

## VIRTUAL EFFECTS OF HIGGS PARTICLES

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The possibility of observing Higgs particles through virtual effects is considered in detail for a general gauge theory. The effect of charged Higgs particles on low-energy weak interaction processes, like muon decay, tau decay, nuclear beta decay, pion decay, and some higher-order processes is analyzed. The effect of flavor-changing neutral Higgs particles on rare decay modes of the muon and kaon,  $\mu e$  conversion,  $K^0-\bar{K}^0$  and  $D^0-\bar{D}^0$  mixing is also studied. We discuss constraints on possible extensions of the Weinberg–Salam model and experiments sensitive to their Higgs particles. In particular, we analyze the neutral Higgs which couple to fermions in the minimal  $SU(2)_L \times SU(2)_R \times U(1)$  model and find that they probably have mass greater than 100 GeV.

### 1. Introduction

Higgs particles are a crucial ingredient of gauge theories of the electro-weak interaction. In these theories, some of the scalar particles become the longitudinal part of massive gauge bosons by the Higgs mechanism, while the other scalar particles, called Higgs particles, will manifest themselves as real particles. This is a general characteristic of this type of spontaneously broken gauge theory. There is an overwhelming amount of evidence for the general idea that gauge theories describe the weak and electromagnetic interaction. Therefore, a complete study of the phenomenology of these particles and detection is of fundamental importance in verifying the validity of gauge theories.

The Weinberg–Salam model [1], which we refer to as the standard model, has proven to be remarkably successful in describing all of the known weak interaction phenomenology. In this model, a single neutral Higgs is left over after symmetry breaking. The phenomenology of the Higgs in this model can be found in the literature [2]. The experimental search for the Higgs particle in the standard model is very difficult since it is electrically neutral, conserves flavor and parity, and it couples very weakly to low-mass fermions like the electron and proton. In summary, its properties do not seem to lead to easily detectable signatures in low-energy processes. However, if the standard model is not the complete theory of the electroweak interaction, there should exist additional Higgs particles which do not have to conserve flavor, might have electric charge, etc. These additional Higgs might

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have some detectable signatures in low-energy processes, which, if seen, would give evidence for existence of Higgs particles and further insight into the structure of the electroweak interaction.

There are many theoretical reasons for extending the standard model. Specifically, some of them are:

- (i) grand unification of the strong, weak, and electromagnetic interactions;
- (ii) ability to calculate mixing angles in the theory;
- (iii)  $CP$  violation;
- (iv) ability to calculate fermion masses in the theory.

A general feature of extensions of the standard model is that more Higgs particles are present and they have a much richer phenomenology than that of the Higgs particle in the standard model.

An often considered extension of the standard model is the  $SU(2)_L \times U(1)$  model with more than a single  $SU(2)_L$  doublet of scalar particles. One of the reasons for this extension is to provide an additional source for the  $CP$  violation besides the usual one in the KM matrix [3]. The motivation for this additional  $CP$  violation is that in the simplest  $SU(5)$  model which unifies the strong and electroweak interactions, the  $CP$  violation is not large enough to explain the observed baryon asymmetry in the universe [4, 5]. One simple way of producing the needed  $CP$  violation is to have a more complicated Higgs structure which will give more than one doublet when the  $SU(5)$  model is reduced to the  $SU(2)_L \times U(1)$  model [6]. In particular, this would imply that charged Higgs particles exist. Furthermore, in models of  $CP$  violation Branco [7] has shown that the requirement of spontaneous  $CP$  breaking and natural flavor conservation lead to a class of theories where  $CP$  non-conservation can occur only through Higgs particles.

Extensions of the standard model have also been considered for calculating the Cabibbo and other mixing angles in the weak interaction. To do this, additional Higgs fields are added and continuous [8] or discrete [9] horizontal symmetries are imposed. Models of this type may manifest themselves by the presence of flavor-changing neutral Higgs [8, 10], which have distinctive signatures in rare decay modes of the muon and kaon and  $D^0$ - $\bar{D}^0$  mixing.

It has been suggested that one should discard elementary Higgs fields and seek a dynamical mechanism for the symmetry breaking in order to avoid the gauge hierarchy problem in the grand unification of the strong and electroweak interactions [11]. In such theories, there must be at least two levels of symmetry breaking at vastly different energy scales. To obtain such a symmetry breaking pattern, one is led to introduce several sets of scalar fields with astutely contrived couplings. Hence, it is felt that unified gauge theories of this type are not natural. The basic idea in dynamically broken theories is to replace elementary scalar fields with composite fields which are Lorentz-invariant spinless bound states of fundamental fermions. It is found that such theories require new, strong interactions at the TeV level, “hyper-color” and “sideways” gauge interactions [12, 13]. Even though there are no

fundamental scalar particles in these theories, one finds, in general, composite Higgs, whose experimental characteristics have been discussed by Bég et al. [14], with the result that composite Higgs are very similar in many ways to elementary Higgs particles. Hence, our analysis of the virtual effects of Higgs particles should also be applicable to dynamical models of the symmetry breaking of the electroweak interaction.

Presently, there is no experimental evidence for the existence of Higgs particles. A general search for these particles in direct processes would be very difficult since their masses are not known and their couplings are model dependent. One reason Higgs particles may be very hard to see is that they could couple very weakly to known particles, as happens in the standard model. Another possibility is that their mass is far above present acceleration energies. As an alternative to direct processes, one could learn about Higgs particles through virtual effects. This is the way we have learned about the W-boson. The success of the standard model implies that virtual effects of Higgs be small; however, high-precision experiments might show their effects. Indeed, high-precision experiments have been proposed for the muon decay [15], and any significant discrepancies with the standard model could indicate the existence of charged Higgs. Furthermore, if the limits on rare decay modes of the muon and kaon are improved and results are inconsistent with the standard model, Higgs particles may well be the reason. More generally, with high-precision experiments on weak decays and other weak effects, one may be able to limit the possible extensions of the standard model.

In this paper we analyze the possible places where virtual effects of charged Higgs particles and flavor-changing neutral Higgs particles might be observed. In sect. 2, we discuss some common properties of Higgs particles in a few different models. In sect. 3, we study contributions of charged Higgs particles to muon decay, tau decay, beta decay, pion decay, and some higher order processes. Then we consider a few different models and the constraints these processes put on the charged Higgs masses. Possible future experiments to look for effects of charged Higgs particles are also discussed. In sect. 4, we consider the effects of flavor changing neutral Higgs particles. Their effect on rare decays of the muon and kaon,  $\mu e$  conversion,  $K^0-\bar{K}^0$  and  $D^0-\bar{D}^0$  mixing are given. Then in a few different models masses of flavor-changing neutral Higgs are estimated. In particular, we put bounds on the masses of neutral Higgs in the minimal  $SU(2)_L \times SU(2)_R \times U(1)$  model of the electroweak interaction.

## 2. Examples of extended Higgs structure

Here we briefly give the salient features of the Higgs particles in a few commonly considered models of the electroweak interaction.

In the minimal WS model based on  $SU(2)_L \times U(1)$  group (the standard model), one introduces a single complex scalar doublet to break the symmetry spontaneously.

The Higgs mechanism allows all but a single spin-zero particle, the Higgs boson, to be eaten up by massive gauge bosons. The coupling of the Higgs particle field,  $\eta(x)$ , to the fermions is given by

$$\mathcal{L}_I = -2^{-1/4} G_F^{1/2} \eta [m_{u_a} \bar{u}_a u_a + m_{d_a} \bar{d}_a d_a + m_{e_a} \bar{e}_a e_a], \quad (2.1)$$

where  $u_a = u, c, t, \dots$ ;  $d_a = d, s, \dots$ ;  $e_a = e, \mu, \tau, \dots$ , etc. The important properties of this Higgs-fermion coupling are that it is proportional to the mass of the fermion, and conserves flavor and parity.

Not much is known about the mass of the Higgs particle in the standard model, but it is believed to be bounded by

$$7.3 \text{ GeV} \leq m_\eta \leq (0.3 - 1) \text{ TeV}. \quad (2.2)$$

The lower bound comes from the condition of a stable vacuum [16]. The upper bound comes from the assumption that perturbation theory does not break down [17]. Another important feature of the standard model is the weak  $\Delta I = \frac{1}{2}$  rule:

$$\rho = M_W^2 / M_Z^2 \cos^2 \theta_W = 1. \quad (2.3)$$

Experimentally, one finds that  $\rho = 0.981 \pm 0.037$  [18]. Therefore, any extension of the standard model should keep  $\rho$  very close to unity.

## 2.1. SU(2) $\times$ U(1) MODEL WITH SEVERAL HIGGS DOUBLETS

One model which maintains the weak  $\Delta I = \frac{1}{2}$  rule is the standard model with more than a single SU(2) doublet of Higgs. In this model we find  $4n - 3$  Higgs particles after spontaneous symmetry breaking, where  $n$  is the number of Higgs doublets. The most general vacuum expectation values of the scalar fields which conserve electric charge are

$$\langle 0 | \phi_j | 0 \rangle = \sqrt{\frac{T}{2}} \begin{pmatrix} 0 \\ v_i e^{i\theta_i} \end{pmatrix}, \quad j = 1, \dots, n. \quad (2.4)$$

In this case, the general Higgs fermion couplings do not *a priori* conserve flavor and parity. Furthermore, their couplings are not necessarily proportional to the mass of the fermion involved, like those in the minimal standard model.

