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FLAVOUR VIOLATION, SCALAR PARTICLES AND LEPTOQUARKS

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General features of neutral Higgs particle-fermion Yukawa couplings, including the relation of these couplings to the charge of the fermion, are discussed. Mass bounds on neutral Higgs particles from different flavour-changing processes are calculated. The $K_L - K_S$ mass difference gives the most stringent bound of 150 TeV. Flavour-changing processes mediated by leptoquarks are also studied. μe conversion and K_L decays give comparable bounds, of order 1-10 TeV for pseudoscalar leptoquarks occurring in technicolour models and of order 10-90 TeV for vector leptoquarks occurring in the Pati-Salam type of grand unification models. μe conversion as well as flavour-violating K_L decays should occur at levels close to present experimental limits if dynamical symmetry breaking as envisaged by technicolour theories is indeed realised in nature.

1. Introduction

A phenomenological study of muon number violating processes is of interest because experiments are currently in progress for setting more stringent limits on the branching ratios, and because extensions of the standard weak interaction model that are being studied at present predict muon number violation at some level. A partial survey of muon number violation was made recently [1], and this work presents more details on some of the models. I also study other flavour-violating processes, namely, the $K^0 - \bar{K}^0$ transition, $K_L \to \mu\bar{\mu}$ and $K_L \to e\bar{e}$. I study flavour violation mediated by Higgs particles, by pseudoscalar leptoquarks and by vector leptoquarks.

2. Higgs particles

Flavour violation in an extension of the standard model incorporating two Higgs doublets was studied by Bjorken and Weinberg [2], Sikivie [3] and Lahanas and Vayonakis [4]. A phenomenological study of flavour violation due to Higgs particles was made by McWilliams and Li [5]. Herczeg [6] pointed out that the constraint on the Higgs particle mass from the $K_L - K_S$ mass difference is so stringent that the muon number violating processes would have uninteresting rates, unless the Higgs particle contribution to the $K_L - K_S$ mass difference is suppressed due to some reason. In the present section I review some aspects of the Higgs sector in the standard model with two Higgs doublets. I study the orders of magnitude for different flavour-violating Higgs-fermion couplings, and derive mass bounds for Higgs particles from different flavour-violating processes.

Let us consider the standard model with two Higgs doublets and spontaneous CP violation. The Higgs doublets are $(\phi_1^+ \phi_1^0)$ and $(\phi_2^+ \phi_2^0)$. For large ranges of the parameters in the Higgs potential, the VEVs of the two doublets are related by a complex proportionality constant. One can then use the $\mathrm{SU}(2)_L$ symmetry to redefine the Higgs doublets such that the VEVs take the form

$$\begin{pmatrix} 0 \\ \sqrt{\frac{1}{2}}\,\rho_1 e^{i\theta} \end{pmatrix}, \qquad \begin{pmatrix} 0 \\ \sqrt{\frac{1}{2}}\,\rho_2 \end{pmatrix}.$$

Thus, even with two Higgs doublets the existence of an unbroken $\mathrm{U}(1)_{\mathrm{EM}}$ continues to be natural. Of the eight Higgs particles that occur in the two doublets, two charged particles and one neutral particle are (would-be) Goldstone bosons which are eaten by the W and Z bosons through the usual Higgs mechanism. Two charged scalar particles and three neutral ones remain as physical particles.

In this model, bare fermion mass terms violate the SU(2) symmetry and the fermion masses are due to the VEVs of the Higgs particles. This mass generating mechanism implies that the fermion mass eigenstates no longer correspond to the fermion gauge eigenstates; they are linear combinations of the gauge eigenstates. Due to fermion mixing the gauge bosons and Higgs particles can mediate flavour-changing processes. In the standard model with one Higgs doublet, only the charged W bosons mediate flavour-changing processes. With two Higgs doublets, in general the physical Higgs particles also mediate such processes.

If we shift the Higgs fields from their VEVs, i.e., if we define new fields by

$$\phi_1^0 = \sqrt{\tfrac{1}{2}}\, e^{i\theta}(\rho_1 + R_1 + iI_1),$$

$$\phi_2^0 = \sqrt{\tfrac{1}{2}}\,(\rho_2 + R_2 + iI_2),$$

the Higgs-fermion Yukawa couplings and the contribution of the VEVs to the mass of the charge $-\frac{1}{3}$ quarks, for example, can be written

$$\overline{U}'_L F D'_R \phi_1^+ + \overline{D}'_L F D'_R \sqrt{\tfrac{1}{2}}\, e^{i\theta}(\rho_1 + R_1 + iI_1) + \overline{U}'_L G D'_R \phi_2^+$$

$$+ \overline{D}'_L G D'_R \sqrt{\tfrac{1}{2}}\,(\rho_2 + R_2 + iI_2) + \text{h.c.},$$

where U' is the notation for the gauge eigenstates $(u'c't')$, D' stands for $(d's'b')$, F and G are constant real 3×3 matrices containing the Higgs-fermion Yukawa couplings, and h.c. stands for hermitian conjugate. In terms of gauge eigenstates the mass matrix for the d-type quarks is

$$\bar{D}'_{\mathrm{L}}\left(\sqrt{\tfrac{1}{2}}\,\rho_1 e^{i\theta}F + \sqrt{\tfrac{1}{2}}\,\rho_2 G\right) D'_{\mathrm{R}}.$$

