and rod of small masses.

- Spring constant of torsion balance was measured from free oscillation

- ten introduced 158kg balls
- measured deflection angle of balance \Rightarrow force
 - using a mini-telescope against Vernier scale
- rearrange Newton's law to get G



Exercise: Show that Cavendish also measured density of Earth as a bonus at the same time

- Modern $G = 6.67384(80) \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2} = \text{kg}^{-1} \text{ m}^3 \text{ s}^{-2}$
- Product GM is known to 1 part in 10^{10} from astrophysics observations
 - \Rightarrow mass is hard to measure gravitationally
- We need a dimensionless number to characterise strength
- Newton: $\Phi = \frac{GM}{r}$ (potential)
- In free fall: $\frac{KE}{mass}, v^2 \frac{GM}{r}$
- We claim gravity is strong if free-fall is relativistic, i.e. $v \sim c$
- This is an order of magnitude estimate