Since constraints on flavor-changing neutral currents are very strong, often when considering the standard model with several doublets one imposes discrete symmetries to eliminate them. In the case of two doublets, this is achieved by the discrete symmetry

$$\begin{aligned} \begin{pmatrix} u_a \\ d_a \end{pmatrix}_L &\rightarrow \begin{pmatrix} u_a \\ d_a \end{pmatrix}_L, & \begin{pmatrix} \nu_a \\ e_a \end{pmatrix}_L &\rightarrow \begin{pmatrix} \nu_a \\ e_a \end{pmatrix}_L, \\ u_{aR} &\rightarrow u_{aR}, & d_{aR} &\rightarrow -d_{aR}, & e_{aR} &\rightarrow -e_{aR}, \\ \phi_1 &\rightarrow \phi_1, & \phi_2 &\rightarrow -\phi_2. \end{aligned} \quad (2.5)$$

In this model, the Higgs-fermion couplings are completely determined, in particular the charged Higgs couplings are given by

$$\mathcal{L}_1^{\text{C.H.}} = -2^{3/4} G_F^{1/2} \{ [\cot \alpha \bar{v}_L M_e e_R + \tan \alpha \bar{u}_R M_u K^+ d_L + \cot \alpha \bar{u}_L K M_d d_R] H^+ + \text{h.c.} \}, \quad (2.6)$$

where  $\tan \alpha = v_2/v_1$ ,  $K$  is the Kobayashi–Maskawa matrix [3], and  $M_e$ ,  $\bar{M}_u$ , and  $M_d$  are the lepton and quark mass matrices. Note that with this discrete symmetry, the Higgs coupling to fermions is again proportional to the fermion mass.

## 2.2. LEFT–RIGHT SYMMETRIC MODELS

Another class of models which are of interest are the left–right symmetric models [19], in which parity is violated spontaneously. In the minimal model based on  $SU(2)_L \times SU(2)_R \times U(1)$ , to break the symmetry properly we need three sets of Higgs fields with the following representation contents:

$$\Phi(2, 2, 0), \quad \chi_L(2, 1, 1), \quad \chi_R(1, 2, 1). \quad (2.7)$$

The left-handed and right-handed fermions are assigned to the representations:

$$\psi_L(2, 1, \frac{1}{3}) = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L, \quad \psi_R(1, 2, \frac{1}{3}) = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_R. \quad (2.8)$$

The Higgs potential can be shown to lead to vacuum expectation values [20]:

$$\langle 0 | \Phi | 0 \rangle = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \quad \langle 0 | \chi_R | 0 \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle 0 | \chi_L | 0 \rangle = 0, \quad (2.9)$$

It has been shown by Senjanovic [20] that this model reproduces the same low-energy phenomenology as the standard model when  $v \gg m_1, m_2$ , in which case  $G_F = \sqrt{2}/(m_1^2 + m_2^2)$  and the mass of the right-handed gauge bosons becomes very large.

In the left–right symmetric models, both the left- and right-handed fermions are put into doublets; therefore, the only gauge-invariant Higgs-fermion couplings are those to the  $\Phi(2, 2, 0)$  Higgs fields. To illustrate the general features of the Higgs-fermion coupling, we can take for simplicity  $m_1 = 0$  in (2.9). This gives for the Higgs-fermion coupling

$$\begin{aligned} \mathcal{L}_1^{\text{L}\times\text{R}} = & -2^{3/4} G_F^{1/2} \{ [\bar{u} M_u u + \bar{d} K M_d K^+ d] (\cos \alpha_0 H_1^0 \\ & - \sin \alpha_0 H_2^0) + [\bar{d} M_u d + \bar{u} K M_d K^+ u] H_3^0 \\ & + i [\bar{d} M_u \gamma_5 d - \bar{u} K M_d K^+ \gamma_5 u] H_4^0 + [(\bar{u}_L M_u d_R \\ & - \bar{u}_R K M_d K^+ d_L) \sin \alpha_+ H^+ + \text{h.c.}] \}, \end{aligned} \quad (2.10)$$

where  $H_1^0$ ,  $H_2^0$ ,  $H_3^0$ , and  $H_4^0$  are hermitian neutral Higgs and  $H^\pm$  are charged Higgs

fields.  $\alpha_0$  and  $\alpha_+$  are mixing angles from the Higgs mass matrix. Unlike the standard model, we see that this model has flavor changing neutral currents in the Higgs sector. This has been shown to be a very general feature of models based on the gauge group  $SU(2)_L \times SU(2)_R \times U(1)$  and does not depend on the particular structure of the Higgs potential [21]. In sect. 4, we analyze the particular constraints this puts on the masses of the Higgs particles in this model.

Finally, we summarize common properties of Higgs sectors in extensions as well as alternatives to the models we have discussed.

(1) In most extensions, additional neutral Higgs particles will occur and they need not conserve flavor.

(2) Most extensions have charged Higgs.

(3) The masses of the Higgs particles are free parameters in the theory.

(4) In general, the Higgs-fermion couplings are not fixed by the theory. But there are two cases of interest:

(a) The Higgs-fermion couplings are roughly of the order of the mass of the fermions they couple to, i.e.,

$$\mathcal{L}_I \simeq \sqrt{G_F} (m_f + m_{\bar{f}}) \bar{\psi}_f(x) \psi_f(x) H(x). \quad (2.11)$$

The models described thus far in this section have been of this type.

(b) The Higgs-fermion couplings are roughly of the order of the mass of some heavy fermion in the theory,  $M_F$ , i.e.,

$$\mathcal{L}_T \simeq \sqrt{G_F} M_F \bar{\psi}_f(x) \psi_f(x) H(x). \quad (2.12)$$

This type of Higgs-fermion coupling occurs in  $SU(3) \times U(1)$  models [22] of the electroweak interaction. It has also been found in extension of the standard model with several Higgs fields having some kind of permutation symmetry among them [23].

Properties (1)–(4) may also apply to composite Higgs in dynamical symmetry breaking schemes; see refs. [13, 14] for details. The main difference in dynamical models is that the Higgs fields,  $H(x)$ , are not elementary but describe a bound state of some very heavy fermions. However, the internal structure of these composite Higgs can be ignored in most low-energy phenomenology allowing one to use an effective lagrangian approach like (2.11) or (2.12).

### 3. Low-energy effects of charged Higgs particles

This section considers the contribution of charged Higgs particles to low-energy weak processes. We look into their effects at the tree level on well-known weak decays and briefly consider their effects at the one-loop level on a few weak processes.

The most general charged Higgs coupling is written as

$$\mathcal{L}^{C.H.} = 2^{3/4} G_F^{1/2} M_H \left\{ \bar{\psi}_f(x) \left[ \alpha_{f\#}^R \left( \frac{1+y_S}{2} \right) + \alpha_{f\#}^L \left( \frac{1-y_S}{2} \right) \right] \psi_f(x) H^+(x) + \text{h.c.} \right\}, \quad (3.1)$$

where  $f$  and  $f'$  are two fermions differing in charge by one unit. Note that the parametrization is such that at the tree level the Higgs mass drops out in the low-energy limit. In theories where the Higgs couple like the mass of the fermion they interact with, this means  $\alpha_{ff'}^{R,L} \sim (m_F + m_{f'})/M_H$ . In theories where the Higgs couplings are roughly determined by the mass of some heavy fermion,  $M_F$ ,  $\alpha_{ff'}^{R,L} \sim M_F/M_H$ .

For the vector interaction coming from gauge bosons, the standard  $V-A$  interaction is assumed. This neglects the possibility that there may be a right-handed component to the vector interaction; these effects on parameters in muon decay and nuclear beta decay have been discussed in the literature [45]. Because of the success of the  $V-A$  interaction, the effect of charged Higgs must be small, or  $\alpha_{ff'}^{L,R} \ll 1$ . In getting bounds on the Higgs coupling to the fermions, it is important to know the higher order electromagnetic and weak corrections to the standard  $V-A$  theory. It turns out that these corrections only modify the total rate significantly and the effects on the decay parameters, being of the order of  $\alpha(m_f^2/m_W^2)$  [24, 25] can be safely neglected. Furthermore, in our analysis we neglect any possible contribution from flavor-changing neutral Higgs since the processes discussed in sect. 4 are more sensitive to their effect.

