

# Lepton flavor violation from supersymmetric grand unified theories: Where do we stand for MEG, PRISM/PRIME, and a super flavor factory

L. Calibbi,<sup>1</sup> A. Faccia,<sup>1</sup> A. Masiero,<sup>1</sup> and S. K. Vempati<sup>2,3</sup><sup>1</sup>*Dipartimento di Fisica “G. Galilei” and INFN, Sezione di Padova, Università di Padova, Via Marzolo 8, I-35131, Padova, Italy*<sup>2</sup>*Centre de Physique Theorique, Unité mixte du CNRS et de l’EP, UMR 7644, Ecole Polytechnique-CPHT, 91128 Palaiseau Cedex, France*<sup>3</sup>*The Institute of Mathematical Sciences, Chennai 600 013, India*

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We analyze the complementarity between lepton flavor violation (LFV) and LHC experiments in probing the supersymmetric (SUSY) grand unified theories (GUT) when neutrinos get a mass via the seesaw mechanism. Our analysis is performed in an SO(10) framework, where at least one neutrino Yukawa coupling is necessarily as large as the top Yukawa coupling. Our study thoroughly takes into account the whole renormalization group running, including the GUT and the right-handed neutrino mass scales, as well as the running of the observable neutrino spectrum. We find that the upcoming (MEG, SuperKEKB) and future (PRISM/PRIME, super flavor factory) LFV experiments will be able to test such SUSY framework for SUSY masses to be explored at the LHC and, in some cases, even beyond the LHC sensitivity reach.

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## I. INTRODUCTION

In this paper we address the issue of detecting low-energy supersymmetry (SUSY) at present, upcoming, and planned lepton flavor violation (LFV) experiments. Moreover, we evaluate the complementarity between these experiments and the CERN Large Hadron Collider (LHC) experiments as probes of supersymmetric grand unified (SUSY-GUT) scenarios.

The study of flavor changing neutral current (FCNC) processes, which are suppressed by the Glashow-Iliopoulos-Maiani mechanism [1] in the standard model (SM) of particle physics, has been considered for a long time a powerful tool in order to shed light on new physics, especially for testing low-energy supersymmetry. Indeed, taking into account the fact that neutrinos have mass and mix [2–6], the standard model predicts lepton flavor violating processes in the charged sector to occur at a negligible rate [e.g.  $BR(\mu \rightarrow e\gamma) \sim \mathcal{O}(10^{-54})$ ] [7,8]. Given the future experimental sensitivities to LFV processes (Table I), the discovery of such processes will open a window to new physics.

It is known that a generic low-energy SUSY model (i.e. a model with arbitrary mixings in the soft breaking parameters sector) would induce unacceptably large flavor violating effects [20]. The unobserved departures from SM in FCNCs make it reasonable to assume flavor universality in the mechanism that breaks SUSY. On the other hand, even taking flavor universal SUSY breaking boundary conditions, renormalization effects can generate sizable flavor mixings in the running of the soft parameters from the SUSY breaking mediation scale down to the SUSY decoupling scales. In the leptonic sector, the relevance of such effects strongly depends on the neutrinos’ parameters.

The existence and smallness of neutrinos’ masses can be simply explained by the seesaw mechanism [21], by introducing right-handed neutrino (RN) fields, that are singlets under the SM gauge transformations. Since there is no gauge symmetry that protects them, the RNs can get a large Majorana mass  $(\hat{M}_R)_{ij}$ , breaking the conservation of lepton number. When they are integrated out, they will give rise to an effective light neutrino Majorana mass matrix

$$m_\nu = -Y_\nu \hat{M}_R^{-1} Y_\nu^T \langle H_u \rangle^2, \quad (1)$$

where  $(Y_\nu)_{ij}$  are the Yukawa couplings between left- and right-handed neutrinos and  $\langle H_u \rangle$  is the vacuum expectation value (VEV) acquired by the up sector Higgs field.

It is known [22] that the marriage between seesaw and SUSY can generate observable LFV rates in the charged lepton sector. In their renormalization group (RG) evolution, the slepton soft masses  $(m_L^2)_{ij}$  acquire LFV entries that are proportional to  $(Y_\nu Y_\nu^\dagger)$ :

TABLE I. Present bounds and expected experimental sensitivities on LFV processes [9–19].

Process	Present bound	Future sensitivity
$BR(\mu \rightarrow e\gamma)$	$1.2 \times 10^{-11}$	$\mathcal{O}(10^{-13}-10^{-14})$
$BR(\mu \rightarrow eee)$	$1.1 \times 10^{-12}$	$\mathcal{O}(10^{-13}-10^{-14})$
$CR(\mu \rightarrow e \text{ in Ti})$	$4.3 \times 10^{-12}$	$\mathcal{O}(10^{-18})^a$
$BR(\tau \rightarrow e\gamma)$	$3.1 \times 10^{-7}$	$\mathcal{O}(10^{-8})-\mathcal{O}(10^{-9})^a$
$BR(\tau \rightarrow eee)$	$2.7 \times 10^{-7}$	$\mathcal{O}(10^{-8})-\mathcal{O}(10^{-9})^a$
$BR(\tau \rightarrow \mu\gamma)$	$6.8 \times 10^{-8}$	$\mathcal{O}(10^{-8})-\mathcal{O}(10^{-9})^a$
$BR(\tau \rightarrow \mu\mu\mu)$	$2 \times 10^{-7}$	$\mathcal{O}(10^{-8})-\mathcal{O}(10^{-9})^a$

<sup>a</sup>Planned or discussed experiment, not yet under construction

$$(m_L^2)_{i \neq j} = -\frac{3m_0^2 + A_0^2}{16\pi^2} \sum_k Y_{\nu ik} Y_{\nu kj}^\dagger \ln\left(\frac{M_X^2}{M_{Rk}^2}\right), \quad (2)$$

where  $M_{Rk}$  is the mass of the  $k$ th right-handed neutrino ( $i, j, k$  being generation indices),  $m_0$  and  $A_0$  are the universal supersymmetry breaking scalar masses and scalar trilinear couplings, respectively. Since the seesaw equation (1) allows large neutrinos' Yukawa couplings, sizable effects can stem from this running. From the above it is obvious that any estimate of  $(m_L^2)_{i \neq j}$  would require a complete knowledge of the neutrino Yukawa matrix  $(Y_\nu)_{ij}$  which is not fixed by the seesaw equation, even with an improved knowledge of the neutrino oscillation parameters, as in (1) there is a mismatch between the number of unknowns and that of low energy observables. This could indeed pose a problem compared to the quark-squark FCNCs sector, where it is possible to make firm predictions of FV entries due to RG evolution in a flavor universal boundary condition. On the other hand, in the quark sector the disentangling of FV effects stemming from SUSY from those coming from the SM is more problematic, as both are driven by the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix and happen to be roughly of the same order of magnitude. In the charged lepton sector, we have the opposite scenario: SM contributions are well below any envisageable experimental sensitivity, but it is not possible to predict SUSY induced FCNC rates without resorting to an ansatz with regard to the form of the Yukawa matrix  $Y_\nu$  or the general framework of the theory. Many groups [23] have addressed this issue in different frameworks.

In this paper we inspect LFV in a SO(10) supersymmetric grand unified (SUSY-GUT) framework. The choice of the GUT scenario stems from the fact that the possible detection of SUSY particles at the LHC will provide an indirect evidence for such a scenario. Moreover, in SO(10) theories the seesaw mechanism is naturally present and the neutrino Yukawa couplings are related to those of the up quarks, making them naturally large [24], so that sizable LFV entries will stem from the (2) RG evolution. Even if the SO(10) framework gives us some hints about the unknown neutrino Yukawa matrix  $Y_\nu$ , telling us that the eigenvalues are related to the ones of the up Yukawa matrix  $Y_u$ , it still leaves uncertainty about the size of mixing angles in  $Y_\nu$ , as the knowledge of the low-energy neutrino parameters (masses and mixings) is not sufficient to set the matrices that diagonalize  $Y_\nu$ . Following the scheme of previous works [24–26], we bypass the ignorance about the mixings by considering two extremal benchmark cases. Such cases are intended as boundary conditions at high scale. As a minimal mixing case, we take the one in which the neutrino and the up Yukawa unify at the high scale, so that the mixing is given by the CKM matrix; this case is named “CKM case.” As a maximal mixing scenario we take the one in which the observed neutrino mixing is coming entirely from the neutrino Yukawa matrix, so that

$Y_\nu = U_{\text{PMNS}} \cdot Y_u^{\text{diag}}$ , where  $U_{\text{PMNS}}$  is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix; in this case the unknown  $U_{e3}$  PMNS matrix element turns out to be crucial in evaluating the size of LFV effects. The maximal case is named “PMNS case.”

The aim of the present work is to study, in a minimal supergravity (mSUGRA) scenario,<sup>1</sup> the influence on LFV from the RG running both above and below the unification scale  $M_{\text{GUT}}$ . A preliminary version of this analysis is already present in the literature [26]. However, that work considered the LFV contributions from the seesaw structure only. It is well known that in a SO(10) scenario there are other contributions stemming from the grand unified structure [27], as on general grounds the SUSY breaking mediation scale and the GUT scale do not coincide. In the present work we detail the various contributions and their relative relevance as a source of LFV in the SO(10) framework. We analyze in detail the impact of LFV experiments in the parameter space region that will be probed by the LHC, both in the minimal (CKM-like) and in the maximal (PMNS-like) cases and for different values of  $\tan\beta$ . Such a *vis-a-vis* analysis allows us to address the issue of the complementarity between LHC and LFV experiments as probes of SUSY-GUTs.

We argue that, even in the presence of a discovery machine like the LHC, flavor physics experiments will still play an important role in the hunt for new physics. Indeed, LHC evidences alone will hardly discriminate among the many possible SUSY realizations, while flavor physics should be, in this sense, more sensitive. Moreover, several flavor physics experiments are currently running or under construction (such as  $B$ -factories, the SuperKEKB upgrade [10], and the upcoming MEG [11] experiment at PSI), and thus some hints of new physics before the LHC era are also possible. Furthermore, there are discussions and plans on very sensitive LFV experiments beyond the LHC era, such as the PRISM/PRIME experiment at J-PARC [17] and a super flavor factory [19]. It is thus timely to ask what will be the capability of such experiments to discriminate between different SUSY-GUT realizations, in the case that the LHC gets a positive evidence for SUSY. Let us note that, even in the case that nothing is seen at the LHC, taking into account that SUSY effects decouple slowly with increasing SUSY masses, flavor physics could still exhibit some indirect SUSY evidence.

The main results of our analysis are:

- (i) The maximal PMNS case is already ruled out by the current MEGA bound on  $\mu \rightarrow e\gamma$  in the case the squark masses are lighter than 1.5 TeV. MEG will improve the situation by testing it well beyond the reach of LHC sensitivity.

<sup>1</sup>We are taking the soft trilinear mass scale  $A_0$  as a free parameter, not linked to the Higgs sector  $B$  parameter as in a strict mSUGRA scenario

- (ii) If the unknown  $U_{e3}$  angle is very small, at present the PMNS case is constrained only in the high  $\tan\beta$  region, by  $B$ -factories  $\text{BR}(\tau \rightarrow \mu\gamma)$  bounds, to lay in the region of squark masses bigger than 800 GeV. In the future, MEG will be able to test this scenario in all the LHC accessible SUSY parameter space if  $\tan\beta$  is high. For small  $\tan\beta$  the best probe will come from SuperKEKB  $\tau \rightarrow \mu\gamma$  BR bounds, testing it for squark masses up to 700 GeV.
- (iii) The minimal CKM case is at present unconstrained. MEG will be able to test it in the high  $\tan\beta$  region and for squark masses lighter than 800 GeV; this scenario will evade detection by SuperKEKB. The low  $\tan\beta$  minimal mixing case will remain unconstrained.
- (iv) The proposed post-LHC era PRISM/PRIME and super flavor factory experiments will much improve the situation. PRISM/PRIME would supersede MEG by testing, by means of  $\mu \rightarrow e$  conversion in Ti, all the scenarios in all the LHC accessible SUSY parameter space. A super flavor factory would be highly complementary, being able to detect the LFV  $\tau \rightarrow \mu\gamma$  process up to 1 TeV squark masses.

The paper is organized as follows: in Sec. II we motivate, in the context of SUSY-GUTs, our  $Y_\nu \sim Y_u$  ansatz; in Sec. III we proceed to estimate the LFV RG induced soft masses and the branching ratios that stem from them. The numerical routine is presented in Sec. IV and the results are discussed in Sec. V: we evaluate LFV rates for the parameter space region within the reach of the LHC and comment on the complementarity between direct SUSY searches and LFV experiments. In Sec. VI we predict LFV rates at the benchmark points defined by the “Snowmass Points and Slopes” (SPS) cases, in our SUSY-GUT frameworks, and argue on the possibility of future experiments to test these scenarios. In Sec. VII we give a summary of our findings and draw the conclusions. Last, in the Appendix we fix the notation and give the  $\text{SU}(5)_{\text{RN}}$  RG equations.

## II. SEESAW AND SUSY $\text{SO}(10)$

An  $\text{SO}(10)$  SUSY-GUT framework naturally incorporates the seesaw mechanism. This is because the matter representation is a 16-dimensional spinor containing right-handed neutrinos which are absent by choice in the standard model spectrum. Further models of  $\text{SO}(10)$  have two salient features which make them interesting to study SUSY seesaw:

- (i) First, they unify the Dirac neutrino Yukawa couplings ( $Y_\nu$ ) and the up-type Yukawa couplings ( $Y_u$ ). Though this unification is exact for smaller Higgs representations, like the **10**s, it can be shown that even in the presence of larger representations like **120** or **126**, at least one of the neutrino Yukawa

couplings is as large as the top Yukawa. This can be shown by a simple analysis of the resultant mass matrices [24].

- (ii) Second, as mentioned in the Introduction, unlike the quark sector, the leptonic sector has the seesaw mechanism that makes it distinct. Particularly, the observed large neutrino mixing does not necessarily mean a large “left” mixing to be present in the neutrino Dirac Yukawa couplings: in fact even CKM-like small mixings in the neutrino Yukawa couplings can lead to large neutrino mixing [28]. This is best depicted by the Casas-Ibarra parametrization [29] that solves the seesaw equation (1) for the neutrino Yukawa matrix  $Y_\nu$ :

$$Y_\nu = \frac{1}{\langle H_u \rangle} U_{\text{PMNS}} \mathcal{D}_\nu R \mathcal{D}_N, \quad (3)$$

where  $\mathcal{D}_N$  and  $\mathcal{D}_\nu$  are the square root of the diagonal right-handed Majorana and the low energy neutrino mass matrices, respectively. The unknown complex orthogonal matrix  $R$  parametrizes the uncertainty of the mixing between Majorana and Dirac right-handed eigenstates. This means that, if  $R$  is the identity the Dirac neutrino Yukawa matrix inherits the PMNS mixing structure, whereas small CKM-like  $Y_\nu$  mixings reflect in a nontrivial structure of the misalignment matrix  $R$ . With this in mind, most flavor models have either of the two situations of (a) small left mixing in  $Y_\nu$  or (b) large left mixing in  $Y_\nu$  (which can also be understood as mixing from the charged lepton sector). In an  $\text{SO}(10)$  GUT, where there is a unification of  $Y_\nu$  and  $Y_u$ , both these situations can be realized by choosing appropriate Higgs representations. We have christened these two cases as the CKM case and the PMNS case, respectively. These two cases have the following relations between the Yukawa matrices, in the basis where charged lepton and down quark mass matrices are diagonal

$$Y_\nu = Y_u (\text{CKM case}) \quad (4)$$

$$Y_\nu = U_{\text{PMNS}} Y_u^{\text{diag}} (\text{PMNS case}). \quad (5)$$

Both the above situations can be realized in  $\text{SO}(10)$  without spoiling the relation between neutrino Yukawa and the top Yukawa. The CKM case can be realized by a simple superpotential involving only ten-plets [30]

$$W_{\text{SO}(10)} = (Y_u)_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_u + (Y_d)_{ii} \mathbf{16}_i \mathbf{16}_i \mathbf{10}_d + (Y_R)_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{126}, \quad (6)$$

where  $i$  and  $j$  are generation indices. The PMNS case is a bit more complicated as it can come from either a renormalizable or nonrenormalizable

couplings. For example, Chang, Masiero, and Murayama [31] have proposed the following superpotential:

$$W_{\text{SO}(10)} = (Y_u)_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_u + (Y_d)_{ii} \mathbf{16}_i \mathbf{16}_i \frac{45 \mathbf{10}}{M_{\text{Planck}}} + (Y_R)_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{126}, \quad (7)$$

which leads to the PMNS-like situation. Note that both superpotentials we have mentioned here are just scenarios but not complete fermion mass models. For our purposes, these two scenarios serve as the *benchmark* points in the seesaw parameter space.