### 3.1. MUON DECAY

The effective hamiltonian density for muon decay in charge-exchange order including both vector and scalar boson exchange is

$$\mathcal{H}^{\mu e} = \sqrt{\frac{1}{2}} G_F \sum_a (\bar{\nu}_\mu(x) \Gamma_a \mu(x)) (\bar{e}(x) \Gamma^a (g_a^{\mu e} + g_a^{\mu e'} \gamma_5) \nu_e(x)) + \text{h.c.}, \quad (3.2)$$

where  $\Gamma_a = 1, \gamma_5, \gamma_\lambda, \gamma_\lambda \gamma_5$  for  $a = S, P, V$ , and  $A$ . The couplings  $g_a$  and  $g_a'$  in terms of those in (3.1) are

$$\begin{aligned} g_S^{\mu e} &= (\alpha_{\mu\gamma}^R + \alpha_{\mu\nu}^L)^* (\alpha_{e\nu}^R + \alpha_{e\nu}^L), & g_S^{\mu e'} &= (\alpha_{\mu\nu}^R + \alpha_{\mu\nu}^L)^* (\alpha_{e\nu}^R - \alpha_{e\nu}^L), \\ g_P^{\mu e} &= (-\alpha_{\mu\nu}^R + \alpha_{\mu\nu}^L) (\alpha_{e\nu}^R - \alpha_{e\nu}^L), & g_P^{\mu e'} &= (-\alpha_{\mu\nu}^R + \alpha_{\mu\nu}^L)^* (\alpha_{e\nu}^R + \alpha_{e\nu}^L), \end{aligned} \quad (3.3)$$

$$g_V = -g_V' = -g_A' = g_A = 1.$$

Note that if the neutrinos are left-handed,  $\alpha_{\mu\nu}^R = \alpha_{e\nu}^R = 0$ . The differential decay rate for muon decay is [24]

$$\begin{aligned} d\Gamma(\mu \rightarrow e \bar{\nu} \nu) &= \frac{G_F^2 m_\mu A}{32 \pi^3} \sqrt{x^2 - x_0^2} dx \{ [x(1-x) \\ &\quad + \frac{2}{9} \rho (4x^2 - 3x - x_0^2) + \eta x_0 (1-x)] \\ &\quad - \frac{1}{3} \xi \sqrt{x^2 - x_0^2} \cos \theta \left[ (1-x) + \frac{2}{3} \delta \left( 4x - 3 - \frac{m_e}{m_\mu} x_0 \right) \right] \\ &\quad - \xi^1 \sqrt{x^2 - x_0^2} \cos \phi \left[ (1-x) + \frac{2}{3} \delta' \left( 4x - 3 - \frac{m_e}{m_\mu} x_0 \right) \right] \}, \end{aligned}$$

TABLE 1  
Muon decay parameters

	Present experiment [27]	V-A interaction	Precision of future experiments [15]
$\rho$	$0.752 \pm 0.003$	$\frac{3}{4}$	$\pm 0.00023$
$\eta$	$-0.12 \pm 0.21$	0	$\pm 0.0061$
$\xi$	$0.972 \pm 0.013$	1	$\pm 0.00099$
$\delta$	$0.755 \pm 0.009$	$\frac{3}{4}$	$\pm 0.00064$
$\xi'$	$1.00 \pm 0.13$	1	
$\delta'$	?	$\frac{1}{4}$	

where  $\cos \theta = \hat{p}_e \cdot \hat{s}_\mu$ ,  $\cos \phi = \hat{p}_e \cdot \hat{s}_e$ ,  $x = E_e/w$  and  $w = (m_\mu^2 + m_e^2)/2m_\mu$ . We have used the standard parameterization for muon decay, and general expressions for  $A, \rho, \eta, \xi, \delta, \xi'$ , and  $\delta'$  can be found in the literature [24]. Table 1 gives the experimental status of measurements of the parameters in (3.4), their value in the standard theory, and precision of future proposed experiments.

For the couplings in (3.3), expressions for  $\rho, \eta, \xi$ , etc., are given below, assuming  $\alpha^{L,R}$  are real, i.e., assuming time-reversal invariance:

$$\rho = \delta = \frac{3}{4}, \quad \delta' = \frac{1}{4},$$

$$A = 16 + 4((\alpha_{\mu\nu}^R)^2 + (\alpha_{\mu\nu}^L)^2)((\alpha_{e\nu}^R)^2 + (\alpha_{e\nu}^L)^2),$$

$$\eta = 8\alpha_{\mu\nu}^L\alpha_{e\nu}^L/A \simeq \frac{1}{2}\alpha_{\mu\nu}^L\alpha_{e\nu}^L, \quad (3.5)$$

$$\xi = [16 - 4((\alpha_{\mu\nu}^L)^2 - (\alpha_{\mu\nu}^R)^2)((\alpha_{e\nu}^R)^2 + (\alpha_{e\nu}^L)^2)]/A \simeq 1 - \frac{1}{2}(\alpha_{\mu\nu}^L)^2((\alpha_{e\nu}^R)^2 + (\alpha_{e\nu}^L)^2),$$

$$\xi' = [16 + 4((\alpha_{\mu\nu}^L)^2 + (\alpha_{\mu\nu}^R)^2)((\alpha_{e\nu}^R)^2 - (\alpha_{e\nu}^L)^2)]/A \simeq 1 - \frac{1}{2}((\alpha_{\mu\nu}^R)^2 + (\alpha_{\mu\nu}^L)^2)(\alpha_{e\nu}^L)^2.$$

Note that  $\rho, \delta$ , and  $\delta'$  are independent of scalar couplings when the V-A vector interaction is assumed. Therefore, their measurement is a test of the V-A interaction and  $\mu e$  universality of the vector boson couplings. The  $\eta$  parameter, which specifies the low-energy part of the electron spectrum, is the most sensitive to the Higgs coupling because of its quadratic dependence on the  $\alpha_i$ 's, rather than the quartic dependence found in the parameters  $\xi$  and  $\xi'$ . If  $\eta$  were measured to be significantly different from zero, it would be evidence for the existence of charged Higgs particles, barring the possibility of tensor couplings in eq. (3.2).  $\xi$  and  $\xi'$  are less sensitive to scalar currents than  $\eta$ ; however,  $\xi$  has been measured more accurately. Assuming  $\xi$  and  $\eta$  are to within one standard deviation of their measured value given in table 1, eq. (3.5) gives the constraints on possible charged Higgs couplings in muon decay of:

$$-0.66 \leq \alpha_{\mu\nu}^L\alpha_{e\nu}^L \leq 0.18, \quad (\eta \text{ constraint}),$$

$$(0.17)^2 \leq [(\alpha_{\mu\nu}^R)^2 + (\alpha_{\mu\nu}^L)^2](\alpha_{e\nu}^L)^2 \leq (0.29)^2, \quad (\xi \text{ constraint}). \quad (3.6)$$



### 3.2. TAU DECAYS

As mentioned in sect. 2 some models have charged Higgs particles which couple with strength proportional to the mass of the fermion it couples to. In such models, looking for effects of charged Higgs particles in muon decay would be fruitless since the mass of the electron is so small. For such models, one must look at decays of heavy fermions to see the effects of the Higgs particles. One place their effect might be seen is in the leptonic tau decays. Using a hamiltonian like eq. (3.3), the decay of the  $\tau$  lepton into leptons,  $\ell = \mu, e$  and neutrinos, is given by

$$\Gamma(\tau \rightarrow \ell \nu \bar{\nu}) = \frac{G_F^2 m_\tau^5}{192 \pi^3} \left(1 + \frac{m_\ell^2}{m_\tau^2}\right)^4 \times \{Q_0(x_\ell)[1 + \frac{1}{4}((\alpha_{\ell\nu}^R)^2 + (\alpha_{\ell\nu}^L)^2)((\alpha_{\tau\nu}^R)^2 + (\alpha_{\tau\nu}^L)^2)] + Q_1(x_\ell)\alpha_{\tau\nu}^L\alpha_{e\nu}^L\}, \quad (3.7)$$

where  $x_\ell = 2m_\ell/[m_\tau(1 + m_\ell^2/m_\tau^2)]$  and

$$Q_0(x) = (1 - \frac{5}{2}x^2)\sqrt{1-x^2} + \frac{3}{2}x^4 \ln \left[ \frac{1 + \sqrt{1-x^2}}{x} \right],$$

$$Q_1(x) = (x + 2x^3)\sqrt{1-x^2} - 3x^3 \ln \left[ \frac{1 + \sqrt{1-x^2}}{x} \right].$$

Experimentally [27],  $R_\tau = \Gamma(\tau \rightarrow \mu \nu \bar{\nu})/\Gamma(\tau \rightarrow e \nu \bar{\nu}) = 0.98 \pm 0.18$ . Using this with eq. (3.7) and assuming the leptons couple to the Higgs like their mass, i.e.,  $\alpha_{\mu\nu} \gg \alpha_{e\nu}$ , we find the couplings are constrained by

$$-0.71 \leq [(\alpha_{\tau\nu}^R)^2 + (\alpha_{\tau\nu}^L)^2][(\alpha_{\mu\nu}^R)^2 + (\alpha_{\mu\nu}^L)^2] + 0.44\alpha_{\tau\nu}^L\alpha_{\mu\nu}^L \leq 0.76. \quad (3.8)$$

If we take  $\alpha_{\ell\nu}^L \approx \alpha_{\ell\nu}^R \approx C(m_\ell/M_H)$ , this gives the constraint  $C^2(m_\tau m_\mu/M_H) \leq 0.33$  or  $M_H \geq (0.71 \text{ GeV})C$ . Clearly, this is not a very strong constraint on the Higgs coupling. Further improvements on the measurement of  $R_\tau$  might yield a useful constraint. Note that for the case where the Higgs-lepton couplings are proportional to the heaviest lepton mass, the measurement of  $R_\tau$  does not give any information.

### 3.3. BETA DECAY

In principle, it is possible to observe effects of charged Higgs in high-precision beta decay experiments in measurements of the following quantities:

- (1) total decay rates;
- (2) Fierz interference terms;
- (3) electron polarization;
- (4) electron-neutrino correlations;
- (5) electron distributions from polarized nuclei.

Of these, we consider the effects of charged Higgs on the total decay rate of

$^{14}\text{O} \rightarrow ^{14}\text{N}^* e^+ \nu_e$  and  $\pi^+ \rightarrow \pi^0 e^+ \nu_e$ , Fierz interference terms in  $0^+ \rightarrow 0^+$  nuclear transitions, and electron polarization measurements in nuclear beta decay.

The beta decay hamiltonian density can be parameterized as

$$\mathcal{H}_{(x)}^{\text{Beta}} = \frac{G_F \cos \theta_C}{\sqrt{2}} \sum_a \bar{u}(x) \Gamma_a d(x) \bar{e}(x) \Gamma^a (h_a + h'_a \gamma_5) \nu(x) + \text{h.c.}, \quad (3.9)$$

where,

$$\begin{aligned} \cos \theta_C h_S &= (\alpha_{du}^R + \alpha_{du}^L)^* (\alpha_{e\nu}^R + \alpha_{e\nu}^L), & \cos \theta_C h'_S &= (\alpha_{du}^R + \alpha_{du}^L)^* (\alpha_{e\nu}^R - \alpha_{e\nu}^L), \\ \cos \theta_C h_P &= (-\alpha_{du}^R + \alpha_{du}^L)^* (\alpha_{e\nu}^R - \alpha_{e\nu}^L), & \cos \theta_C h'_P &= (-\alpha_{du}^R + \alpha_{du}^L)^* (\alpha_{e\nu}^R + \alpha_{e\nu}^L), \\ h_V &= h_A = -h'_V = -h'_A = 1. \end{aligned} \quad (3.10)$$

In our analysis, we consider only pure Fermi transitions; therefore, the only quark matrix elements needed are

$$\langle p(p') | \bar{u}(0) \gamma_\lambda d(0) | n(p) \rangle = g_V(t) \bar{u}_p(p') \gamma_\lambda u_n(p), \quad (3.11)$$

$$\langle p(p') | \bar{u}(0) d(0) | n(p) \rangle = g_S(t) \bar{u}_p(p') u_n(p), \quad t = (p' - p)^2. \quad (3.12)$$

From CVC,  $g_V(0) = 1$ . In the standard quark model, the divergence of the vector current is given by

$$i \partial_\lambda (\bar{u}(x) \gamma^\lambda d(x)) = -(m_u - m_d) \bar{u}(x) d(x) + e \bar{u}(x) \gamma^\lambda d(x) A_\lambda(x). \quad (3.13)$$

Using the equation of motion to eliminate the photon field and taking the matrix element between nucleon states, we get

$$\begin{aligned} g_S(t) \bar{u}_p(p') u_n(p) &= \frac{1}{m_d - m_u} \left[ (m_n - m_p) \bar{u}_p(p') u_n(p) \right. \\ &\quad \left. - e^2 \int d^4x D_F^{\lambda\nu}(x) \langle p(p') | J_\lambda^{\text{em}}(x) \bar{u}(0) \gamma_\nu d(0) | n(p') \rangle \right], \end{aligned} \quad (3.14)$$

where  $J_\lambda^{\text{em}}(x) = \frac{2}{3} \bar{u}(x) \gamma_\lambda u(x) - \frac{1}{3} \bar{d}(x) \gamma_\lambda d(x)$  and  $D_F^{\lambda\nu}(x)$  is the photon propagator. To make a rough estimate of the effect of electromagnetic interaction in the 2nd term, we will approximate it by the nucleon intermediate state and use the standard form factors for the vector current. The result is

$$g_S(0) \approx 1.05. \quad (3.15)$$

Now consider the nuclear beta decay  $^{14}\text{O} \rightarrow ^{14}\text{N}^* e^+ \nu_e$  and pion beta decay,  $\pi^+ \rightarrow \pi^0 e^+ \nu_e$ . The matrix elements of both these decays are predicted by CVC. We will consider the ratio,  $R_\beta$ :

$$R_\beta = \frac{\Gamma(^{14}\text{O} \rightarrow ^{14}\text{N}^* e^+ \nu_e)}{\Gamma(\pi^+ \rightarrow \pi^0 e^+ \nu_e)}, \quad (3.16)$$

which eliminates Cabibbo angles. Therefore, we can use eq. (3.16) to set limits on possible scalar couplings in beta decay.

Using the hamiltonian, eq. (3.9), we find that the pion beta decay rate is given by

$$\Gamma(\pi^+ \rightarrow \pi^0 e^+ \nu_e) \simeq \frac{G_F^2 \cos^2 \theta_C}{30\pi^3} (m_{\pi^+} - m_{\pi^0})^5 \left( \frac{m_{\pi^+} + m_{\pi^0}}{2m_{\pi^+}} \right)^3 \\ \times \{R_0(x)[1 + \frac{1}{8}f_s^2(0)(h_s^2 + h_s'^2)] - \frac{5}{4}f_s(0)R_1(x)(h_s - h_s')\}, \quad (3.11)$$

where  $x = m_e/(m_{\pi^+} - m_{\pi^0})$ ,

$$R_0(x) = (1 - \frac{9}{2}x^2 - 4x^4)\sqrt{1-x^2} + \frac{15}{2}x^4 \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) \\ R_1(x) = (x - \frac{13}{2}x^3)\sqrt{1-x^2} - \frac{3}{2}(4x^3 + x^5) \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right),$$

and

$$\langle \pi^0(p') | \bar{d}(0)u(0) | \pi^+(p) \rangle = \sqrt{2}f_s(t)m_{\pi^+}. \quad (3.18)$$

However,  $f_s(0)$  should be small since eq. (3.18) vanishes in the limit where  $G$  parity is conserved. In particular, since  $G$  parity is broken by the violation of isospin symmetry, we expect  $f_s(0) \sim O(\alpha) + O((m_d - m_u)/m_\pi)$ . Therefore, pion beta decay in general is expected to be much less sensitive to effects of charged Higgs than nuclear beta decay.

For  $^{14}\text{O}$  beta decay, we find using eq. (3.9) that the decay width is given by:

$$\Gamma(^{14}\text{O} \rightarrow ^{14}\text{N}^* e^+ \nu_e) \simeq \frac{G^2 \cos^2 \theta_C m_e^5}{\pi^3} F(\rho_0) [1 + \frac{1}{2}g_s^2(0)(h_s^2 + h_s'^2)], \quad (3.19)$$

where  $F(\rho_0)$  is the standard phase-space integral found in nuclear beta decay [28] given in terms of “ $ft$ ” values:

$$F(\rho_0) = \frac{ft(^{14}\text{O})\Gamma(^{14}\text{O})}{\ln(2)}. \quad (3.20)$$

If we put eqs. (3.16), (3.17), and (3.19) together with the experimentally observed quantity  $\Gamma(\pi^+ \rightarrow \pi^0 e^+ \nu_e)$   $ft(^{14}\text{O}) = (1.205 \pm 0.087) \cdot 10^3$  [27, 28], we find when assuming  $f_s(0) \simeq 0$  and  $g_s(0) \simeq 1$ , that

$$(\alpha_{du}^R + \alpha_{du}^L)^2 ((\alpha_{e\nu}^R)^2 + (\alpha_{e\nu}^L)^2) \leq (0.30)^2. \quad (3.21)$$

Now we consider constraints on charged Higgs from Fierz interference terms in nuclear beta decay. The Fierz interference term specifies the low-energy part of the electron spectrum in beta decay like the  $\eta$  parameter does in muon decay. For pure Fermi transitions it is given by

$$b_{\text{Fierz}} = \frac{\pm g_s(0)(h_s - h_s')}{1 + \frac{1}{2}g_s^2(0)(h_s^2 + h_s'^2)} \simeq \pm 2g_s(0)(\alpha_{du}^R + \alpha_{du}^L)\alpha_{e\nu}^L, \quad (3.22)$$

for  $\beta^\mp$  decay. Hardy and Towner [29], from an analysis of the  $0^+ \rightarrow 0^+$  transitions in

nuclear beta decay, found that  $b_{\text{Fierz}} = -0.001 \pm 0.006$ , which with eq. (3.19) and  $g_s(0) = 1$  gives the constraint:

$$-0.0025 \leq (\alpha_{du}^R + \alpha_{du}^L) \alpha_{e\nu}^2 \leq 0.0035. \quad (3.23)$$

Another parameter in beta decay which depends on scalar couplings is the electron polarization given by [28]

$$P = \pm \frac{p_e}{E_e} \left[ \frac{1 + g_s^2(0) h'_s h_s}{1 + \frac{1}{2} g_s^2(0) (h_s^2 + h'^2_s)} \right], \text{ for } \beta^\mp \text{ decay} \quad (3.24)$$

For  $^{14}\text{O} \rightarrow ^{14}\text{N}^* e^+ \nu_e$  the electron polarization has been measured to be  $P(E_e/P_e) = 0.97 \pm 0.19$  [30]. This gives the constraint on the Higgs couplings in beta decay of:

$$|\alpha_{du}^R + \alpha_{du}^L| \alpha_{e\nu}^L \leq 0.33. \quad (3.25)$$

In summary, the strongest constraint on charged Higgs couplings in beta decay comes from measurements of  $b_{\text{Fierz}}$  which is an interference term between vector and scalar couplings.