The LFV soft mass entries are generated in the RG evolution from the universality scale down to the SUSY decoupling scale  $M_{\text{SUSY}} = 1$  TeV. The breaking of SUSY SO(10) down to the SM can be achieved in several different ways [32]. Within the two benchmark scenarios chosen above, we envisage a breaking chain (Fig. 1) of  $\text{SO}(10) \rightarrow \text{SU}(5)_{\text{RN}} \rightarrow \text{MSSM}_{\text{RN}} \rightarrow \text{MSSM}$ . Such a breaking can be achieved if the singlet under the SU(5) component of a  $\mathbf{16}$  or of a  $\mathbf{126}$  attains a VEV. The scale of  $\text{SU}(5)_{\text{RN}}$  is taken to be the scale of the gauge coupling unification  $M_{\text{GUT}} \sim 2 \times 10^{16}$  GeV. The SO(10) scale is considered to be slightly higher about  $M_X \sim 10^{17}$ . This scale can be considered to be the string unification scale,  $M_{\text{string}}$ , which roughly turns out to be a factor 20–25 from the gauge coupling unification scale after considering string loop effects [33]. One interesting aspect is that, while we set the scale of the right-handed neutrinos from low energy neutrino data and  $Y_\nu$  (as we will detail in Sec. IV), it turns out that the required right-handed neutrino masses are close to the GUT scale, which fits our scheme naturally.

Let us note in passing that, as a consequence of RH down quarks and lepton superfields sitting in the same  $\bar{5}$  representation of SU(5), we expect that the neutrino Yukawa couplings contribute to the RGE evolution of the soft mass matrix  $m_d^2$ . This determines the arising of new contributions to flavor changing processes in the hadronic sector (contributions that are consequently linked to the LFV ones) [31, 34]. Here we focus on the reach implications for LFV, while postponing a detailed discussion of the correlated implications for hadronic and leptonic sector to a forthcoming publication [35].

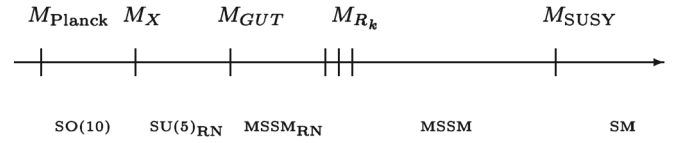


FIG. 1. Schematic picture of the energy scales involved in the model.

Before proceeding to next section where we detail the various lepton flavor violating terms generated in these two schemes, we would like to make some comments on the recent progress in SO(10) model building. In the recent years a new view regarding SO(10) model building is being developed, where construction of more realistic and complete models is being pursued [36]. In these “minimal” complete models, it is perhaps for the first time possible to compute the entire SO(10) spectrum, study realistically precision observables such as running of fermion mass spectrum including threshold effects, gauge coupling running, proton decay, etc. [37]. In the present work, we are more concerned with the effect of SO(10) seesaw couplings on the *soft* supersymmetry breaking sector of the theory. We do not resort to a complete model building of SO(10), but just consider schemes of SO(10). This is sufficient for our purposes, as we aim to compute the flavor violating entry in the slepton and sneutrino mass matrices at the weak scale generated through the seesaw mechanism within both these schemes. We use 1-loop RGE equations for this purpose and perform scatter plots in the SUSY breaking parameter space.

### III. LFV SOURCES IN SUSY SO(10)

In the present section, we elaborate on the various contributions to the lepton flavor violating entries in the SO(10) SUSY-GUT framework. Perhaps the best way to understand them is in terms of the low-energy parameters. We use the so-called mass insertion (MI) [38] notation to denote the various flavor violating entries of the slepton mass matrix. These flavor violating entries are zero at the high scale, where SUSY breaking soft scalar masses are universal. At the weak scale, the universality is broken by the RG evolution and the  $6 \times 6$  slepton squared-masses matrix  $\mathcal{M}_\ell^2$  takes the form

$$\mathcal{M}_\ell^2 = \begin{pmatrix} m_\ell^2(1 + \delta_{\text{LL}}) + Y_e Y_e^\dagger v_d^2 + \mathcal{O}(g^2) & v_d(A_e^\dagger - Y_e^\dagger \mu \tan \beta) + \delta_{\text{LR}} \hat{m}_\ell^2 \\ v_d(A_e - Y_e \mu \tan \beta) + \delta_{\text{RL}} \hat{m}_\ell^2 & m_\ell^2(1 + \delta_{\text{RR}}) + Y_e^\dagger Y_e v_d^2 + \mathcal{O}(g^2) \end{pmatrix}, \quad (8)$$

where the flavor violation is coded in the  $\delta$ s given by

$$\delta_{ij} = \frac{\Delta_{ij}}{\hat{m}_\ell^2}, \quad (9)$$

with  $\hat{m}_\ell^2$  being the geometric mean of the slepton squared

masses [39] and  $\Delta_{i \neq j}$  flavor nondiagonal entries of the slepton mass matrix generated at the weak scale by RG evolution. The mass insertions are divided into the LL/LR/RL/RR types, according to the chirality of the corresponding SM fermions. Detailed bounds on each of these types



TABLE II. Masses (in GeV) of the lightest SUSY particles and of the Higgs boson, corresponding to the SUSY-GUT point  $m_0 = 500$  GeV,  $M_{1/2} = 500$  GeV,  $A_0 = 0$ .

Mass	$\tan\beta = 10$	$\tan\beta = 40$
$m_{\tilde{\tau}_1}$	574	447
$m_{\tilde{t}_1}$	845	838
$m_{\tilde{g}}$	1225	1225
$m_{\tilde{\chi}_1^0}$	234	234
$m_{\tilde{\chi}_1^\pm}$	431	432
$m_h$	123	124

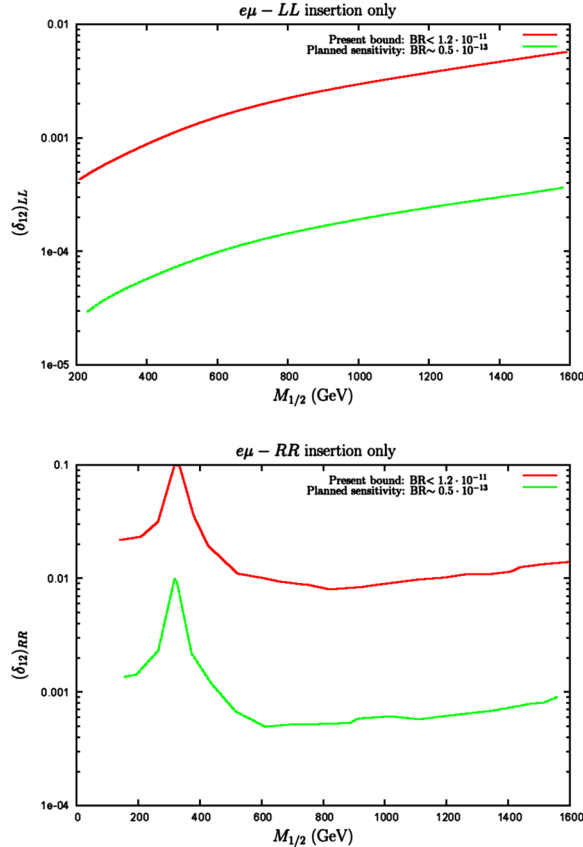


FIG. 2 (color online). Points in the  $(M_{1/2}, (\delta_{12})_{LL})$  and  $(M_{1/2}, (\delta_{12})_{RR})$  planes that fulfill the  $\mu \rightarrow e\gamma$  branching ratio experimental limit. The plots are for  $\tan\beta = 10$  and  $m_0 = 500$  GeV.

TABLE III. Bounds on the  $\delta$ s from the present and future  $\text{BR}(\ell_i \rightarrow \ell_j \gamma)$  experimental limits; by unconstrained we mean that the  $\delta$  is  $\mathcal{O}(1)$ . In the  $\tau\mu$  sector the future sensitivity is given both for SuperKEKB and for the proposed super flavor factory. The  $\delta$ s are calculated at  $t_\beta = 10$ ,  $m_0 = 500$  GeV, and  $M_{1/2} = 500$  GeV; as can be seen from Fig. 2 no particular cancellation is occurring at this point.

Process	LL		RR		LR	
	Present	Future	Present	Future	Present	Future
$\mu \rightarrow e\gamma$	$1.4 \times 10^{-3}$	$8.5 \times 10^{-5}$	$1.4 \times 10^{-2}$	$9 \times 10^{-4}$	$9.2 \times 10^{-6}$	$5.9 \times 10^{-7}$
$\tau \rightarrow \mu\gamma$	$2.4 \times 10^{-1}$	$1.3 \times 10^{-1}/3 \times 10^{-2}$	Unconstrained	Unconstrained/ $2.9 \times 10^{-1}$	$2.8 \times 10^{-2}$	$1.5 \times 10^{-2}/4.7 \times 10^{-3}$
$\tau \rightarrow e\gamma$	$4.7 \times 10^{-1}$	$1.4 \times 10^{-1}$	Unconstrained	Unconstrained	$5.9 \times 10^{-2}$	$1.5 \times 10^{-2}$

of  $\delta$ s already exist in the literature [40,41]. Note that these bounds are obtained by considering one  $\delta$  at time to be the source of the flavor violating effects, and putting all the other  $\delta$ s to zero.

In our case, these  $\Delta_{ij}$  are generated by RG evolution either through the seesaw mechanism or through the GUT evolution. This means that there exist several  $\delta$ s at the same time, so that the interplay between them should be evaluated. However, as an illustration, we compare these resultant  $\Delta$ s generated by RGEs with the existing MI bounds, conveying us the *power* of each individual contribution at the weak scale in constraining the SUSY breaking parameter space. We further elaborate the cases where double mass insertions could be important compared to the single mass insertions.

For the discussion of this section, we choose a point in the SUSY parameter space:  $m_0 = 500$  GeV;  $M_{1/2} = 500$  GeV;  $A_0 = 0$ ;  $\tan\beta = 10$  (some comments will be made also on a high  $\tan\beta$  scenario, with  $t_\beta = 40$ ). The SUSY spectrum for this point is given in Table II. As can be seen from Fig. 2 there is no fine-tuned cancellation in LFV amplitudes at this point, so that we can take it as a “safe” benchmark point.

By turning on a  $\delta$  at time, we get the present and future bounds on each single LFV  $\delta$ , as given in Table III. From the table and from Fig. 2 it is clear that the RR  $\delta$ s are less constrained than the LL ones by the LFV branching ratio bounds; this is due to two reasons: (i) the amplitudes involving only  $\delta_{RR}$  MI do not have chargino contributions, and (ii) in certain regions of the parameter space, there could be cancellations between the bino and the higgsino-bino-higgsino contributions [41]. It is also clear that the LR entries are much suppressed. This is mainly due to the fact that, as can be seen by comparing Eq. (8) and (9), in the normalization procedure (9) the  $(\delta_{LR})_{ij}$  entry pays a factor  $\frac{m_i}{m_e}$ , where  $m_i$  is the mass of the  $i$ th lepton.

### A. LL insertions from the running

To compute the  $\Delta$ 's from the RGEs, in this section we use the leading log approximation. Taking the soft masses to be flavor universal at the input scale, off-diagonal entries in the LL sector are generated by right-handed neutrinos

running in the loops; in our framework where  $Y_{\nu_3} = Y_l$  we can estimate

$$(\Delta_{LL})_{i \neq j} = -\frac{3m_0^2 + A_0^2}{16\pi^2} Y_l^2 V_{i3} V_{j3} \ln\left(\frac{M_X^2}{M_{R_3}^2}\right), \quad (10)$$

where  $V$  can be either  $V_{CKM}^T$  or  $U_{PMNS}$ , depending on the case.

### 1. CKM case

To use the leading log expression (10), we need to know the mass of the heaviest right-handed neutrino. By using the seesaw formula (1) we can estimate it to be in the CKM case [26]:

$$M_{R_3} \approx \frac{m_t^2}{4m_{\nu_1}}; \quad (11)$$

taking  $m_{\nu_1} \approx 10^{-3}$  eV, we get  $M_{R_3} \sim 10^{15}-10^{16}$  GeV. The induced off-diagonal entries relevant to  $\ell_i \rightarrow \ell_j$ ,  $\gamma$  are of the order of (putting  $A_0$  to zero)

$$\begin{aligned} (\delta_{LL})_{\mu e} &= -\frac{3}{8\pi^2} Y_l^2 V_{td} V_{ts} \ln\frac{M_X}{M_{R_3}} \\ (\delta_{LL})_{\tau\mu} &= -\frac{3}{8\pi^2} Y_l^2 V_{tb} V_{ts} \ln\frac{M_X}{M_{R_3}} \\ (\delta_{LL})_{\tau e} &= -\frac{3}{8\pi^2} Y_l^2 V_{tb} V_{td} \ln\frac{M_X}{M_{R_3}}. \end{aligned} \quad (12)$$

In these expressions, the CKM angles are small but one would expect the presence of the large top Yukawa coupling  $Y_t$  to compensate such a suppression, giving rise to sizable  $\delta$ s. We see from Table IV that all the  $\delta$ s will be outside the reach of planned experiments. Let us note that the  $\mu e$  sector entry is almost at the boundary of MEG sensitivity: from Fig. 2 it is clear that MEG will test it for  $M_{1/2} \lesssim 250$  GeV.

### 2. PMNS case

In the PMNS case the  $R$  matrix is the identity; the seesaw formula (1) can be straightforwardly inverted to get

$$\hat{M}_R = \text{diag}\left\{\frac{m_u^2}{m_{\nu_1}}, \frac{m_c^2}{m_{\nu_2}}, \frac{m_t^2}{m_{\nu_3}}\right\}. \quad (13)$$

TABLE IV. CKM case: leading log estimates of off-diagonal entries in the slepton mass matrices. The bounds are calculated at  $\tan\beta = 10$ ,  $m_0 = 500$  GeV, and  $M_{1/2} = 500$  GeV. For the  $\tau\mu$  sector we give the sensitivity for both SuperKEKB and a super flavor factory.