### 3.4. PION DECAY

As we have seen, all of the constraints in beta decay considered are on scalar quark type couplings, i.e.,  $|\alpha_{ud}^R + \alpha_{ud}^L|$ , to put limits on pseudoscalar quark couplings,  $|\alpha_{ud}^R - \alpha_{ud}^L|$ , we consider pion decay. It is well known that the ratio of pion decays,  $R_\pi^0 = \Gamma(\pi \rightarrow e\nu)/\Gamma(\pi \rightarrow \mu\nu)$  is very sensitive to possible pseudoscalar couplings. Using the hamiltonian, eq. (3.9), one finds it to be given by:

$$R_\pi^0 = \left( \frac{m_e}{m_\mu} \right)^2 \left( \frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2} \right)^2 \left[ \frac{(1 + (m_\mu/m_e) \gamma h_p)^2 + (1 - (m_\mu/m_e) \gamma h'_p)^2}{(1 + \gamma h_p^\mu)^2 + (1 - \gamma h_p^{\mu'})^2} \right], \quad (3.26)$$

where  $\gamma = m_\pi^2/m_\mu(m_u + m_d)$  and  $h_p^\mu$  and  $h_p^{\mu'}$  are the same as  $h_p$  and  $h'_p$  given in eq. (3.10) except  $\alpha_{e\nu}^{R,L}$  is replaced by  $\alpha_{\mu\nu}^{R,L}$ .

When comparing  $R_\pi$  to experiment, one should take into account radiative corrections. Goldman and Wilson [31] have computed them in the Weinberg–Salam model by treating the pion as a fundamental field and find

$$R_\pi^{\text{V-A}} = \frac{\Gamma(\pi \rightarrow e\nu) + \Gamma(\pi \rightarrow e\nu\gamma)}{\Gamma(\pi \rightarrow \mu\nu) + \Gamma(\pi \rightarrow \mu\nu\gamma)} = (1 + \Delta_\pi) R_\pi^{\text{V-A}} \approx 1.23 \cdot 10^{-4}, \quad (3.27)$$

where

$$\Delta_\pi \equiv -15.1 \left( \frac{\alpha}{\pi} \right), \quad R_\pi^{\text{V-A}} = \left( \frac{m_e}{m_\mu} \right)^2 \left( \frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2} \right)^2.$$

The current experimental value [32],  $R_\pi^{\text{exp}} = (1.274 \pm 0.024) \cdot 10^{-4}$ , is almost two standard deviations from the radiative corrected value given in eq. (3.27).

It is interesting to try to explain the discrepancy between the experimental value of  $R_\pi$  and the standard models prediction of it by possible existence of charged Higgs

particles. In models where the Higgs particles couple in proportion to the fermions mass it couples to, i.e., those described by a lagrangian like eq. (2.11),  $R_\pi^0$  in eq. (3.26) is equal to  $R_\pi^{0V-A}$ ; i.e., it is independent of possible Higgs coupling. However, if the Higgs couplings are not of the type described by eq. (2.11), we find that  $R_\pi$  will be very sensitive to their existence. In particular, if the charged Higgs couples like the mass of some heavy fermion,  $M_F$ , in the theory, i.e.,  $\alpha_{ff}^L \approx M_F/M_H$ , we can put some bounds on possible Higgs couplings. Assuming that the Higgs couple only to left-handed neutrinos, i.e.,  $\alpha_{\mu\nu}^R = \alpha_{e\nu}^R = 0$ , we find that the discrepancy between  $R_\pi^{V-A}$  and  $R_\pi^{\text{exp}}$  is explained by letting

$$|\alpha_{du}^R - \alpha_{du}^L| |\alpha_{e\nu}^L| \approx 6.5 \cdot 10^{-4}. \quad (3.28)$$

Furthermore, for a rough estimate let  $|\alpha_{du}^R - \alpha_{du}^L| \approx \alpha_{e\nu}^L \approx M_F/M_H$ , where  $M_F$  is the mass of a heavy lepton in the theory. For  $M_F = 2 \text{ GeV}$  we find that eq. (3.28) implies that the charged Higgs mass is roughly  $M_H = 80 \text{ GeV}$ .

### 3.5. HIGHER-ORDER EFFECTS OF CHARGED HIGGS

Finally, we consider places where charged Higgs particles might manifest themselves at the one-loop level. Since we know that charged Higgs couplings are at least smaller than the gauge couplings, to observe their effects at the one-loop level we have to consider processes that are forbidden in the standard model at the tree level. We will consider  $K^0 - \bar{K}^0$  and  $D^0 - \bar{D}^0$  mixing and weak corrections to “ $g - 2$ ” for the electron and muon.

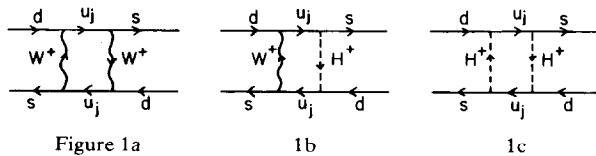
The charged Higgs contribution to the  $K_L - K_S$  mass difference shown in fig. 1 has been discussed in detail by Abbott et al. [33]. For completeness, we summarize their result here for the standard model with two Higgs doublets as given in sect. 2. They find the approximate constraint that

$$M_H \geq 2m_c \tan^2 \alpha \quad \text{for } \tan^2 \alpha \gg 1, \quad (3.29)$$

where  $m_c$  is the charmed quark mass and  $\tan \alpha$  is the ratio of vacuum expectation values of the Higgs particles as defined in sect. 2. They also consider the effects of Higgs on  $D^0 - \bar{D}^0$  mixing and find that current experimental limits on the mixing lead to the constraint:

$$M_H \geq (0.15 \text{ GeV}) \cot^2 \alpha, \quad \text{if } \cot^2 \alpha \gg 1. \quad (3.30)$$

As can be seen, these constraints, eqs. (3.29) and (3.30) are not very strong. They only help in ruling out the possibility that there is an extremely light charged Higgs or



one which couples very strongly so that its effects might be observed in D and K meson decays.

Charged Higgs particles as well as neutral ones can contribute to “ $g-2$ ” of the electron and muon. Their contribution in an arbitrary gauge model has been worked out by Leveille [34]. It is found that all the weak corrections to “ $g-2$ ” are of the order  $G_F m_\ell^2$ . Therefore, for the electron they are too small to be measurable. The muon anomalous magnetic moment, “ $g-2$ ”, has been measured to be [35]  $[a_\mu]_{\text{exp}} = (1165922 \pm 9) \cdot 10^{-9}$ . The QED contribution [35] is  $[a_\mu]_{\text{QED}} = (1165851.8 \pm 2.4) \cdot 10^{-9}$  and the hadronic contribution is  $[a_\mu]_{\text{hadronic}} = (66.7 \pm 9.4) \cdot 10^{-9}$ . For charged Higgs, the contribution is

$$\begin{aligned} [a_\mu]_{\text{C.H.}} &= -\frac{G_F m_\mu^2}{24\sqrt{2}\pi^2} ((\alpha_{\mu\nu}^L)^2 + (\alpha_{\mu\nu}^R)^2) \\ &\simeq 0.38 \cdot 10^{-9} ((\alpha_{\mu\nu}^L)^2 + (\alpha_{\mu\nu}^R)^2). \end{aligned} \quad (3.31)$$

Unfortunately, unless estimates of hadronic contributions and experimental measurements of “ $g-2$ ” of the muon are improved by at least a few orders of magnitude, the effects of possible charged Higgs cannot be isolated. One should note that in a complete analysis of “ $g-2$ ”, other weak corrections such as the effects of vector bosons and neutral Higgs should be included.

### 3.6. SUMMARY AND DISCUSSION

To conclude the analysis of charged Higgs particles, we summarize their effect on low-energy phenomenology and compare with two classes of models of interest. In each of these types of models, we consider bounds that current experiments put on the masses of charged Higgs particles by specifying the value of the couplings  $\alpha_{ff'}^{L,R}$ . Then we discuss experiments that could be done to improve our knowledge about possible charged Higgs.

As mentioned in sect. 2, the Higgs-fermion couplings may be related to fermion masses in one of two ways.

(i) The Higgs-fermion coupling is roughly proportional to the masses of the fermions they are interacting with, i.e.,

$$\alpha_{ff'}^{L,R} = \frac{m_f + m_{f'}}{M_H} C_{ff'}^{L,R}, \quad (3.32)$$

where  $C_{ff'}^{L,R}$  is roughly of the order of unity. This is the type of coupling found in the WS model with 2 or 3 Higgs doublets with a discrete symmetry like eq. (2.5) and the left-right symmetric model considered in sect. 2. In these two models, the parameters  $C_{ff'}^{L,R}$  in eq. (3.32) are roughly some combinations of Cabibbo-type mixing angles and mixing angles in the Higgs sector.

(ii) The Higgs-fermion couplings are all comparable and proportional to some heavy fermion mass,  $M_F$

$$\alpha_{ff'}^{L,R} = \frac{M_F}{M_H} C_{ff'}^{L,R}, \quad (3.33)$$

where  $C_{ff'}^{L,R}$  is roughly of the order of unity.

In table 2, the different low-energy weak interaction processes and their parameter sensitive to charged Higgs are given. Furthermore, lower bounds put by experiment on the mass of a charged Higgs particle contributing to these processes are given with the Higgs-fermion couplings given in eqs. (3.32) and (3.33). It should be pointed out that the  $\xi$  parameter in muon decay may deviate from 1 by the presence of right-handed vector currents [45].

From table 2 one can see that a charged Higgs particle could exist with a mass accessible at current or near future acceleration energies, i.e., 5–40 GeV. In particular, the constraints on theories where the charged Higgs couples like eq. (3.32)

TABLE 2.  
The lower bounds put on the mass of a charged Higgs particle by various processes

Process and parameter sensitive to charged Higgs	Type of Higgs coupling	
	$\alpha_{ff'}^{L,R} = \left(\frac{M_F + M_{F'}}{M_H}\right) C_{ff'}^{L,R}$	$\alpha_{ff'}^{L,R} = \frac{M_F}{M_H} C_{ff'}^{L,R}$
$\mu \rightarrow e \bar{\nu} \nu$ “ $\eta$ ” and “ $\xi$ ” parameter	very weak constraint	$M_H \geq 2.6 M_F  C_{\mu\nu} ^{1/2}  C_{e\nu} ^{1/2}$
$\tau \rightarrow \ell \bar{\nu} \nu$ $R_\tau = \frac{\Gamma(\tau \rightarrow \mu \nu \bar{\nu})}{\Gamma(\tau \rightarrow e \nu \bar{\nu})}$	$M_H \geq 2 \text{ GeV}  C_{\mu\nu} ^{1/2}  C_{e\nu} ^{1/2}$	no constraint
Nuclear beta decay $b_{\text{Fierz}}$ in $0^+ \rightarrow 0^+$ transitions	very weak constraint	$M_H \geq 18 M_F  C_{du}^S ^{1/2}  C_{e\nu}^L ^{1/2}$
$\pi \rightarrow \ell \bar{\nu}$ $R_\pi = \frac{\Gamma(\pi \rightarrow e \nu)}{\Gamma(\pi \rightarrow \mu \nu)}$	no constraint	$M_H \geq 40 M_F  C_{du}^P ^{1/2}  C_{e\nu}^L ^{1/2}$ if $C_{e\nu}^R = 0$
$K^0 - \bar{K}^0$ mixing $\delta m_K = m_L - m_S$	$M_H \geq 10 \text{ GeV}  C_{cd}^L   C_{cs}^L $	pion decay and beta decay offer stronger constraint

The first column gives the specific process considered. The second and third columns give the specific bounds on the Higgs mass for two different parametrizations of the Higgs-fermion couplings given at the top of each column. For the quark masses in the second column, the Weinberg current algebra masses [9] are used. In the table,  $C_{ff'} = \frac{1}{2}(|C_{ff'}^L|^2 + |C_{ff'}^R|^2)^{1/2}$ ,  $C_{ff'}^S = \frac{1}{2}(C_{ff'}^R + C_{ff'}^L)$ , and  $C_{ff'}^P = \frac{1}{2}(C_{ff'}^R - C_{ff'}^L)$ .

(column 2 in table 2) allow a charged Higgs to exist with a mass as low as a few GeV because heavy fermion decay branching ratios have not been well measured yet. In particular, if  $R_\pi$ , which is only known to within 20% at present, could be greatly improved, one could put much stronger constraints on possible charged Higgs couplings and possibly even see their effects, if they exist. Constraints on models with couplings like eq. (3.33) (column 3 in table 2) are a little more stringent. If we take  $M_F = 2$  GeV, we expect that  $M_H$  is at least 20 GeV. One experiment which can be very helpful is a better measurement of  $R_\pi$  which is currently in slight disagreement with the standard models prediction with radiative corrections. If the discrepancy persisted in better measurements of  $R_\pi$ , this might be taken as possible evidence for a charged Higgs particle. Furthermore, the  $\eta$  parameter is the most sensitive parameter to charged Higgs in muon decay, and it has not yet been well measured. If accurate measurements of  $\eta$  could be performed, we would have a much better handle on the pure leptonic Higgs interaction.

#### 4. Flavor-changing neutral Higgs particles

Flavor-changing neutral Higgs particles have very distinguishable characteristics to those particles found in the standard model of the electroweak interaction. In the standard model, lepton number is strictly conserved, and neutral Higgs as well as neutral gauge bosons conserve flavor at the tree level. In the standard model, flavor-changing neutral currents at the one-loop level in the quark sector are suppressed by the GIM mechanism [36]. Hence, flavor-changing neutral Higgs particles could have significant effects on weak processes, even if their masses are very large or couplings are very weak.

Experimentally, there is no evidence for flavor-changing neutral currents of the order  $G_F$ . In particular, no lepton-number violating processes have been observed. In the quark sector, strangeness changing neutral currents have been observed such as  $K_L \rightarrow \mu \bar{\mu}$  and the  $K_L - K_S$  mass difference, on the level  $G_F \alpha$  or smaller. Therefore, if there are neutral Higgs particles that change flavor, either their masses are very large or their coupling to known fermions are very weak.

The presence of flavor-changing neutral currents comes from extensions of the standard model to calculate mixing angles like the Cabibbo angle and fermion masses. In fact, it has been shown that [10, 37] the Cabibbo angle is not calculable in extensions of the standard model that do not have flavor-changing neutral currents. Therefore, if we want quantities like the Cabibbo angle to be calculable in terms of other parameters in the theory, we have to consider flavor-changing neutral Higgs. This section considers the implications of flavor-changing neutral Higgs particles at the tree level and current constraints on their couplings.

We parameterize their couplings by the interaction lagrangian:

$$\mathcal{L}_1^{N.H.}(x) = 2^{-1/4} G_F^{1/2} M_H^0 \sum_{f,f'} \bar{\psi}_f(x) \left[ \eta_{ff'}^L \left( \frac{1 - \gamma_5}{2} \right) + \eta_{ff'}^R \left( \frac{1 + \gamma_5}{2} \right) \right] \psi_{f'}(x) H^0(x). \quad (4.1)$$



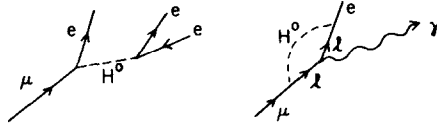


Figure 2a

2b

$M_{H^0}$  is the Higgs mass whose field is  $H^0(x)$ .  $H^0(x)$  is assumed to be a hermitian field in our analysis. The generalization of our results to several Higgs fields and those which are not hermitian is straightforward.

#### 4.1. MUON-NUMBER VIOLATING DECAYS

Experiments on  $\mu \rightarrow 3e$  and  $\mu \rightarrow e\gamma$  are currently under way to search for these decays at levels two orders of magnitude less than present upper bounds on their branching ratio [38]. The standard model has no mechanism for muon-number violation and if one of these decays were seen it would have to be modified.

The decay  $\mu \rightarrow 3e$  can proceed through neutral Higgs particles as shown by fig. 2a. In terms of the couplings given in eq. (4.1), the branching ratio for  $\mu \rightarrow 3e$  is:

$$B(\mu \rightarrow 3e) = \frac{\Gamma(\mu \rightarrow 3e)}{\Gamma(\mu)} = \frac{1}{4} \lambda_{\mu e}^2 \lambda_{ee}^2, \quad (4.2)$$

where we have defined  $\lambda_{ff} = \frac{1}{2} \sqrt{|\eta_{ff}^L|^2 + |\eta_{ff}^R|^2}$ . The current experimental limits are that  $B(\mu \rightarrow 3e) \leq 2 \cdot 10^{-9}$ , therefore

$$\sqrt{\lambda_{ee} \lambda_{e\mu}} \leq 10^{-2}. \quad (4.3)$$

The decay  $\mu \rightarrow e\gamma$  must come from one-loop diagrams like fig. 2b or higher-loop diagrams; therefore, it cannot be calculated without the details of the model, see for example Bjorken and Weinberg [39].