Generations	$ \delta_{LL} $	Present bound	Future sensitivity
$\mu e$	$3.4 \times 10^{-5}$	$1.4 \times 10^{-3}$	$8.5 \times 10^{-5}$
$\tau\mu$	$6.2 \times 10^{-3}$	$2.4 \times 10^{-1}$	$1.3 \times 10^{-1}/3.0 \times 10^{-2}$
$\tau e$	$8.5 \times 10^{-4}$	$4.7 \times 10^{-1}$	$1.4 \times 10^{-1}$

TABLE V. PMNS case: leading log estimates of off-diagonal entries in the slepton mass matrices. The values are for  $m_0 = 500$  and  $M_{1/2} = 500$  GeV,  $U_{e3} = 0.07$  and  $\tan\beta = 10$ . In the  $\tau\mu$  sector we give the sensitivities for both SuperKEKB and a super flavor factory.

Generations	$ \delta_{LL} $	Present bound	Future sensitivity
$\mu e$	$1.2 \times 10^{-2}$	$1.4 \times 10^{-3}$	$8.5 \times 10^{-5}$
$\tau\mu$	$1.2 \times 10^{-1}$	$2.4 \times 10^{-1}$	$1.3 \times 10^{-1}/3.0 \times 10^{-2}$
$\tau e$	$1.2 \times 10^{-2}$	$4.7 \times 10^{-1}$	$1.4 \times 10^{-1}$

Taking the neutrino spectrum to be hierarchical so that  $m_{\nu_3} \approx \sqrt{\Delta m_{\text{atm}}^2}$ , we can estimate the third right-handed neutrino to have mass  $M_{R_3} \sim 10^{14}$ . Plugging the value in the equation (10)

$$\begin{aligned} (\delta_{LL})_{\mu e} &= -\frac{3}{8\pi^2} Y_l^2 U_{e3} U_{\mu 3} \ln\frac{M_X}{M_{R_3}} \\ (\delta_{LL})_{\tau\mu} &= -\frac{3}{8\pi^2} Y_l^2 U_{\mu 3} U_{\tau 3} \ln\frac{M_X}{M_{R_3}} \\ (\delta_{LL})_{\tau e} &= -\frac{3}{8\pi^2} Y_l^2 U_{e3} U_{\tau 3} \ln\frac{M_X}{M_{R_3}} \end{aligned} \quad (14)$$

and taking  $U_{e3} = 0.07$  at about half of the current CHOOZ bound, we get the estimates in Table V. We see from the table that the  $\mu e$  sector is already ruled out by the present bound and that the upcoming bound will be able to test it up to

$$U_{e3} = 0.07 \frac{8.5 \times 10^{-5}}{1.4 \times 10^{-3}} \sim 10^{-3}. \quad (15)$$

Moreover, the  $\tau\mu$  sector will be probed by the SuperKEKB machine and thoroughly tested by the proposed super flavor factory.

### B. LR/RL insertions from the running

The flavor violating terms in the LR sector are given by the off-diagonal terms of the sleptons' soft trilinear  $(A_e)_{ij}$ ; the RG generated entries in (8) are

$$\begin{aligned} (\Delta_{LR})_{i \neq j} &= \langle H_d \rangle [(A_e)_{ij} (M_X \rightarrow M_{\text{GUT}}) \\ &\quad + (A_e)_{ij} (M_{\text{GUT}} \rightarrow M_R)] \\ &= -\frac{3m_i A_0}{32\pi^2} \sum_k Y_{\nu ik} Y_{\nu kj}^\dagger \ln\left(\frac{M_X^2}{M_{Rk}^2}\right) \\ &\quad - \frac{9m_j A_0}{32\pi^2} \sum_k Y_{uik} Y_{ukj}^\dagger \ln\left(\frac{M_X^2}{M_{\text{GUT}}^2}\right), \end{aligned} \quad (16)$$

where  $m_i$  is the mass of the  $i$ th lepton and the last line is coming from the fact that the left-handed sleptons and the  $d^c$  squarks are hosted together in the **5** of SU(5). Taking into account only the third generation order one Yukawa coupling, we have

$$(\Delta_{\text{LR}})_{ij} = -\frac{3A_0}{32\pi^2} \left[ m_i Y_{\nu i3} Y_{\nu j3}^* \ln\left(\frac{M_X^2}{M_{R_3}^2}\right) + 3m_j Y_{ui3} Y_{uj3}^* \ln\left(\frac{M_X^2}{M_{\text{GUT}}^2}\right) \right] \quad (17)$$

so that the seesaw driven contribution and the GUT driven one give the dominant contribution to different (transposed) entries. Let us note that these entries are roughly equal, as the color factor 3 is almost compensated by the longer running of the seesaw driven entries. As a consequence, the remarks we are going to do about the  $ij$  entry will at the same time apply to the transposed  $ji$  one. Comparing Eq. (17) with Eq. (10) we have

$$(\Delta_{\text{LR}})_{ij} = \frac{3m_i A_0}{3m_0^2 + A_0^2} (\Delta_{\text{LL}})_{ij} \quad (18)$$

switching to the adimensional  $\delta$ s we get

$$|(\delta_{\text{LR}})_{ij}| = \frac{3|a_0|}{3 + a_0^2} \frac{m_i}{m_0} |(\delta_{\text{LL}})_{ij}| < \frac{m_i}{m_0} |(\delta_{\text{LL}})_{ij}|, \quad (19)$$

where  $a_0 = A_0/m_0$ . A crucial point is that the RG generated LR insertion (19) is not the main contribution to the LR flavor violating insertion [42]. Indeed it is possible to build an effective LR flavor violating insertion by combining together the LL RG generated flavor violating entry (10) with a LR flavor conserving  $m_i \mu \tan\beta$  chirality flip

$$(\delta_{\text{LR}})_{ij}^{\text{eff}} = \frac{m_i \mu \tan\beta}{\hat{m}_\ell^2} (\delta_{\text{LL}})_{ij}. \quad (20)$$

Comparing Eq. (19) to Eq. (20),

$$\left| \frac{(\delta_{\text{LR}})_{ij}}{(\delta_{\text{LR}})_{ij}^{\text{eff}}} \right| < \frac{\hat{m}_\ell^2}{m_0 |\mu| \tan\beta} \approx (\tan\beta)^{-1}, \quad (21)$$

we see that the effective LR insertion stemming from the RG generated LL one is enhanced by a factor  $\tan\beta$  with respect to the RG generated pure LR insertion, so that the effective insertion always dominates.

### C. RR insertions from the running

The  $\text{SU}(5)_{\text{RN}}$  running from the soft breaking scale  $M_X$  to the GUT scale already breaks the universality of the soft spectrum by generating LFV entries at  $M_{\text{GUT}}$ . The renormalization group equation for  $\text{SU}(5)_{\text{RN}}$  are calculated at the 1-loop level in the Appendix; the most interesting consequence of the  $\text{SU}(5)_{\text{RN}}$  running is that, as both  $Q$  and  $e^c$  are hosted in the **10**, the CKM matrix mixing the left handed quarks will give rise to off-diagonal entries in the running of the right-handed slepton soft masses [27,34,43–45]

TABLE VI. Estimates of the  $\delta$ s from leading log at  $A_0 = 0$ .

Generations	$ \delta_{\text{LL}} $ CKM	$ \delta_{\text{LL}} $ PMNS	$ \delta_{\text{RR}} $
$\mu e$	$3.4 \times 10^{-5}$	$1.2 \times 10^{-2}$	$7.8 \times 10^{-5}$
$\tau \mu$	$6.2 \times 10^{-3}$	$1.2 \times 10^{-1}$	$1.4 \times 10^{-2}$
$\tau e$	$8.8 \times 10^{-4}$	$1.2 \times 10^{-2}$	$2 \times 10^{-3}$

$$(\Delta_{\text{RR}})_{i \neq j} = (m_{\tilde{10}}^2)_{ij} (M_X \rightarrow M_{\text{GUT}}) = -3 \frac{3m_0^2 + a_0^2}{16\pi^2} V_{ii} V_{ij} \ln\left(\frac{M_X^2}{M_{\text{GUT}}^2}\right), \quad (22)$$

where we have used the fact that  $Y_i \approx 1$  and  $V_{ik}$ ,  $k = i, j$  is the  $tk$  entry of the CKM matrix. Let us note that this effect is due to the GUT structure, and is so independent on any ansatz on the form of the neutrino Yukawa matrix: in this sense the  $\delta_{\text{RR}}$  and the BRs stemming from them form a guaranteed minimum on the LFV effects from a SUSY-GUT.

In Table VI we present a comparison of all the  $\delta$ s, other than the LR ones, for the two cases of minimal and maximal mixing. We see that in the PMNS case the main source of LFV violation are the LL insertions, whereas in the CKM case the  $\text{SU}(5)_{\text{RN}}$  running gives rise to a sizable right slepton off-diagonal mass

$$(m_{\tilde{E}}^2)_{i \neq j} \approx 2(m_{\tilde{L}}^2)_{i \neq j}, \quad (23)$$

which could give a significant contribution to LFV amplitudes. However, as we mentioned at the beginning of this section, the  $\delta_{\text{RR}}$  contribution to the branching ratio is suppressed by the cancellations in the neutralino sector. In Fig. 3 we point out that the full BR in the CKM case is of

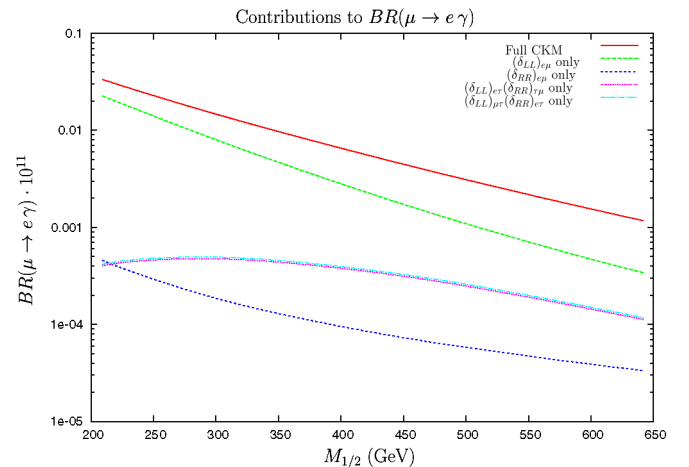


FIG. 3 (color online). Comparison of the  $\mu \rightarrow e \gamma$  BRs occurring from a full CKM case with those from just the LL entries, the GUT generated RR entries and the double mass insertions. The plots are done at  $\tan\beta = 40$ ,  $m_0 = 500$  GeV, and varying  $M_{1/2}$  between 200 and 650 GeV. Details about the numerical procedure will be given in the next section.

TABLE VII. Estimates of the branching ratios versus present bounds and future sensitivities.  $m_0$  and  $M_{1/2}$  are taken to be 500 GeV.

Generations	$t_\beta = 40$ CKM	$t_\beta = 10$ PMNS	Experimental bound	Future sensitivity
$\mu e$	$6 \times 10^{-14}$	$5 \times 10^{-10}$	$1.2 \times 10^{-11}$	$10^{-13}-10^{-14}$
$\tau\mu$	$2 \times 10^{-9}$	$5 \times 10^{-8}$	$6.8 \times 10^{-8}$	$10^{-8}$
$\tau e$	$4 \times 10^{-11}$	$5 \times 10^{-10}$	$3.1 \times 10^{-7}$	$10^{-8}$

the same order of magnitude of the one calculated by taking into account the LL entries only; on the other hand, the ratio between the  $\delta_{LL}$  and the  $\delta_{RR}$  generated BRs is more than a factor 10.

Following these results we can safely neglect the RR contributions and estimate the branching ratios according to the formula [46] (note that this already incorporates the effective LR insertion)

$$\begin{aligned} \text{BR}(l_i \rightarrow l_j \gamma) &= \frac{\alpha^3}{G_F^2} \frac{(\delta_{LL})_{ij}^2}{m_{\text{SUSY}}^4} \tan^2 \beta \\ &\approx 4.5 \times 10^{-6} \left( \frac{500 \text{ GeV}}{m_{\text{SUSY}}} \right)^4 (\delta_{LL})_{ij}^2 \left( \frac{t_\beta}{10} \right)^2, \end{aligned} \quad (24)$$

where  $m_{\text{SUSY}}$  is linked to the high energy inputs parameters  $m_0$  and  $M_{1/2}$  by the best fit relation [47]

$$m_{\text{SUSY}}^4 = 0.5 \left( \frac{M_{1/2}^2}{m_0^2} \right) (m_0^2 + 0.6 M_{1/2}^2)^2. \quad (25)$$

The estimated  $\text{BR}(\ell_i \rightarrow \ell_j \gamma)$  are given in Table VII. We see that for  $m_0$  and  $M_{1/2}$  at 500 GeV the PMNS case is already ruled out even at small  $\tan\beta$  by the  $\mu \rightarrow e \gamma$  branching ratio, so that we expect the upcoming MEG experiment to be able to test it even for soft masses as big as 5 TeV. On the other hand, we expect that the MEG experiment will be able to test the small mixing angle case only for high values of  $\tan\beta$ . As for the  $\tau$  sector, we see that the only channel that offers interesting rates is the  $U_{e3}$  independent  $\tau \rightarrow \mu \gamma$  process: the SuperKEKB bound of  $10^{-8}$  will be able to test the PMNS case even in the small  $\tan\beta$  region, whereas a super flavor factory, with a planned sensitivity of at least  $\mathcal{O}(10^{-9})$ , is expected to address even the issue of small mixing angles, provided that  $\tan\beta$  is large.

It is interesting to note from Fig. 3 that the subleading contribution to the  $\mu \rightarrow e \gamma$  process is not arising from a pure  $(\delta_{RR})_{e\mu}$  insertion but from the double FV mass insertion  $(\delta_{LL})_{e\tau}(\delta_{RR})_{\tau\mu} + (e \leftrightarrow \mu)$ : this is an effective LR MI, which is enhanced by the flavor conserving  $m_\tau \mu \tan\beta$  contribution. This allows us to give a rough estimate for the subleading contribution to the BR to be

$$\text{BR}(\mu \rightarrow e \gamma)_{2\delta} = \left( \frac{h_\tau}{h_\mu} \right)^2 (\delta_{RR})_{\mu 3}^2 \text{BR}(\tau \rightarrow \mu \gamma), \quad (26)$$

where the suffix  $2\delta$  represents the 2 flavor violating effective MI.

## IV. NUMERICAL ANALYSIS OF LFV PROCESSES

### A. Parameter space of SUSY-GUTs

As mentioned above, we consider mSUGRA boundary conditions for the soft masses. At the high scale, the parameters of the model are the universal scalar mass  $m_0$ , universal trilinear couplings  $A_0$ , and unified gaugino masses  $M_{1/2}$ . In addition to these there are the two Higgs potential parameters  $\mu$  and  $B$  and the undetermined ratio of the Higgs VEVs,  $\tan\beta$ . The entire supersymmetric mass spectrum is determined once these parameters are given. However, all these parameters are not independent. Incorporating electroweak symmetry breaking gives rise to two conditions, reducing the number of independent parameters by two. In our case, we determine  $\mu$  and  $B$  by electroweak symmetry breaking conditions. The two conditions of the electroweak symmetry breaking are

$$\begin{aligned} |\mu|^2 &= \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} m_Z^2 \\ \sin 2\beta &= \frac{2B\mu}{m_{H_u}^2 + m_{H_d}^2 + 2\mu^2}, \end{aligned} \quad (27)$$

where  $m_{H_u}^2$  and  $m_{H_d}^2$  are the up- and down-type Higgs soft mass squared parameters determined at the weak scale, using the RG equations from the  $M_X$  scale to the weak scale. At the weak scale, all the supersymmetric soft parameters are thus known, enabling us to compute the complete supersymmetric mass spectrum.

We impose two main “theoretical” constraints on the SUSY parameter space: (a) radiative electroweak symmetry breaking (REWSB) [48] should take place; (b) no tachyonic particles and that the lightest supersymmetric particle (LSP) should be a neutralino.<sup>2</sup> The experimental constraints are detailed in the next subsection. In contrast to the minimal supersymmetric standard model (MSSM),<sup>3</sup> both these constraints are significantly modified in the SO(10) framework we discuss in the present paper. As it is well known, in the MSSM, radiative electroweak symmetry breaking is driven by the top Yukawa coupling. In the SO(10) model we are considering, two further effects

<sup>2</sup>We do not impose any additional constraints requiring unification of the Yukawa couplings in the present work.

<sup>3</sup>In the present paper, we call simply MSSM what some authors define “constrained MSSM” (CMSSM), i.e. MSSM with universal conditions for the soft SUSY breaking parameters imposed at  $M_X$ .



are present: (i) the range of the logarithmic running is a bit larger as  $M_X$  is taken to be  $\sim 5 \times 10^{17}$  compared to typical MSSM studies, which consider the scale to be  $\sim 2 \times 10^{16}$ ; (ii) the neutrino Yukawa couplings  $Y_\nu$ , one of which is necessarily as large as the top Yukawa coupling, also contribute to driving the up-type Higgs soft mass squared  $m_{H_u}^2$  negative in the running from the scales  $M_X$  to  $M_{R_3}$ . These two contributions can significantly alter the parameter space which is viable under the electroweak breaking constraint.

A similar effect takes place for the region of the parameter space in which the lightest slepton, which is typically the right-handed stau  $\tilde{\tau}_R$ , is the LSP. In contrast to MSSM, in the GUT framework, the stau also receives corrections from the “pre-GUT scale” running from the scale  $M_X$  to the  $M_{\text{GUT}}$ . In SU(5), as we consider in the present scenario, the stau sits in the ten-plet **10** which also hosts the strongly interacting sector, leading to “strong” contributions

through the gaugino loops. A leading log estimate of these contributions for  $m_0 \approx 0$  is given by

$$m_{\tilde{\tau}_R}^2(M_{\text{GUT}}) = \frac{96}{80\pi^2} M_{1/2}^2 \ln\left(\frac{M_X}{M_{\text{GUT}}}\right) \approx 0.4 M_{1/2}^2. \quad (28)$$

These positive contributions can thus offset the negative contributions from the Yukawa running. Both these effects are best demonstrated in the Fig. 4, where the difference in the allowed parameter space of the MSSM and of the SO(10) framework is evident.

Finally, as mentioned in the Introduction, we would like to do a complementarity study between the region of the parameter space probed at the LHC *vis-a-vis* the LFV experiments. For this, we first need to determine the region of the parameter space probed by the LHC considering various detection channels, putting the appropriate background cuts, detector response functions, etc. We do not intend to do such detailed analysis in this work. For mSUGRA, it is already present in the literature [49]. The typical estimate for the mass of the gluino and squarks to be detected at the LHC is about 2–3 TeV. We define the parameter space region that allows a squark mass to be below 2.5 TeV to be the region probed by the LHC. In Fig. 4 the contours for the masses are shown in the  $(m_0, M_{1/2})$  plane. We call this region the LHC accessible region. However, we also further consider other regions of the parameter space which, though not accessible at the LHC, can be relevant for the reach of flavor physics experiments. With this in mind, we scan the total parameter space in the following ranges

$$m_0 \in (0, 5000) \text{ GeV}$$

$$M_{1/2} \in (0, 1500) \text{ GeV}$$

$$A_0 \in (-3m_0, +3m_0)$$

$$\tan\beta = 10, 40$$

$$\text{sign}\mu \in \{+, -\}.$$

### B. Integration procedure

In the present section, we detail the integration procedure we have incorporated in our work. A schematic diagram is presented in Fig. 5. As inputs at the weak scale, we consider the Yukawa couplings of the up-type quarks, down-type quarks, charged leptons, the CKM mixing matrix, and  $\tan\beta$ . We employ a hierarchical scheme for the neutrino masses. The lightest neutrino is taken to be around  $10^{-3}$  eV. The other two neutrino masses are determined by the square roots of the solar and atmospheric mass squared differences, respectively. The leptonic mixing matrix  $U_{\text{PMNS}}$  has two large mixing angles, and the unknown third mixing angle  $U_{e3}$  is left as a free parameter.<sup>4</sup> Unless otherwise stated, we take  $U_{e3} = 0.07$ , half of the current

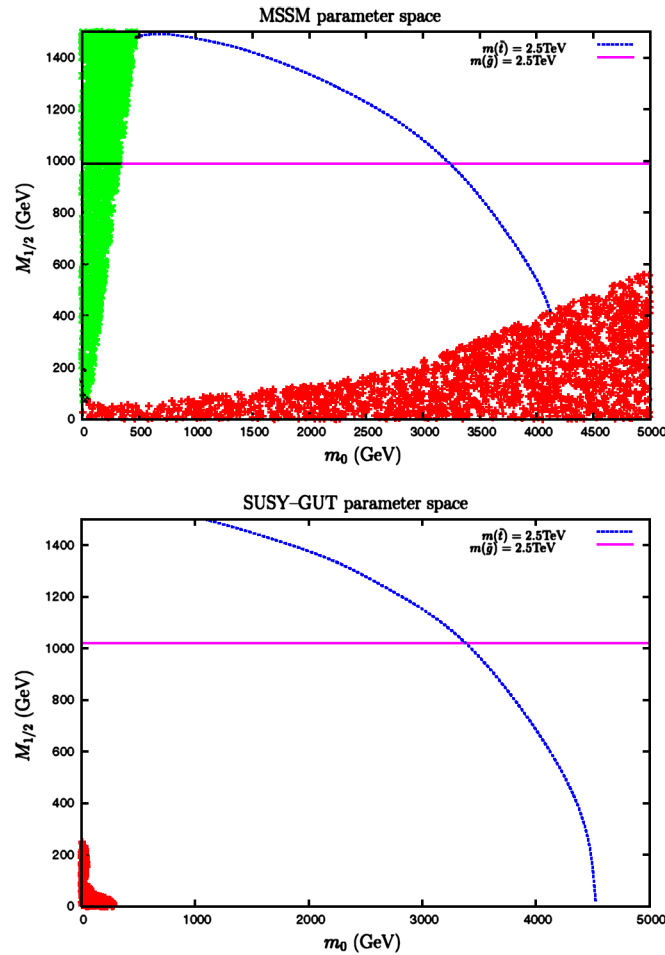


FIG. 4 (color online). Comparison of MSSM and SUSY-GUT parameter spaces. The colored areas are ruled out: green (light grey) one corresponds to points where the LSP is not a neutralino; red ones (dark grey) to points where the vacuum is not viable (either because of no REWSB or tachyonic particles). The plots are for  $\tan\beta = 30$ ,  $A_0 = 0$ , and  $m_t = 173$  GeV.

<sup>4</sup>In the present work we consider only real parameters. The phases of CKM and PMNS matrices are set to be zero.

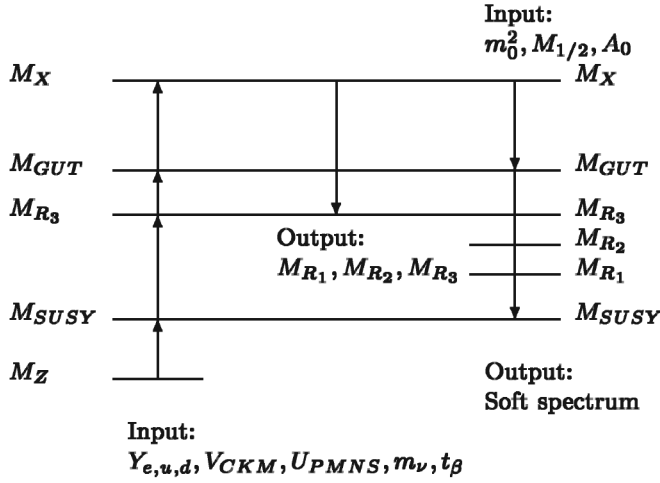


FIG. 5. Pictorial explanation of the running routine. See the text for the details.

upper limit from the CHOOZ experiment. We use 1-loop RGEs to run all the Yukawa couplings up to the high scale. For the neutrino masses and mixing we use the RGE given in the literature [50,51].

As a first step, we run the neutrino mass matrix and the Yukawa and gauge couplings up to the right-handed neutrino masses, using an estimated  $M_{R_3}$ , given by (11) in the CKM case and by (13) in the PMNS one. At that scale we assign the neutrino Yukawa matrix: in the CKM case we evaluate it to be  $Y_\nu = Y_u$ ; in the PMNS case we first extract  $U_{\text{PMNS}}(M_{R_3})$  and then define  $Y_\nu = U_{\text{PMNS}}(M_{R_3})Y_u^{\text{diag}}$ . We then run up to the  $M_X$  scale and we redefine  $Y_\nu(M_X)$  to be equal to  $Y_u(M_X)$  in the CKM case, or to  $U_{\text{PMNS}}(M_X)Y_u^{\text{diag}}$  in the PMNS case. Once the neutrino Yukawa matrix is known at  $M_X$ , we are able to use the seesaw formula (1) in order to calculate the right-handed neutrinos mass matrix and, thus, the energies at which each heavy neutrino should decouple; we have only to use again the RGEs down to the estimated  $M_{R_3}$  and do the calculation.<sup>5</sup> We thus use the iterative method to check if our results are right.

With this information, high energy inputs and the intermediate energy scales, we are now ready to compute the running of the soft spectrum from the high scale to the weak scale. We do this using 1-loop RGEs [52]. At the weak scale, we compute the full  $6 \times 6$  mass matrices of all the scalars and the neutralino and chargino mass matrices. In the Higgs sector we employ the full 1-loop effective potential [53] to determine the parameters and compute the spectrum. Finally, we impose various direct experimental constraints as well as the theoretical constraints on the SUSY parameter space:

- (i) LEP mass limit on the lightest Higgs;

- (ii) direct search limits on charginos and sfermions;
- (iii) neutral LSP;
- (iv) viable vacuum: REWSB at  $M_Z$  and no tachyonic particles.

For every point which passes through all these constraints, we compute leptonic flavor violating decay rates by using the exact mixing matrices [45] as well as masses for the sleptons, neutralinos, and charginos.

## V. RESULTS

From our leading log estimates, we expect that the most promising sectors for finding SUSY-GUT induced LFV are the  $\mu e$  and the  $\tau \mu$  ones. Given that the planned sensitivities (Table I) to all the LFV processes will be of the same order ( $\sim \mathcal{O}(10^{-13}-10^{-14})$  in the  $\mu e$  sector and  $\sim \mathcal{O}(10^{-8})$  in the  $\tau \mu$  one), we concentrate on the two body decays,  $\mu \rightarrow e \gamma$ , to be probed by the MEG experiment at PSI, and  $\tau \rightarrow \mu \gamma$ , that is under study at beauty factories. Indeed, the three body decays are weaker probes of SUSY-GUTs, as the leading penguin contribution leads to a BR that is suppressed by a factor  $\sim \alpha$  with respect to the two body decay. The  $\mu \rightarrow e$  conversion in the nuclei process suffers from a similar suppression, but due to the well-defined experimental signal the PRISM/PRIME aims to a huge improvement in the sensitivity to offset this factor.

In this section we display the results from the numerical routine for the processes of interest. All the plots are done for positive  $\mu$  as there is no sensible difference with the negative  $\mu$  case as far as lepton flavor violating processes are concerned.<sup>6</sup>

### A. The MEG experiment at PSI

Given the planned astonishing sensitivity of the upcoming MEG [11] experiment at PSI, we expect that the muon decay  $\mu \rightarrow e \gamma$  will be a very interesting probe of LFV in a SUSY-GUT scenario. This statement is quantified in Figs. 6 and 7: the PMNS case high  $\tan \beta$  scenario is already ruled out by the current MEGA [12] bound on the  $\text{BR}(\mu \rightarrow e \gamma)$ ; the low  $\tan \beta$  regime is already severely constrained for not too high  $M_{1/2}$  and will be completely probed by the upcoming MEG experiment. The CKM case, instead, is below the present bounds in all the parameter space, but a sensible portion of the high  $\tan \beta$  regime will be within the reach of MEG sensitivity (Fig. 7).

This allows us to draw the conclusion that (Table VIII), for not too big values of the soft breaking parameters [i.e.:  $(m_0, m_{\tilde{g}}) \lesssim 1$  TeV], the MEG experiment will be able to find evidence of SUSY induced lepton flavor violation, unless we are in a low  $\tan \beta$ , small mixing SUSY-GUT: as a consequence, if the LHC finds supersymmetry to be at

<sup>5</sup>This last step is necessary only in the CKM case, as the relations (13) are exact at the scale  $M_R$ .

<sup>6</sup>Let us note that the  $\mu < 0$  scenario is strongly disfavored by bounds on the FCNC  $b \rightarrow s \gamma$  amplitude and by the SUSY corrections to  $(g_\mu - 2)$ .

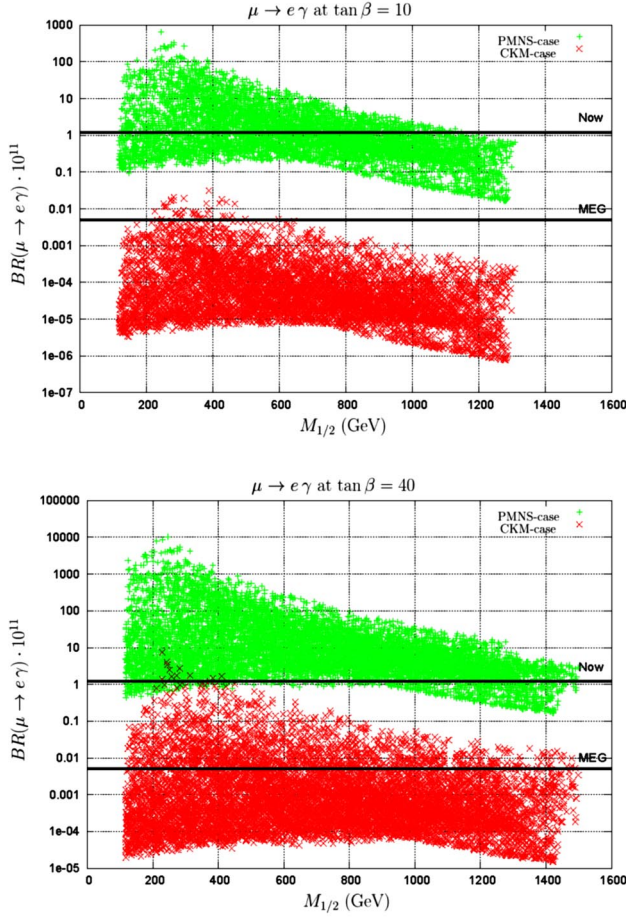


FIG. 6 (color online). Scaled  $BR(\mu \rightarrow e\gamma)$  vs  $M_{1/2}$ . The plots are obtained by scanning the LHC accessible SUSY-GUT parameter space at fixed values of  $\tan\beta$ . The horizontal lines are the present (MEGA) and the future (MEG) experimental sensitivities. Note that MEG will test the PMNS case and, for high  $\tan\beta$ , constrain the CKM one.