One should mention that if the Higgs couple like the mass of the fermions they couple to, one would practically never see Higgs effect in  $\mu \rightarrow 3e$ , while their effects on decays like  $\tau \rightarrow 3\mu$  would be more important. Furthermore,  $\mu \rightarrow e\gamma$  might have a somewhat larger rate than  $\mu \rightarrow 3e$  since the intermediate leptons,  $\ell$ , in fig. 2b might have much larger couplings to Higgs than those in  $\mu \rightarrow 3e$ . In addition, it has been pointed out that two-loop contributions to  $\mu \rightarrow e\gamma$  could be much larger than the one-loop contributions [39].

#### 4.2. $\mu e$ CONVERSION IN NUCLEI

Another place one can look for lepton-number violation is  $\mu e$  conversion in nuclei, that is, the process  $\mu + (A, Z) \rightarrow e + (A, Z)$ . The advantage of  $\mu e$  conversion over the decays  $\mu \rightarrow 3e$  and  $\mu \rightarrow e\gamma$  is the nucleus coherence effect.

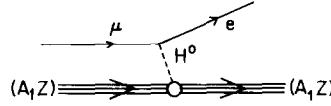


Figure 3

Neutral Higgs particles can lead to  $\mu e$  conversion as shown by the diagram in fig. 3. The rates for  $\mu e$  conversion in different nuclei has been considered in detail by Shanker [40]. Experiments looking for  $\mu e$  conversion have been performed on sulphur [41] and copper [42] nuclei. The sulphur experiment gives the constraint on neutral Higgs couplings of

$$\sqrt{\lambda_{e\mu} |\eta_{uu}^S + \eta_{dd}^S|} \leq 0.7 \cdot 10^{-3}, \quad (4.4)$$

where we define  $\eta_{ff'}^S = \frac{1}{2}(\eta_{ff'}^R + \eta_{ff'}^L)$ . The copper experiment gives the constraint

$$\sqrt{\lambda_{e\mu} |\eta_{uu}^S - \eta_{dd}^S|} \leq 1.4 \cdot 10^{-2}. \quad (4.5)$$

#### 4.3. KAON DECAYS

The small observed rate for  $K_L \rightarrow \mu \bar{\mu}$  and the small  $K_L - K_S$  mass difference offer strict constraints on strangeness-changing neutral currents. In fact, this was one of the motivations for introducing the charm quark through the GIM mechanism. In the standard model the GIM mechanism also suppresses second-order weak contributions to these processes. Thus, they are sensitive to possible Higgs contributions.

The muon-number violating decays  $K_L \rightarrow \mu e$  and  $K^+ \rightarrow \pi^+ \mu e$  are forbidden in the standard model and, if seen, could be explained by flavor changing neutral Higgs.

We have calculated using eq. (4.1) the Higgs particle contribution to branching ratios of  $K_L$  into two leptons,  $\ell$  and  $\ell'$  and find

$$\frac{\Gamma(K_L \rightarrow \ell \bar{\ell}')}{\Gamma(K_L \rightarrow \mu^+ \nu)} \approx \frac{2}{\sin^2 \theta_K} \left( \frac{m_K^2}{m_\mu(m_s + m_d)} \right)^2 \lambda_{\ell\ell'}^2 |\eta_{ds}^P|^2, \quad (4.6)$$

where  $\eta_{ff'}^P = \frac{1}{2}(\eta_{ff'}^R - \eta_{ff'}^L)$ , and  $m_s$  and  $m_d$  are the  $s$  and  $d$  quark current algebra masses. Experiments give the bound that  $B(K_L \rightarrow \mu e) < 2 \cdot 10^{-9}$  [27] If we assume that the Higgs contribution to  $K_L \rightarrow \mu \bar{\mu}$  is no greater than the observed rate, we find with eq. (4.6) the set of constraints:

$$\begin{aligned} \sqrt{\lambda_{e\mu} |\eta_{ds}^P|} &\leq 5.2 \cdot 10^{-4}, \\ \sqrt{\lambda_{ee} |\eta_{ds}^P|} &\leq 5.2 \cdot 10^{-4}, \\ \sqrt{\lambda_{\mu\mu} |\eta_{ds}^P|} &\leq 7.5 \cdot 10^{-4}. \end{aligned} \quad (4.7)$$

Another place we can search for flavor-changing neutral Higgs is in the decay  $K^+ \rightarrow \pi^+ \mu e$ . The calculation of this decay is straightforward and we find:

$$\frac{\Gamma(K^+ \rightarrow \pi^+ \mu e)}{\Gamma(K^+ \rightarrow \pi^0 \mu^+ \nu)} \simeq \frac{16}{\sin^2 \theta_C} |f_0(0)|^2 |\eta_{ds}^S|^2 \lambda_{\mu e}^2, \quad (4.8)$$

where  $\langle \pi^+(p') | \bar{s}(0) d(0) | K^+(p) \rangle = f_0(t) m_K$ . The experimental upper bound is  $B(K^+ \rightarrow \pi^+ \mu e) < 7 \cdot 10^{-9}$  [27]. With eq. (4.8), this gives the rough constraint:

$$\sqrt{\lambda_{\mu e} |\eta_{ds}^S|} \leq 2 \cdot 10^{-3}. \quad (4.9)$$

#### 4.4. THE $K_L - K_S$ MASS DIFFERENCE AND $D^0 - \bar{D}^0$ MIXING

Often in a model with flavor-changing neutral currents, the strongest constraint comes from the small observed  $K_L - K_S$  mass difference and possible  $D^0 - \bar{D}^0$  mixing. Flavor-changing neutral Higgs can contribute at the tree level to  $K^0 - \bar{K}^0$  and  $D^0 - \bar{D}^0$  mixing as illustrated by fig. 4. In models where the Higgs couple like the mass of the fermion they couple to,  $D^0 - \bar{D}^0$  mixing is more sensitive to flavor-changing neutral currents since heavy quarks are involved.

It is well known that the  $K_L - K_S$  mass difference is given by the formula

$$\delta m_K = m_L - m_S = \frac{1}{m_K} \text{Re} [\langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2}(0) | K^0 \rangle]. \quad (4.10)$$

Using the couplings given in eq. (4.1), the  $\Delta S = 2$  effective hamiltonian coming from the tree diagram involving a neutral Higgs exchange is given by

$$\mathcal{H}_{\text{eff}}^{\Delta S=2} = \sqrt{\frac{1}{2}} G_F \bar{s}(x) (\eta_{sd}^S + \eta_{sd}^P \gamma_5) d(x) \bar{s}(x) (\eta_{sd}^S + \eta_{sd}^P \gamma_5) d(x). \quad (4.11)$$

Therefore, to evaluate the effect of neutral Higgs particles on the mass difference  $\delta m_K$ , we need to know the matrix elements

$$\begin{aligned} \mathcal{M}_S &= \frac{1}{2m_K} \langle \bar{K}^0 | \bar{s}(0) d(0) \bar{s}(0) d(0) | K^0 \rangle, \\ \mathcal{M}_P &= \frac{1}{2m_K} \langle \bar{K}^0 | \bar{s}(0) \gamma_5 d(0) \bar{s}(0) \gamma_5 d(0) | K^0 \rangle. \end{aligned} \quad (4.12)$$

It is easy to see for the case of more than one flavor-changing neutral Higgs we have

$$\delta m_K|_{\text{Higgs}} = \sqrt{2} G_F \sum_{a=1}^N [\text{Re} (\eta_{sd}^S)^2_a \mathcal{M}_S + \text{Re} (\eta_{sd}^P)^2_a \mathcal{M}_P]. \quad (4.13)$$

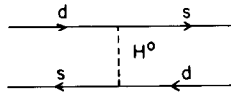


Figure 4

The four-quark matrix elements given in eq. (4.12) have been evaluated in the MIT bag model [43] and are found to be  $M_S = -7.3 \cdot 10^{-3} \text{ GeV}^3$  and  $M_P \approx 8.5 \cdot 10^{-2} \text{ GeV}^3$ . Assuming that the magnitude of the Higgs contribution is no larger than the observed value for  $\delta m_K$ , we get the rough constraint:

$$\sqrt{|(\eta_{sd}^P)^2 - 0.086(\eta_{sd}^S)^2|} \leq 5 \cdot 10^{-5}. \quad (4.14)$$

Now consider constraints from  $D^0$ - $\bar{D}^0$  mixing on the couplings  $\eta_{uc}^S$  and  $\eta_{uc}^P$ . Current experiments indicate that  $|\delta m_D| \leq 2.5 \cdot 10^{-12} \text{ GeV}$  [44]. The standard model predicts a much smaller mixing for  $D^0$ - $\bar{D}^0$  than  $K^0$ - $\bar{K}^0$  since in the  $D^0$  case the GIM suppression is two orders of magnitude stronger. Therefore, as argued in ref. [44], it is a good place to look for unambiguous effects of flavor-changing neutral Higgs. We calculated the mass difference using formulas similar to eq. (4.13) for  $\delta m_D$  with the  $D^0$ - $\bar{D}^0$  matrix elements computed in the MIT bag model given in ref. [43]. This gives the constraint:

$$\sqrt{|(\eta_{uc}^P)^2 - 0.13(\eta_{uc}^S)^2|} \leq 1.5 \cdot 10^{-3}. \quad (4.15)$$

#### 4.5. SUMMARY AND DISCUSSION

We now summarize our results on flavor-changing neutral Higgs and discuss the two special cases of interest mentioned in sect. 2. For the couplings defined at the end of sect. 3, we give the constraints on the Higgs mass in table 3 for the various processes we considered.