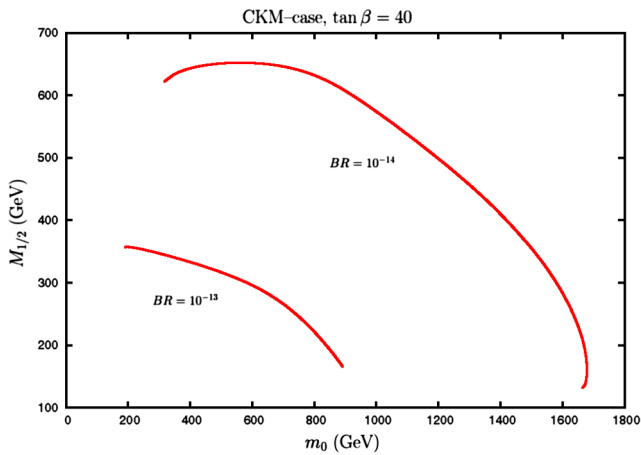


FIG. 7 (color online). Contour plots at fixed  $BR(\mu \rightarrow e\gamma)$  in the  $(m_0, M_{1/2})$  plane, at  $A_0 = 0$  in a CKM high  $\tan\beta$  case. Note that, while the plane is presently unconstrained, the MEG experiment sensitivity of  $\mathcal{O}(10^{-13}-10^{-14})$  will be able to probe it in the  $(m_0, m_{\tilde{g}}) \lesssim 1$  TeV region.

TABLE VIII. Reach in  $(m_0, m_{\tilde{g}})$  of the past and upcoming experiments from their  $\mu \rightarrow e\gamma$  sensitivity. LHC means that all the LHC testable parameter space will be probed; all means that soft masses as high as  $(m_0, m_{\tilde{g}}) \lesssim 5$  TeV will be probed.

Experiment	PMNS		CKM	
	$t_\beta = 40$	$t_\beta = 10$	$t_\beta = 40$	$t_\beta = 10$
MEGA	LHC	2 TeV	No	No
MEG	All	All	1.3 TeV	No

the TeV scale but  $\mu \rightarrow e\gamma$  escapes MEG detection, this will be the only viable SUSY SO(10) seesaw scenario. Moreover, as depicted in Fig. 8, in the PMNS case the sensitivity of MEG will outreach that of the LHC, being able to probe soft masses as high as  $(m_0 = 5, M_{1/2} = 1.6)$  TeV—so that if MEG gets positive evidence but the LHC fails in its aim to detect superpartners the viable SUSY-GUTs will be restricted to the high soft mass regime with large mixing angle in the neutrino Yukawa sector.

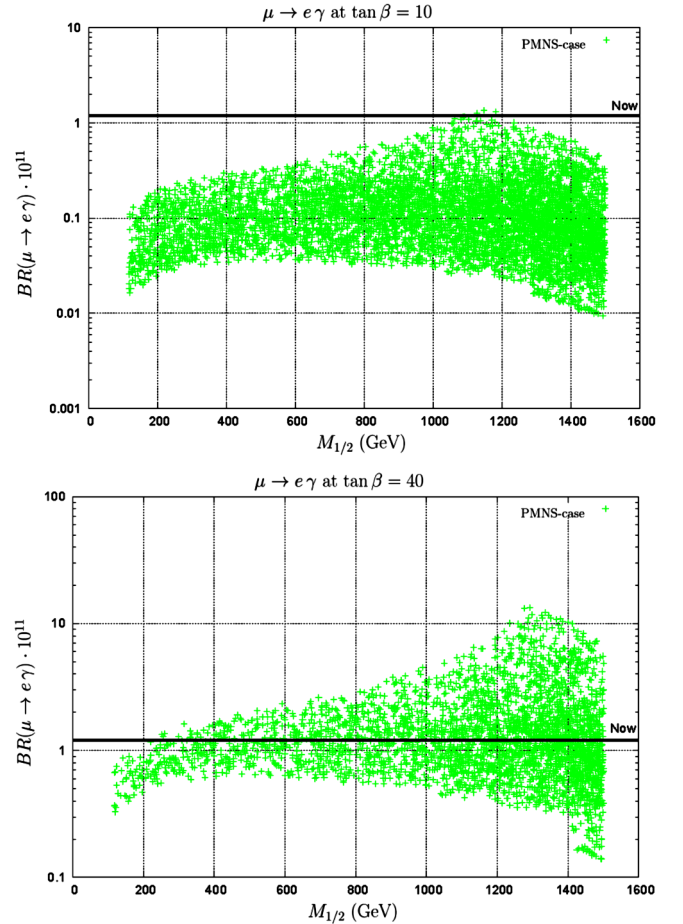


FIG. 8 (color online). Scaled  $BR(\mu \rightarrow e\gamma)$  vs  $M_{1/2}$  outside LHC experiments' reach for low and high  $\tan\beta$ . The horizontal line is the present MEGA bound. The upcoming MEG sensitivity will test all the points.



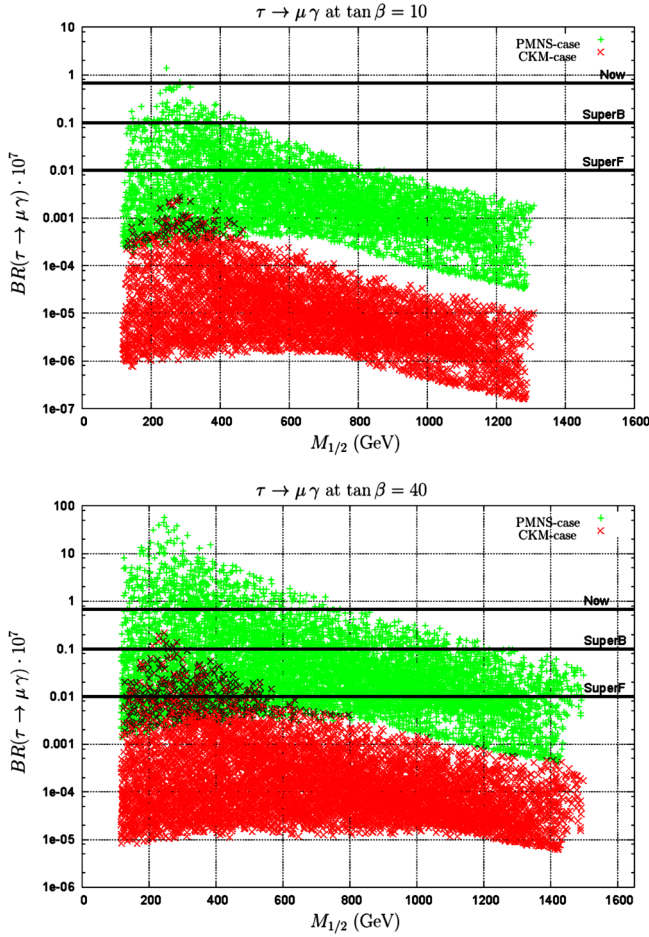


FIG. 9 (color online). Scaled  $BR(\tau \rightarrow \mu\gamma)$  vs  $M_{1/2}$ . The plots are obtained by scanning the LHC accessible SUSY-GUT parameter space at fixed  $\tan\beta$ . The horizontal lines are the present ( $B$  factories), future (SuperKEKB), and planned (super flavor factory) experimental sensitivities.

### B. $B$ factories, SuperKEKB, and super flavor factory

The  $\tau\mu$  sector poses promising prospects of discovery of SUSY-GUT induced lepton flavor violation in the case that the planned super flavor factory [19] will be realized: let us note that this machine is planned to reach a sensitivity of at least  $BR(\tau \rightarrow \mu\gamma) \sim \mathcal{O}(10^{-9})$ , with an improvement of the present bound by nearly 2 orders of magnitude. The main theoretical interest for such a process arises from the fact that the dominant LFV insertion  $(\delta_{LL})_{\tau\mu}$  does not depend on the unknown PMNS angle  $U_{e3}$ .

As far as beauty factories [14–16] are concerned, we see from Fig. 9, that even with the present bound it is possible to rule out part of the PMNS high  $\tan\beta$  regime; the planned accuracy of the SuperKEKB [10] machine  $\sim \mathcal{O}(10^{-8})$  will allow to test much of high  $\tan\beta$  region and will start probing the low  $\tan\beta$  PMNS case, with a sensitivity to soft masses as high as  $(m_0, m_{\tilde{g}}) \lesssim 900$  GeV. The situation changes dramatically if one takes into account the possibility of a super flavor factory (Figs. 9 and 10): taking the

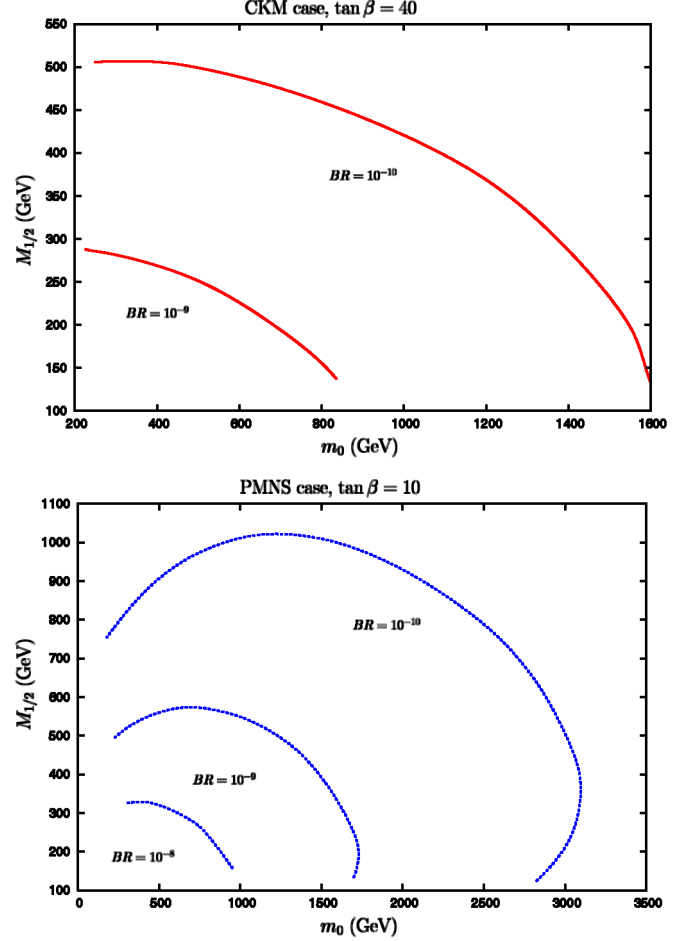


FIG. 10 (color online). Contour plots at fixed  $BR(\tau \rightarrow \mu\gamma)$  in the  $(m_0, M_{1/2})$  plane, at  $A_0 = 0$  in the CKM high  $\tan\beta$  case, and in the PMNS  $t_\beta = 10$  one. Note that, while the planes are presently unconstrained, the super flavor factory sensitivity of  $\mathcal{O}(10^{-9})$  would be able to probe much of the PMNS case at low  $t_\beta$  and the  $(m_0, m_{\tilde{g}}) < 900$  GeV portion of the high  $\tan\beta$  CKM case.

sensitivity of the most promising  $\tau \rightarrow \mu\gamma$  process to  $\sim \mathcal{O}(10^{-9})$ , the PMNS case will be nearly ruled out in the high  $\tan\beta$  regime and severely constrained in the low  $\tan\beta$  one; as for the CKM case it would be tested in the  $(m_0, m_{\tilde{g}}) \lesssim 900$  GeV region, provided that  $\tan\beta$  is high.

The conclusions (Table IX) are that with the planned improvements of the KEK facility the  $U_{e3}$  independent

TABLE IX. Reach in  $(m_0, m_{\tilde{g}})$  of the present and planned experiment from their  $\tau \rightarrow \mu\gamma$  sensitivity.

Experiment	PMNS		CKM	
	$t_\beta = 40$	$t_\beta = 10$	$t_\beta = 40$	$t_\beta = 10$
BABAR, Belle	1.2 TeV	No	No	No
SuperKEKB	2 TeV	0.9 TeV	No	No
Super flavor <sup>a</sup>	2.8 TeV	1.5 TeV	0.9 TeV	No

<sup>a</sup>Post-LHC era proposed/discussed experiment.



$\tau \rightarrow \mu \gamma$  process will allow us to test much of the PMNS scenario. A super flavor factory would much improve the situation, as it would be able to almost completely probe the PMNS case and to test the minimal mixing, high  $\tan\beta$  scenario up to soft masses of 600 GeV.

### C. Probing the PMNS case with $U_{e3} \approx 0$ at MEG

We have seen that, if a super flavor factory will be built, the  $\tau \rightarrow \mu \gamma$  process will be highly complementary to the  $\mu \rightarrow e \gamma$  one as a probe of SUSY-GUT scenarios, with the added bonus of being  $U_{e3}$  independent. As a super flavor factory is just a proposed experiment, whereas MEG will surely be operating, it is nevertheless interesting to ask what is the probing capability of such an experiment in the PMNS case, if  $U_{e3}$  happens to be vanishing small, or even 0.

In the case that  $U_{e3} = 0$ , Eq. (10) is no more a good approximation to the running of the off-diagonal LL entries, as we have to resort to the 2nd generation entries:

$$(\delta_{LL})_{\mu e} = -\frac{3}{8\pi^2} Y_c^2 U_{e2} U_{\mu 2} \ln \frac{M_X}{M_{R_2}}. \quad (29)$$

Here the off-diagonal contribution in slepton masses, now being proportional to the square of the charm Yukawa  $Y_c$  are much smaller, in fact even smaller than the CKM contribution by a factor

$$\frac{Y_c^2 U_{\mu 2} U_{e2} \ln(M_X/M_{R_2})}{Y_t^2 V_{td} V_{ts} \ln(M_X/M_{R_3})} \sim \mathcal{O}(10^{-2}). \quad (30)$$

The point is that the estimate (30) misses an important point. The PMNS case is the case where the  $R$  matrix is the identity; but we should keep in mind at what scale we should enforce this. Because the angle  $U_{e3}$  runs with the energy scale and  $U_{e3} \approx 0$  at the weak scale does not necessarily mean  $U_{e3} \approx 0$  at high scale. Even for hierarchical spectra, where the running effects are small, the induced RG effects in the soft spectrum could be large, leading to large enough  $\mu \rightarrow e \gamma$ . The running effect of the neutrino mixing angle can be estimated by using the neutrino RG [50,51] equations.

Moreover, as we have seen in Sec. III, in a SUSY-GUT framework we have also a sizable subleading contribution to the amplitude of the  $\mu \rightarrow e \gamma$  process coming from the  $(\delta_{RR})_{e\mu}$  insertion and from the double insertions  $(\delta_{RR})_{e\tau} \times (\delta_{LL})_{\tau\mu}$ ; the interplay between the RG enhancement of  $U_{e3}$  and the amplitudes coming from the subleading insertions has been thoroughly discussed in a recent publication [54].