From table 3, one can see that a Higgs particle which violates only lepton number has at least an order of magnitude less stringent bounds on it than one which changes strangeness or charm. This is because the  $K_L - K_S$  mass difference is linear in the amplitude while all the decay rates are quadratic in the amplitude. Therefore, unless something suppresses the Higgs effect on the  $K_L - K_S$  mass difference and  $D^0$ - $\bar{D}^0$  mixing, they will in general give the strongest limit on possible flavor changing neutral Higgs.

Finally, we consider the implications of the bounds given in table 3 for the mass of neutral Higgs in the left-right symmetric model given in sect. 2. From the lagrangian given in eq. (2.10) the Higgs which lead to  $d \leftrightarrow s$  and  $u \leftrightarrow c$  transitions have the couplings given by

$$\begin{aligned} \mathcal{L}_1 = & -2^{3/4} G_F^{1/2} \sin \theta_C \cos \theta_C (m_s - m_d) \{ \bar{s} d [ \cos \alpha_0 H_1^0 - \sin \alpha_0 H_2^0 ] \\ & + \bar{c} u H_3^0 - i \bar{c} \gamma_5 u H_4^0 \}, \end{aligned} \quad (4.16)$$

where  $\alpha_0$  is a mixing angle in the Higgs sector. We take:

$$\begin{aligned} |C_{sd}^S| &= 2 \sin \theta_C \cos \theta_C \left( \frac{m_s - m_d}{m_s + m_d} \right) \approx 0.42, \\ |C_{sd}^P| &= 0 \quad \text{and} \quad \cos \alpha_0 \approx \sin \alpha_0. \end{aligned} \quad (4.17)$$

TABLE 3  
The lower bounds put on the mass of a flavor-changing neutral Higgs particle by various processes

Process	Type of neutral Higgs couplings	
	$\eta_{ff}^{L,R} = \frac{M_F + M_F}{M_H} C_{ff}^{L,R}$	$\eta_{ff}^{L,R} = \frac{M_F}{M_H} C_{ff}^{L,R}$
$\mu \rightarrow 3e$	$M_H \geq 1 \text{ GeV}  C_{ee} ^{1/2}  C_{e\mu} ^{1/2}$	$M_H \geq 100 M_F  C_{ee} ^{1/2}  C_{e\mu} ^{1/2}$
$\mu e$ conversion		
Sulphur	$M_H \geq 40 \text{ GeV}  C_{\mu e} ^{1/2}  C_{dd}^S + 0.6 C_{uu}^S ^{1/2}$	$M_H \geq 10^3 M_F  C_{\mu e} ^{1/2}  C_{dd}^S + 0.6 C_{uu}^S ^{1/2}$
Copper	$\geq 2.3 \text{ GeV}  C_{\mu e} ^{1/2}  C_{dd}^S - 0.6 C_{uu}^S ^{1/2}$	$\geq 59 M_F  C_{\mu e} ^{1/2}  C_{dd}^S - 0.6 C_{uu}^S ^{1/2}$
Kaon decays		
$K_L \rightarrow \mu\mu$	$M_H \geq 240 \text{ GeV}  C_{\mu\mu} ^{1/2}  C_{ds}^P ^{1/2}$	$M_H \geq 1.3 \cdot 10^3 M_F  C_{\mu\mu} ^{1/2}  C_{ds}^P ^{1/2}$
$K_L \rightarrow \mu e$	$\geq 250 \text{ GeV}  C_{\mu e} ^{1/2}  C_{ds}^P ^{1/2}$	$\geq 2 \cdot 10^3 M_F  C_{\mu e} ^{1/2}  C_{ds}^P ^{1/2}$
$K_L \rightarrow ee$	$\geq 25 \text{ GeV}  C_{ee} ^{1/2}  C_{ds}^P ^{1/2}$	$\geq 2 \cdot 10^3 M_F  C_{ee} ^{1/2}  C_{ds}^P ^{1/2}$
$K^+ \rightarrow \pi^+ \mu e$	$\geq 64 \text{ GeV}  C_{\mu e} ^{1/2}  C_{ds}^S ^{1/2}$	$\geq 5 \cdot 10^2 M_F  C_{\mu e} ^{1/2}  C_{ds}^S ^{1/2}$
$K_L - K_S$ mass difference	$M_H \geq 3 \cdot 10^3 \text{ GeV}  C_{sd}^P ^2 - 0.086  C_{sd}^S ^2 ^{1/2}$	$M_H \geq 2 \cdot 10^4 M_F  C_{sd}^P ^2 - 0.086  C_{sd}^S ^2 ^{1/2}$
$D^0 - \bar{D}^0$ mixing	$\geq 7.7 \cdot 10^2 \text{ GeV}  C_{uc}^P ^2 - 0.13  C_{uc}^S ^2 ^{1/2}$	$\geq 6.7 \cdot 10^2 M_F  C_{uc}^P ^2 - 0.13  C_{uc}^S ^2 ^{1/2}$

The first column gives the specific process considered. The second and third columns give the specific bounds on the Higgs mass for the two different parametrizations of the Higgs-fermion couplings given at the top of each column. For the quark masses in the second column, the Weinberg current algebra masses [9] are used. The lepton couplings are parameterized by  $C_{ff} = \frac{1}{2}(|C_{ff}^L|^2 + |C_{ff}^R|^2)$  and the quark couplings by  $C_{ff}^S = \frac{1}{2}(C_{ff}^R + C_{ff}^L)$  and  $C_{ff}^P = \frac{1}{2}(C_{ff}^R - C_{ff}^L)$ .

This gives roughly that:  $M_{H_{1,2}} \gtrsim 350$  GeV. Similarly, for  $H_3^0$  and  $H_4^0$  we find the rough bounds,  $M_{H_3} \gtrsim 120$  GeV and  $M_{H_4} \gtrsim 330$  GeV from limits on  $D^0$ - $\bar{D}^0$  mixing. Hence, the neutral Higgs which couples to fermions in left-right symmetric models must roughly all be greater than about 100 GeV in mass. This can be avoided only if there is a cancellation between the  $H_3^0$  and  $H_4^0$  contributions to the  $D^0 \leftrightarrow \bar{D}^0$  transition amplitude or the mixing angle  $\alpha_0$  is very small, allowing one of the neutral Higgs to be lighter.

## 5. Conclusion

There are many theoretical reasons for extending the Weinberg-Salam model of the electroweak interaction. Some of them are to calculate the Cabibbo angle and fermion masses, grand unification, and  $CP$  violation. All of these extensions require additional Higgs particles. In particular, extensions usually have charged Higgs particles may have flavor-changing neutral ones as well.

We considered the possible virtual effects of charged Higgs particles on low-energy weak processes. In muon decay, the  $\eta$  parameter, which determines the low-energy part of the electron spectrum, is an interference term between possible charged Higgs particles and the W-boson contributions. Therefore, if it were found to be different from zero, it would provide evidence for the existence of charged Higgs particles. The tau leptonic decays provide a test for possible charged Higgs which couple like the mass of the fermions they are coupled to since they involve heavy fermions. In beta decay, the Fierz interference term,  $b_{\text{Fierz}}$ , in  $0^+ \rightarrow 0^+$  transitions is sensitive to the possible presence of charged Higgs. The pion leptonic decays can also be very sensitive to charged Higgs particles. The present experimental measurement of  $R_\pi$  does not seem to be in good agreement with the standard model after radiative corrections are taken into account. If the discrepancies were to remain in better experiments, charged Higgs could well be the reason.

We analyzed the effects of flavor-changing neutral Higgs. For muon number violating couplings, the best constraints come from  $\mu e$  conversion. In the quark sector, the  $K_L - K_S$  mass difference and  $D^0$ - $\bar{D}^0$  mixing provide the strongest constraints on possible Higgs couplings. Finally, we analyzed constraints on neutral Higgs particles in the minimal  $SU(2)_L \times SU(2)_R \times U(1)$  model and found that neutral Higgs in this model that couple to fermions must all be roughly greater than 100 GeV in mass.

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