The results for the PMNS mixing with  $U_{e3} = 0$  (defined at the weak scale) are shown in Fig. 11. We see that even for low  $\tan\beta$  the branching ratio is never lower than that of the CKM case, giving a proof that the CKM case is really a representative of a “minimal mixing” case. We see (Table X) that, given the present experimental LFV rates bounds, the  $U_{e3} = 0$  PMNS case is better constrained by the  $\tau \rightarrow \mu \gamma$  than by the  $\mu \rightarrow e \gamma$  process. MEG will be

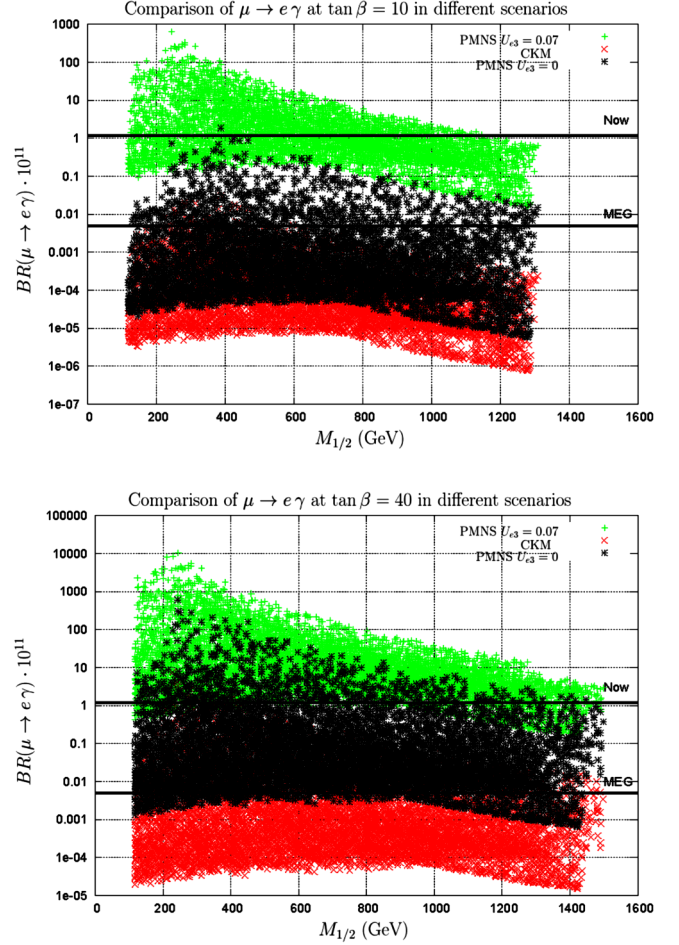


FIG. 11 (color online).  $BR(\mu \rightarrow e \gamma)$  as a probe of different SUSY-GUT scenarios. The plots are obtained by scanning the LHC accessible parameter space at fixed  $\tan\beta$ . The lines are the present (MEGA) and future (MEG) experimental sensitivities. We see that MEG will completely test the PMNS scenario for  $U_{e3}$  close to the CHOOZ bound and severely constrain it for  $U_{e3} = 0$ .

able to probe much of this scenario: for high values of  $\tan\beta$  almost all the LHC accessible parameter space will be probed, whereas if  $\tan\beta$  happens to be small it will be probed up to  $(m_0, m_{\tilde{g}}) \lesssim 1100$  GeV. We thus can state that, if  $\tan\beta$  is high the MEG experiment will probe the PMNS case better than the  $\tau\mu$  sector experiments, irrespectively of the value of  $U_{e3}$ , and with an accuracy comparable to that of the SuperKEKB machine if  $\tan\beta$  is small. On the

TABLE X. Reach in  $(m_0, m_{\tilde{g}})$  of the past and upcoming experiments from their  $\mu \rightarrow e \gamma$  sensitivity. LHC means that all the LHC testable parameter space will be probed.

Experiment	PMNS, $U_{e3} = 0$	
	$t_\beta = 40$	$t_\beta = 10$
MEGA	1.1 TeV	No
MEG	LHC	1.1 TeV

other hand, a super flavor factory would for sure supersede MEG.

#### D. The PRISM/PRIME experiment at J-PARC

Since the experimental signal is very well defined, the  $\mu \rightarrow e$  conversion in nuclei poses very good prospects as a probe of lepton flavor violating scenarios. In SUSY-GUT frameworks the main contribution to the amplitude comes from the penguin diagram that is also responsible for the FV  $\mu \rightarrow e\gamma$  amplitude. There is thus a strong correlation between these two processes, the  $\mu \rightarrow e$  conversion being suppressed by a factor  $\sim Z\alpha/\pi$  with respect to the flavor violating decay  $\mu \rightarrow e\gamma$ .

The present bounds on  $\mu \rightarrow e$  conversion come from the SINDRUM II experiment at PSI, that gave bounds on conversion rates in different nuclei. For instance, the bound for the conversion in titanium ( $4.3 \times 10^{-12}$ ) is almost as

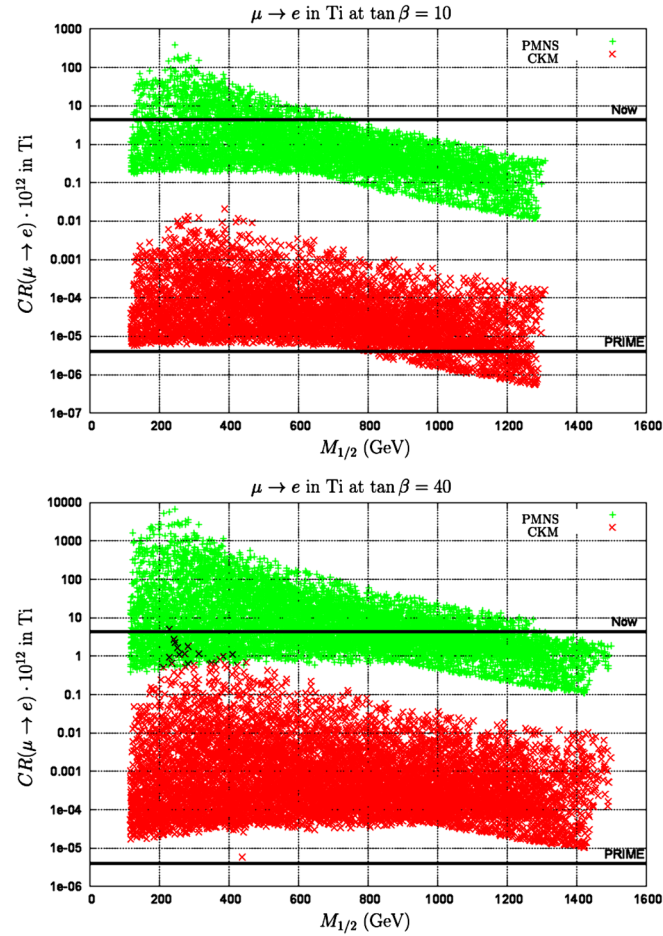


FIG. 12 (color online).  $\mu \rightarrow e$  in Ti as a probe of SUSY-GUT scenarios. The plots are obtained by scanning the LHC accessible parameter space. The horizontal lines are the present (SINDRUM II) bound and the planned (PRISM/PRIME) sensitivity to the process. We see that PRIME would be able to severely constrain the low  $\tan\beta$ , low mixing angles case and to completely test the other scenarios.

good as the current MEG bound on  $\mu \rightarrow e\gamma$  ( $1.1 \times 10^{-11}$ ) in constraining the SUSY-GUT parameter space, but it will be superseded by the future MEG sensitivity. To achieve a sensitivity to SUSY-GUTs scenarios that is comparable to the MEG experiment, a  $\mu \rightarrow e$  conversion experiment in titanium would need a sensitivity of  $\mathcal{O}(10^{-15})$ . This would require an high intensity muon source and an experimental apparatus that provides a very good resolution in the energy of the emitted electron, to discriminate with high accuracy the  $\mu \rightarrow e$  conversion versus the  $\mu$  decay in orbit. The J-PARC experiment PRISM/PRIME [17] addresses these issues by means of an innovative  $\mu$  source (phase rotated intense slow muons, PRISM), with an intensity of  $10^{11}$ – $10^{12}$   $\mu/s$ , and its  $\mu \rightarrow e$  conversion in the Ti dedicated experiment (PRIME: PRISM  $\mu - e$  conversion experiment); the planned sensitivity of the experiment is of  $4 \times 10^{-18}$ , with the possibility of improving it by upgrading the PRISM machine intensity to  $10^{14}$   $\mu/s$ .

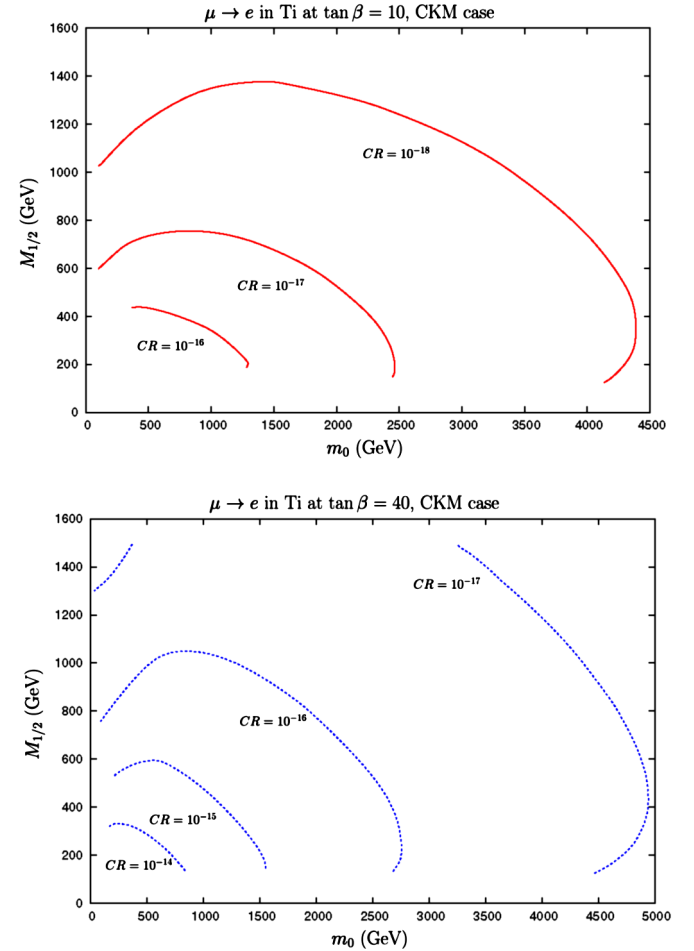


FIG. 13 (color online). Contour plots at  $A_0 = 0$  of the parameter space region within reach of different  $\mu \rightarrow e$  in Ti CR sensitivities in the CKM case for low and high  $\tan\beta$ . We see that the PRIME experiment will be able to test the CKM  $t_\beta = 10$  case for  $(m_0, m_g) \lesssim 2800$  GeV and the  $t_\beta = 40$  even beyond LHC reach.

TABLE XI. Reach in  $(m_0, m_{\tilde{g}})$  of the present and planned experiment from their  $\mu \rightarrow e$  conversion sensitivity. LHC means that all the LHC testable parameter space will be probed. All means that masses as high as  $(m_0, m_{\tilde{g}}) \lesssim 5$  TeV will be probed.

Experiment	PMNS		CKM	
	$t_\beta = 40$	$t_\beta = 10$	$t_\beta = 40$	$t_\beta = 10$
SINDRUM II	2 TeV	1.3 TeV	No	No
MECO <sup>a</sup>	All	All	2.6 TeV	1.3 TeV
PRISM/PRIME <sup>b</sup>	All	All	LHC	2.8 TeV

<sup>a</sup>MECO [55] was terminated by the NSF in 2004. The values are given as a reference comparison.

<sup>b</sup>Post-LHC era proposed experiment.

Although the experiment has not yet been approved, the construction of the PRISM machine has already begun and should be completed in five years [18]. It is thus timely to ask what will be the power of the post-LHC PRIME experiment to discriminate between the different SUSY-GUT scenarios in the case that the LHC finds evidence for SUSY. As can be seen from Figs. 12 and 13, the PRIME experiment would be able to really test our SUSY-GUT ansatz (Table XI): the high  $\tan\beta$  case would be tested in both the large and small mixing angles scenarios, even beyond the reach of the LHC. As for the low  $\tan\beta$  scenario, the PMNS case would be completely tested and much of the CKM case would be within reach: masses as high as  $(m_0, m_{\tilde{g}}) \lesssim 2800$  GeV could be probed.

As the PRIME experiment would be a post-LHC era experiment, its capability of testing and ruling out so many different SUSY-GUT scenarios is most interesting. It would be an ideal complement to the findings of the LHC in the case that it gets positive evidence for low energy supersymmetry.

## VI. LFV RATES AT SPS BENCHMARK POINTS

In this section we discuss the possibility of detecting supersymmetry at the SPS benchmark points [56] by means of LFV experiments. We concentrate on the SPS points defined for mSUGRA/MSSM framework. These take into consideration various constraints, including relic density requirements, in addition to what we have considered here. We note that some of these points will be ruled out in the light of new WMAP data if one requires a purely Bino dark matter. As of now, there is no corresponding definition of SPS points within SUSY-GUTs. In the present work, we consider the input values of the mSUGRA SPS points in our SO(10) model and study the impact of flavor violation for that spectra.<sup>7</sup> We note that, for all the points,

<sup>7</sup>In some points, we notice the need for modifying these numbers within a SUSY-GUT framework. For example, in **SPS 3**, the LSP and  $\tilde{\tau}_1$  are no longer degenerate, whereas **SPS 4** and **SPS 5** are already in conflict with experimental measurements.

the PMNS framework is ruled out by the present MEGA bound on  $\mu \rightarrow e\gamma$ . Furthermore, the PRISM/PRIME experiment would be able to test all the scenarios.

The “typical” mSUGRA scenario is represented by SPS points **1a** at low  $\tan\beta$  and **1b** at relatively high  $t_\beta$ :

$$\begin{aligned} \text{SPS 1a: } m_0 &= 100, & M_{1/2} &= 250, & A_0 &= -100, \\ t_\beta &= 10, & m_h &= 112, & m_{\tilde{t}} &= 375, \\ m_{\tilde{g}} &= 612, \\ \text{SPS 1b: } m_0 &= 200, & M_{1/2} &= 400, & A_0 &= 0, \\ t_\beta &= 30, & m_h &= 120, & m_{\tilde{t}} &= 636, \\ m_{\tilde{g}} &= 980, \end{aligned}$$

where the values are given in GeV and we have also given the values of three low energy observable ( $m_h, m_{\tilde{t}}, m_{\tilde{g}}$ ) as obtained from the routine.<sup>8</sup> We see that point **1a** is already ruled out by the bound on the lightest Higgs mass. We are including it as it lays at the boundary of the experimentally ruled out region, so that a further improved version of our code could give the small correction that is needed to satisfy the present bound. The CKM scenario and the PMNS case at  $U_{e3} = 0$  are unscathed by the present bounds. We see (Table XII) that the PMNS  $U_{e3} = 0$  scenario will be within reach of both MEG and SuperKEKB, for the two benchmark points, while the CKM case could escape MEG detection, as the predicted BR for both points are at the boundary of the planned sensitivity.

The **SPS 2** benchmark point lies in the so-called “focus point” region<sup>9</sup> [57]

$$\begin{aligned} \text{SPS 2: } m_0 &= 1450, & M_{1/2} &= 300, & A_0 &= 0, \\ t_\beta &= 10, & m_h &= 124, & m_{\tilde{t}} &= 940, \\ m_{\tilde{g}} &= 735, \end{aligned}$$

where all the masses are given in GeV. From Table XII we see that the PMNS  $U_{e3} = 0$  scenario will be within reach of the proposed super flavor factory; as for the other processes they will escape detection.

The mSUGRA/MSSM “coannihilation region” [58] has its representative in point **SPS 3**. In this region a rapid coannihilation between the neutralino LSP and the stau next-to-lightest supersymmetric particle will give rise to a sufficiently low relic abundance: for this reason, we are also giving  $m_{\tilde{\tau}}$  and  $m_{\text{LSP}}$  as low energy observables (all masses in GeV):

<sup>8</sup>All the SPS points have  $\mu > 0$ .

<sup>9</sup>Here “focus point” is just used as a conventional name, but does not really correspond to the dark matter region as in MSSM/mSUGRA.

TABLE XII. LFV rates for points **SPS 1a** and **SPS 1b** in the CKM case and in the  $U_{e3} = 0$  PMNS case. The processes that are within reach of the future experiments (MEG, SuperKEKB) have been highlighted in boldface. Those within reach of the post-LHC era planned/discussed experiments (PRISM/PRIME, super flavor factory) are highlighted in italics.

Process	SPS 1a		SPS 1b		SPS 2		SPS 3		Future sensitivity
	CKM	$U_{e3} = 0$	CKM	$U_{e3} = 0$	CKM	$U_{e3} = 0$	CKM	$U_{e3} = 0$	
$\text{BR}(\mu \rightarrow e\gamma)$	$3.2 \times 10^{-14}$	$3.8 \times 10^{-13}$	$4.0 \times 10^{-13}$	$1.2 \times 10^{-12}$	$1.3 \times 10^{-15}$	$8.6 \times 10^{-15}$	$1.4 \times 10^{-15}$	$1.2 \times 10^{-14}$	$\mathcal{O}(10^{-14})$
$\text{BR}(\mu \rightarrow eee)$	$2.3 \times 10^{-16}$	$2.7 \times 10^{-15}$	$2.9 \times 10^{-16}$	$8.6 \times 10^{-15}$	$9.4 \times 10^{-18}$	$6.2 \times 10^{-17}$	$1.0 \times 10^{-17}$	$8.9 \times 10^{-17}$	$\mathcal{O}(10^{-14})$
$\text{CR}(\mu \rightarrow e \text{ in Ti})$	$2.0 \times 10^{-15}$	$2.4 \times 10^{-14}$	$2.6 \times 10^{-15}$	$7.6 \times 10^{-14}$	$1.0 \times 10^{-16}$	$6.7 \times 10^{-16}$	$1.0 \times 10^{-16}$	$8.4 \times 10^{-16}$	$\mathcal{O}(10^{-18})$
$\text{BR}(\tau \rightarrow e\gamma)$	$2.3 \times 10^{-12}$	$6.0 \times 10^{-13}$	$3.5 \times 10^{-12}$	$1.7 \times 10^{-12}$	$1.4 \times 10^{-13}$	$4.8 \times 10^{-15}$	$1.2 \times 10^{-13}$	$4.1 \times 10^{-14}$	$\mathcal{O}(10^{-8})$
$\text{BR}(\tau \rightarrow eee)$	$2.7 \times 10^{-14}$	$7.1 \times 10^{-15}$	$4.2 \times 10^{-14}$	$2.0 \times 10^{-14}$	$1.7 \times 10^{-15}$	$5.7 \times 10^{-17}$	$1.5 \times 10^{-15}$	$4.9 \times 10^{-16}$	$\mathcal{O}(10^{-8})$
$\text{BR}(\tau \rightarrow \mu\gamma)$	$5.0 \times 10^{-11}$	$1.1 \times 10^{-8}$	$7.3 \times 10^{-11}$	$1.3 \times 10^{-8}$	$2.9 \times 10^{-12}$	$7.8 \times 10^{-10}$	$2.7 \times 10^{-12}$	$6.0 \times 10^{-10}$	$\mathcal{O}(10^{-9})$
$\text{BR}(\tau \rightarrow \mu\mu\mu)$	$1.6 \times 10^{-13}$	$3.4 \times 10^{-11}$	$2.2 \times 10^{-13}$	$3.9 \times 10^{-11}$	$8.9 \times 10^{-15}$	$2.4 \times 10^{-12}$	$8.7 \times 10^{-15}$	$1.9 \times 10^{-12}$	$\mathcal{O}(10^{-8})$

$$\begin{aligned} \text{SPS 3: } m_0 &= 90, & M_{1/2} &= 400, & A_0 &= 0, \\ t_\beta &= 10, & m_h &= 119, & m_{\tilde{t}} &= 631, \\ m_{\tilde{g}} &= 980, & m_{\tilde{\tau}} &= 270, & m_{\text{LSP}} &= 185. \end{aligned}$$

$$\begin{aligned} \text{SPS 5: } m_0 &= 150, & M_{1/2} &= 300, & A_0 &= -1000, \\ t_\beta &= 5, & m_h &= 102, & m_{\tilde{t}} &= 275, \\ m_{\tilde{g}} &= 735, \end{aligned}$$

This point will be within reach of the proposed super flavor factory (Table XII) in the PMNS  $U_{e3} = 0$  scenario.

The mSUGRA scenario at high  $\tan\beta$  has it benchmark in point **SPS 4**

$$\begin{aligned} \text{SPS 4: } m_0 &= 400, & M_{1/2} &= 300, \\ A_0 &= 0, & t_\beta &= 50, \end{aligned}$$

where all masses are in GeV. This point is ruled out, because it gives a nonviable vacuum.

The point **SPS 5**, that corresponds to a scenario of relatively light stop, is ruled out because it predicts a too light Higgs boson:

TABLE XIII. Capability of past, present, and future experiment to detect LFV at the SPS benchmark points. When two experiments are able to detect the same process, only the less sensitive experiment is displayed.

Point	CKM	PMNS	PMNS, $U_{e3} = 0$
<b>SPS 1a</b>	MEG (maybe) PRIME <sup>a</sup>	MEGA SINDRUM II SuperKEKB	MEG PRIME <sup>a</sup> SuperKEKB
<b>SPS 1b</b>	MEG (maybe) PRIME <sup>a</sup>	MEGA SINDRUM II SuperKEKB	MEG PRIME <sup>a</sup> SuperKEKB
<b>SPS 2</b>	PRIME <sup>a</sup>	MEGA SINDRUM II Super flavor <sup>a</sup>	PRIME <sup>a</sup> Super flavor <sup>a</sup>
<b>SPS 3</b>	PRIME <sup>a</sup>	MEG SINDRUM II Super flavor <sup>a</sup>	PRIME <sup>a</sup> Super flavor <sup>a</sup>

<sup>a</sup>Post-LHC era, planned/discussed experiment.

where the dimensional parameters are given in GeV.

As a conclusion (Table XIII), we can state that the only scenarios that will for sure escape detection are the CKM focus point **SPS 2** and CKM coannihilation region **SPS 3** cases. The **SPS 1a** and **SPS 1b** CKM scenario are at the boundary of MEG sensitivity so that probing these scenario, though hard, is nevertheless a possibility. The PRISM/PRIME experiment would much improve the situation, as it would be able to test all the scenarios; these results would be complemented by that from a super flavor factory.

## VII. CONCLUSIONS

In this paper we addressed the capability of past (MEGA, SINDRUM II), present (*BABAR*, Belle), upcoming (MEG, SuperKEKB), and proposed (PRISM/PRIME, super flavor factory) lepton flavor violation experiments to probe SUSY-GUT scenarios. We have found that these experiments have strong capabilities to detect SUSY induced LFV, in some cases even outreaching the LHC.

The more interesting feature of such experiments is the possibility to give hints about the viable SUSY-GUT scenarios, by constraining the neutrino Yukawa sector. The reach of such experiments as probes of different scenarios are summarized in Table XIV and displayed in Fig. 14, where we compare the scope of  $\tau \rightarrow \mu\gamma$  and  $\mu \rightarrow e\gamma$  experiments.

Suppose that the LHC does find signals of low-energy supersymmetry, then grand unification becomes a very appealing scenario, because of the successful unification of gauge couplings driven by the SUSY partners. Among SUSY-GUT models, an SO(10) framework is much favored as it is the “minimal” GUT to host all the fermions in a single representation and it accounts for the smallness of the observed neutrino masses by naturally including the



TABLE XIV. Reach in  $(m_0, m_{\tilde{g}})$  of the past, present, and upcoming experiments from their LFV sensitivity. LHC means that all the LHC testable parameter space will be probed; all means that soft masses up to  $(m_0, m_{\tilde{g}}) \lesssim 5$  TeV will be probed.

Experiment	PMNS		CKM	
	$t_\beta = 40$	$t_\beta = 10$	$t_\beta = 40$	$t_\beta = 10$
$\mu e$ sector				
MEGA	LHC	2 TeV	No	No
	1.1 TeV <sup>a</sup>	No <sup>a</sup>		
MEG	All	All	1.3 TeV	No
	LHC <sup>a</sup>	1.1 TeV <sup>a</sup>		
PRISM/PRIME <sup>b</sup>	All	All	All	2.8 TeV
		LHC <sup>a</sup>		
$\tau\mu$ sector				
BABAR, Belle	1.2 TeV	No	No	No
SuperKEKB	2 TeV	0.9 TeV	No	No
Super flavor <sup>b</sup>	2.8 TeV	1.5 TeV	0.9 TeV	No

<sup>a</sup> $U_{e3} = 0$ .

<sup>b</sup>Post-LHC era, planned/discussed experiment.

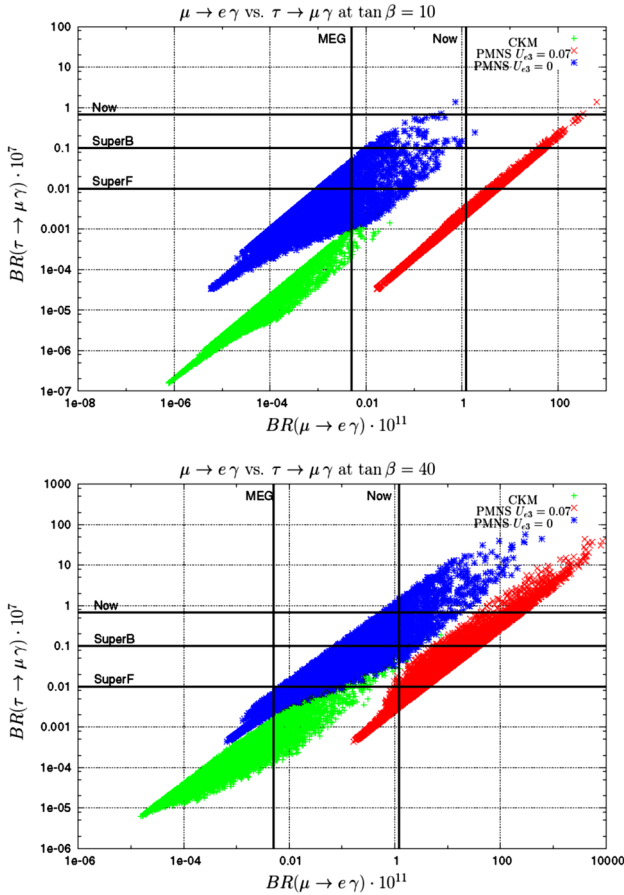


FIG. 14 (color online). Comparison of  $\mu \rightarrow e\gamma$  and  $\tau \rightarrow \mu\gamma$  as a probe of SUSY-GUTs scenarios. The plots are done by scanning the LHC accessible parameter space at fixed  $\tan\beta$ . The lines are the present bounds and future sensitivities. Let us note that the interplay between MEG and a super flavor factory will leave unscathed only the low  $\tan\beta$  CKM case.

seesaw mechanism. Moreover, in recent years, SO(10) SUSY models have spurred much interest as in this framework it is possible to build realistic fermion mass model and to account for the proton lifetime bounds. In this paper we have addressed the issue by a generic benchmark analysis, within the ansatz that there is no fine-tuning in the neutrino Yukawa sector.

From our analysis we can state that lepton flavor violation experiments should be able to tell us much about the structure of such a SUSY-GUT scenario. If they detect LFV processes, by their rate and exploiting the interplay between different experiments, we would be able to get hints of the structure of the unknown neutrinos' Yukawas. In this sense, the capability of a super flavor factory to discriminate between the minimal mixing case and the  $U_{e3} = 0$  PMNS case is a most interesting feature.

On the contrary, in the case that both MEG and a future super flavor factory happen not to see any LFV process, only two possibilities should be left: (i) the minimal mixing, low  $\tan\beta$  scenario; (ii) mSUGRA-SO(10) seesaw without fine-tuned  $Y_\nu$  couplings is not a viable framework of physics beyond the standard model. Moreover, if the planned, high sensitivity PRISM/PRIME conversion experiment, able to test even the minimal mixing low  $\tan\beta$  region, does not manage to find LFV evidences, the latter conclusion should be the most sensible one and there should be no room left for the no fine-tuning framework we studied in this paper. Actually, one should remark that LFV experiments will be able to falsify some of the SUSY-GUT scenarios even in regions of the SUGRA parameter space that are beyond the reach of LHC experiments. In this sense, the power of LFV experiments of testing/discriminating among different SUSY-GUTs models results very interesting and highly complementary to the direct searches at the LHC.

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**APPENDIX: NOTATION AND RG EQUATIONS****1. The model**

The model consists in a supersymmetric SO(10) framework with the following breaking pattern:

$$\text{SO}(10) \xrightarrow{M_X} \text{SU}(5)_{\text{RN}} \xrightarrow{M_{\text{GUT}}} \text{MSSM}_{\text{RN}}, \quad (\text{A1})$$

where SO(10) is broken at the scale  $M_X = 5 \times 10^{17}$  GeV where we equate it to the SUSY breaking mediation scale and the GUT scale is  $M_{\text{GUT}} = 2 \times 10^{16}$  GeV. Below the scale  $M_X$ , the model is given by the following SU(5)<sub>RN</sub> superpotential:

$$W_{\text{SU}(5)_{\text{RN}}} = Y_{10ij} 10_i 10_j 5_H + Y_{5ij} 10_i \bar{5}_j \bar{5}_H + Y_{1ij} \bar{5}_i 1_j 5_H \\ + M_{ij} 1_i 1_j + \mu \bar{5}_H 5_H, \quad (\text{A2})$$

while the corresponding soft SUSY breaking potential is

$$V_{\text{SU}(5)_{\text{RN}}} = (A_{10ij} 10_i 10_j 5_H + A_{5ij} 10_i \bar{5}_j \bar{5}_H + A_{1ij} \bar{5}_i 1_j 5_H \\ + \tilde{M}_{ij} 1_i 1_j + B \mu 5_H \bar{5}_H + \text{H.c.}) + m_{\tilde{5}_{ij}}^2 \bar{5}_i^* \bar{5}_j \\ + m_{10ij}^2 10_i^* 10_j + m_{1ij}^2 1_i^* 1_j + m_H^2 5_H^* 5_H \\ + m_{\bar{H}}^2 \bar{5}_H^* \bar{5}_H + M_5 \tilde{2} \tilde{2}. \quad (\text{A3})$$

After reaching the GUT scale, the theory is broken to the MSSM (plus right-handed neutrinos) Lagrangian:

$$W_{\text{MSSM}_{\text{RN}}} = Y_{ij}^u Q_i U_j^c H_2 + Y_{ij}^d Q_i D_j^c H_1 + Y_{ij}^e L_i E_j^c H_1 \\ + Y_{ij}^\nu L_i \nu_j^c H_2 + M_{ij} \nu_i^c \nu_j^c + \mu H_1 H_2, \quad (\text{A4})$$

$$V_{\text{MSSM}_{\text{RN}}} = (A_{ij}^u \tilde{Q}_i \tilde{U}_j^c H_2 + A_{ij}^d \tilde{Q}_i \tilde{D}_j^c H_1 + A_{ij}^\nu \tilde{L}_i \tilde{N}_j^c H_2 \\ + \tilde{M}_{ij} \tilde{N}_i \tilde{N}_j + B \mu H_1 H_2 + \text{H.c.}) + m_{\tilde{Q}_{ij}}^2 \tilde{Q}_i^* \tilde{Q}_j \\ + m_{\tilde{U}_{ij}}^2 \tilde{U}_i^* \tilde{U}_j + m_{\tilde{D}_{ij}}^2 \tilde{D}_i^* \tilde{D}_j + m_{\tilde{L}_{ij}}^2 \tilde{L}_i^* \tilde{L}_j \\ + m_{\tilde{E}_{ij}}^2 \tilde{E}_i^* \tilde{E}_j + m_{\tilde{N}_{ij}}^2 \tilde{N}_i^* \tilde{N}_j + m_H^2 H_1^* H_1 \\ + m_{\bar{H}}^2 H_2^* H_2 + M_1 \tilde{B} \tilde{B} + M_2 \tilde{W} \tilde{W} + M_3 \tilde{G} \tilde{G}. \quad (\text{A5})$$

The matching between SU(5) parameters and MSSM ones at  $M_{\text{GUT}}$  is given by

$$Y_{ij}^\nu = Y_{ij}^1 \quad Y_{ij}^u = 4Y_{ij}^{10} \\ Y_{ij}^d = \frac{1}{\sqrt{2}} Y_{ij}^5 \quad Y_{ij}^e = \frac{1}{\sqrt{2}} Y_{ji}^5. \quad (\text{A6})$$

The matching of the soft A-matrices is the same as the Yukawas, whereas for the soft mass matrices is

$$m_{\tilde{U}}^2 = m_{10}^2; \quad m_{\tilde{Q}}^2 = m_{10}^2; \quad m_{\tilde{D}}^2 = m_{\tilde{5}}^2; \\ m_{\tilde{L}}^2 = m_{\tilde{5}}^2; \quad m_{\tilde{E}}^2 = m_{10}^2; \quad m_{\tilde{N}}^2 = m_{\tilde{1}}^2 \quad (\text{A7})$$

and

$$M_1 = M_2 = M_3 = M_5. \quad (\text{A8})$$

**2. SU(5)<sub>RN</sub> RGE**

Conventions:

$$\tilde{Y} = \frac{Y}{4\pi}; \quad \tilde{A} = \frac{A}{4\pi}; \\ \tilde{\alpha} = \frac{\alpha}{4\pi} = \frac{g^2}{(4\pi)^2}; \quad t = \ln \frac{M_X^2}{Q^2}.$$

Yukawas:

$$\frac{d}{dt} \tilde{Y}_{10ij} = \frac{48}{5} \tilde{\alpha}_5 \tilde{Y}_{10ij} - 24 \text{Tr}(\tilde{Y}_{10}^\dagger \tilde{Y}_{10}) \tilde{Y}_{10ij} \\ - \frac{1}{2} \text{Tr}(\tilde{Y}_1^\dagger \tilde{Y}_1) \tilde{Y}_{10ij} - 48(\tilde{Y}_{10} \tilde{Y}_{10}^\dagger \tilde{Y}_{10})_{ij} \\ - \frac{1}{2} [(\tilde{Y}_5 \tilde{Y}_5^\dagger \tilde{Y}_{10})_{ij} + (\tilde{Y}_{10} \tilde{Y}_5^* \tilde{Y}_5^T)_{ij}], \quad (\text{A9})$$

$$\frac{d}{dt} \tilde{Y}_{5ij} = \frac{42}{5} \tilde{\alpha}_5 \tilde{Y}_{5ij} - \text{Tr}(\tilde{Y}_5^\dagger \tilde{Y}_5) \tilde{Y}_{5ij} - \frac{3}{2} (\tilde{Y}_5 \tilde{Y}_5^\dagger \tilde{Y}_5)_{ij} \\ - 24(\tilde{Y}_{10} \tilde{Y}_{10}^\dagger \tilde{Y}_5)_{ij} - \frac{1}{2} (\tilde{Y}_5 \tilde{Y}_1^* \tilde{Y}_1^T)_{ij}, \quad (\text{A10})$$

$$\frac{d}{dt} \tilde{Y}_{1ij} = \frac{24}{5} \tilde{\alpha}_5 \tilde{Y}_{1ij} - \frac{1}{2} \text{Tr}(\tilde{Y}_1^\dagger \tilde{Y}_1) \tilde{Y}_{1ij} \\ - 24 \text{Tr}(\tilde{Y}_{10}^\dagger \tilde{Y}_{10}) \tilde{Y}_{1ij} - 3(\tilde{Y}_1 \tilde{Y}_1^\dagger \tilde{Y}_1)_{ij} \\ - (\tilde{Y}_5^T \tilde{Y}_5^* \tilde{Y}_1)_{ij}. \quad (\text{A11})$$

Majorana mass:

$$\frac{d}{dt} M_{ij} = -\frac{5}{2} [(M \tilde{Y}_1^\dagger \tilde{Y}_1)_{ij} + (M \tilde{Y}_1^T \tilde{Y}_1^*)_{ij}]. \quad (\text{A12})$$

Soft masses:

$$\frac{d}{dt} (m_{\tilde{5}}^2)_{ij} = \frac{48}{5} \tilde{\alpha}_5 M_5^2 \delta_{ij} - [(m_{\tilde{5}}^2 \tilde{Y}_5^\dagger \tilde{Y}_5)_{ij} + (\tilde{Y}_5^\dagger \tilde{Y}_5 m_{\tilde{5}}^2)_{ij}] \\ - \frac{1}{2} [(m_{\tilde{5}}^2 \tilde{Y}_1^* \tilde{Y}_1^T)_{ij} + (\tilde{Y}_1^* \tilde{Y}_1^T m_{\tilde{5}}^2)_{ij}] \\ - 2[(\tilde{Y}_5^\dagger \tilde{Y}_{10}^{2T} \tilde{Y}_5)_{ij} + (\tilde{Y}_5^\dagger \tilde{Y}_5)_{ij} m_H^2 + (\tilde{A}_5^\dagger \tilde{A}_5)_{ij}] \\ - [(\tilde{Y}_1^* m_{\tilde{1}}^{2T} \tilde{Y}_1^T)_{ij} + (\tilde{Y}_1^* \tilde{Y}_1^T)_{ij} m_H^2 + (\tilde{A}_1^* \tilde{A}_1^T)_{ij}]. \quad (\text{A13})$$

$$\begin{aligned}
\frac{d}{dt}(m_{10}^2)_{ij} = & \frac{72}{5} \tilde{\alpha}_5 M_5^2 \delta_{ij} - 24[(m_{10}^2 \tilde{Y}_{10}^* \tilde{Y}_{10}^T)_{ij} \\
& + (\tilde{Y}_{10}^* \tilde{Y}_{10}^T m_{10}^2)_{ij}] - \frac{1}{2}[(m_{10}^2 \tilde{Y}_5^* \tilde{Y}_5^T)_{ij} \\
& + (\tilde{Y}_5^* \tilde{Y}_5^T m_{10}^2)_{ij}] - 48[(\tilde{Y}_{10}^* m_{10}^{2T} \tilde{Y}_{10}^T)_{ij} \\
& + (\tilde{Y}_{10}^* \tilde{Y}_{10}^T)_{ij} m_H^2 + (\tilde{A}_{10}^* \tilde{A}_{10}^T)_{ij}] \\
& - [(\tilde{Y}_5^* m_5^{2T} \tilde{Y}_5^T)_{ij} + (\tilde{Y}_5^* \tilde{Y}_5^T)_{ij} m_H^2 + (\tilde{A}_5^* \tilde{A}_5^T)_{ij}],
\end{aligned} \tag{A14}$$

$$\begin{aligned}
\frac{d}{dt}(m_1^2)_{ij} = & -\frac{5}{2}[(m_1^2 \tilde{Y}_1^\dagger \tilde{Y}_1)_{ij} + (\tilde{Y}_1^\dagger \tilde{Y}_1 m_1^2)_{ij}] \\
& - 5[(\tilde{Y}_1^\dagger m_5^2 \tilde{Y}_1)_{ij} + (\tilde{Y}_1^\dagger \tilde{Y}_1)_{ij} m_H^2 + (\tilde{A}_1^\dagger \tilde{A}_1)_{ij}],
\end{aligned} \tag{A15}$$

$$\begin{aligned}
\frac{d}{dt}(m_H^2) = & \frac{48}{5} \tilde{\alpha}_5 M_5^2 \delta_{ij} - 48[\text{Tr}(\tilde{Y}_{10}^\dagger \tilde{Y}_{10}) m_H^2 \\
& + 2 \text{Tr}(\tilde{Y}_{10} m_{10}^2 \tilde{Y}_{10}^\dagger) + \text{Tr}(\tilde{A}_{10}^\dagger \tilde{A}_{10})] \\
& - [\text{Tr}(\tilde{Y}_1^\dagger \tilde{Y}_1) m_H^2 + \text{Tr}(\tilde{Y}_1^\dagger m_5^2 \tilde{Y}_1) \\
& + \text{Tr}(\tilde{Y}_1 m_1^2 \tilde{Y}_1^\dagger) + \text{Tr}(\tilde{A}_1^\dagger \tilde{A}_1)],
\end{aligned} \tag{A16}$$

$$\begin{aligned}
\frac{d}{dt}(m_H^2) = & \frac{48}{5} \tilde{\alpha}_5 M_5^2 \delta_{ij} - 2[\text{Tr}(\tilde{Y}_5^\dagger \tilde{Y}_5) m_H^2 + \text{Tr}(\tilde{Y}_5 m_5^2 \tilde{Y}_5^\dagger) \\
& + \text{Tr}(\tilde{Y}_5^\dagger m_{10}^{2T} \tilde{Y}_5) + \text{Tr}(\tilde{A}_5^\dagger \tilde{A}_5)].
\end{aligned} \tag{A17}$$

A-terms:

$$\begin{aligned}
\frac{d}{dt} \tilde{A}_{10ij} = & \frac{48}{5} \tilde{\alpha}_5 (\tilde{A}_{10ij} - 2M_5 \tilde{Y}_{10ij}) - 24 \text{Tr}(\tilde{Y}_{10}^\dagger \tilde{Y}_{10}) \tilde{A}_{10ij} \\
& - \frac{1}{2} \text{Tr}(\tilde{Y}_1^\dagger \tilde{Y}_1) \tilde{A}_{10ij} - 48 \text{Tr}(\tilde{Y}_{10}^\dagger \tilde{A}_{10}) \tilde{Y}_{10ij} \\
& - \text{Tr}(\tilde{Y}_1^\dagger \tilde{A}_1) \tilde{Y}_{10ij} - 72[(\tilde{Y}_{10} \tilde{Y}_{10}^\dagger \tilde{A}_{10})_{ij} \\
& + (\tilde{A}_{10} \tilde{Y}_{10}^\dagger \tilde{Y}_{10})_{ij}] - \frac{1}{2}[(\tilde{Y}_5 \tilde{Y}_5^\dagger \tilde{A}_{10})_{ij} \\
& + (\tilde{A}_{10} \tilde{Y}_5^\dagger \tilde{Y}_5)_{ij}] - (\tilde{Y}_{10} \tilde{Y}_5^* \tilde{A}_5^T)_{ij} - (\tilde{A}_5 \tilde{Y}_5^\dagger \tilde{Y}_{10})_{ij},
\end{aligned} \tag{A18}$$

$$\begin{aligned}
\frac{d}{dt} \tilde{A}_{5ij} = & \frac{42}{5} \tilde{\alpha}_5 (\tilde{A}_{5ij} - 2M_5 \tilde{Y}_{5ij}) - \text{Tr}(\tilde{Y}_5^\dagger \tilde{Y}_5) \tilde{A}_{5ij} \\
& - 2 \text{Tr}(\tilde{Y}_5^\dagger \tilde{A}_5) \tilde{Y}_{5ij} - \frac{5}{2} (\tilde{Y}_5 \tilde{Y}_5^\dagger \tilde{A}_5)_{ij} \\
& - 2(\tilde{A}_5 \tilde{Y}_5^\dagger \tilde{Y}_5)_{ij} - 24(\tilde{Y}_{10} \tilde{Y}_{10}^\dagger \tilde{A}_5)_{ij} \\
& - \frac{1}{2}(\tilde{A}_5 \tilde{Y}_1^* \tilde{Y}_1^T)_{ij} - (\tilde{Y}_5 \tilde{Y}_1^* \tilde{A}_1^T)_{ij} - 48(\tilde{A}_{10} \tilde{Y}_{10}^* \tilde{Y}_5)_{ij},
\end{aligned} \tag{A19}$$

$$\begin{aligned}
\frac{d}{dt} \tilde{A}_{1ij} = & \frac{24}{5} \tilde{\alpha}_5 (\tilde{A}_{1ij} - 2M_5 \tilde{Y}_{1ij}) - \frac{1}{2} \text{Tr}(\tilde{Y}_1^\dagger \tilde{Y}_1) \tilde{A}_{1ij} \\
& - 24 \text{Tr}(\tilde{Y}_{10}^\dagger \tilde{Y}_{10}) \tilde{A}_{1ij} - 48 \text{Tr}(\tilde{Y}_{10}^\dagger \tilde{A}_{10}) \tilde{Y}_{1ij} \\
& - \text{Tr}(\tilde{Y}_1^\dagger \tilde{A}_1) \tilde{Y}_{1ij} - \frac{11}{2} (\tilde{Y}_1 \tilde{Y}_1^\dagger \tilde{A}_1)_{ij} \\
& - \frac{7}{2} (\tilde{A}_1 \tilde{Y}_1^\dagger \tilde{Y}_1)_{ij} - 2(\tilde{A}_5^T \tilde{Y}_5^* \tilde{Y}_1)_{ij} - (\tilde{Y}_5^T \tilde{Y}_5^* \tilde{A}_1)_{ij}.
\end{aligned} \tag{A20}$$

$\mu$  terms:

$$\begin{aligned}
\frac{d}{dt} \mu^2 = & 2 \left[ \frac{24}{5} \tilde{\alpha}_5 - 12 \text{Tr}(\tilde{Y}_{10} \tilde{Y}_{10}^\dagger) - \frac{1}{2} \text{Tr}(\tilde{Y}_1 \tilde{Y}_1^\dagger) \right. \\
& \left. - \text{Tr}(\tilde{Y}_5 \tilde{Y}_5^\dagger) \right] \mu^2,
\end{aligned} \tag{A21}$$

$$\begin{aligned}
\frac{d}{dt} B\mu = & - \left[ \frac{48}{5} \tilde{\alpha}_5 M_5 + 12 \text{Tr}(\tilde{A}_{10} \tilde{Y}_{10}^\dagger) \right. \\
& + \frac{1}{2} \text{Tr}(\tilde{A}_1 \tilde{Y}_1^\dagger) + \text{Tr}(\tilde{A}_5 \tilde{Y}_5^\dagger) \left. \right] \mu \\
& + \left[ \frac{24}{5} \tilde{\alpha}_5 - 12 \text{Tr}(\tilde{Y}_{10} \tilde{Y}_{10}^\dagger) - \frac{1}{2} \text{Tr}(\tilde{Y}_1 \tilde{Y}_1^\dagger) \right. \\
& \left. - \text{Tr}(\tilde{Y}_5 \tilde{Y}_5^\dagger) \right] B\mu.
\end{aligned} \tag{A22}$$

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