To find the physical fermion states, one has to diagonalise the above matrix by expressing it in terms of new fields $D_{\mathrm{R}} = C_{\mathrm{R}}^{\dagger} D'_{\mathrm{R}}$ and $D_{\mathrm{L}} = C_{\mathrm{L}}^{\dagger} D'_{\mathrm{L}}$. C_{R} and C_{L} are 3×3 unitary matrices which have the property that

$$\sqrt{\tfrac{1}{2}}\, C_{\mathrm{L}}^{\dagger}\left(\rho_1 e^{i\theta}F + \rho_2 G\right) C_{\mathrm{R}} = M_{\mathrm{d}}, \tag{1}$$

where M_{d} is a real positive diagonal matrix containing the fermion masses. D_{R} and D_{L} are the mass eigenstates and the angles in C_{L} and C_{R} are related to the Kobayashi-Maskawa mixing angles. The material presented so far is well known, but the discussion helps to fix the notation. In what follows, we use the matrices $F' = C_{\mathrm{L}}^{\dagger} F C_{\mathrm{R}}$ and $G' = C_{\mathrm{L}}^{\dagger} G C_{\mathrm{R}}$.

Lahanas and Vayonakis studied *CP* violation due to Higgs particles in this model. They find that the Higgs particle contribution to the imaginary part of the $\mathrm{K_L} - \mathrm{K_S}$ mass difference is proportional to the neutral Higgs particle mass differences. The origin of this cancellation is interesting to study, because muon number violating processes may not be affected by it. In that case muon number violating rates could be close to present experimental limits, in spite of the stringent limit on the $\mathrm{K_L} - \mathrm{K_S}$ mass difference. (A similar situation occurs in horizontal gauge models [7].) However, we will see that such a situation doesn't occur for the model of ref. [4]. This is because the cancellation doesn't hold for the real part of the $\mathrm{K_L} - \mathrm{K_S}$ mass difference, and also seems to depend on the assumptions of ref. [4] regarding the mixing angles. They assume that the fermion mixing matrix C_{L} is real and that $C_{\mathrm{R}} = PC_{\mathrm{L}}$, where P is a diagonal phase matrix (i.e., $P_{ij} = e^{i\theta_i}\delta_{ij}$). This is possible only when some strong limitations are placed on the Yukawa coupling matrices F and G, namely, $G = F *$ real diagonal matrix and $F =$ real symmetric matrix $*$ real diagonal matrix*. This can be derived by considering the real and imaginary parts of eq. (1).

We will now write down the Higgs-fermion couplings. We define the Higgs fields

$$G = (\rho_1 I_1 + \rho_2 I_2)/\rho,$$

$$I = (\rho_2 I_1 - \rho_1 I_2)/\rho,$$

$$R' = (\rho_1 R_1 + \rho_2 R_2)/\rho,$$

$$R = (\rho_2 R_1 - \rho_1 R_2)/\rho,$$

where $\rho = \sqrt{\rho_1^2 + \rho_2^2}$. G is not a physical Higgs particle, it is a would-be Goldstone

* It is this fact which permits Lahanas and Vayonakis to completely eliminate these couplings from their equations.

boson which is eaten up by the Z boson. The fields I, R' and R are linear combinations of the physical (mass-eigenstate) Higgs particle fields. The derivation of the general properties of flavour-violating Higgs-fermion couplings that follows does not depend on the Higgs sector mixing angles that implicitly occur in the equations below. In terms of the fermion mass eigenstates, the d-type quark couplings to the fields G, I, R', R are

$$
\begin{aligned}
& iG\overline{D}_L M_d D_R/\rho + \text{h.c.}, \\
& iI\overline{D}_L(\rho_2 e^{i\theta}F' - \rho_1 G')D_R/(\sqrt{2}\,\rho) + \text{h.c.}, \\
& R'\overline{D}_L M_d D_R/\rho + \text{h.c.}, \\
& R\overline{D}_L(\rho_2 e^{i\theta}F' - \rho_1 G')D_R/(\sqrt{2}\,\rho) + \text{h.c.}
\end{aligned} \tag{2}
$$

The only flavour violating couplings are to the R and I fields. The rotation which diagonalised the fermion mass matrix also made the couplings of the R' Higgs field flavour conserving (similar to the situation in the standard model with one Higgs doublet). The couplings of the R and I fields are identical, except for a factor of i. It is this fact which leads to the suppression factor $\Delta M_H/M_H$ for CP violation in the $K^0 - \overline{K}^0$ transition. Consider fig. 1f which contributes to this process. When both d quarks are in left helicity states, a cancellation occurs between the diagrams involving the exchange of the R Higgs particle and the I Higgs particle. When the d quarks have different helicities the R and I diagrams add instead of cancelling. However, with the mixing angles of ref. [4] these diagrams contribute only to the real part of the $K_L - K_S$ mass difference. Thus, the contribution of neutral Higgs particles to CP violation in the $K^0 - \overline{K}^0$ transition is proportional to $\Delta M_H/M_H$ in ref. [4].

Let us abstract some general features of Higgs-fermion couplings, in order to derive phenomenological bounds on Higgs boson masses. Firstly we note that the VEVs of the Higgs particles are related to the weak interaction Fermi coupling constant. In this model $\rho^2 = 1/(\sqrt{2}\,G_F)$. Secondly, the Higgs-fermion Yukawa couplings in each charged fermion sector are of the order of m_f/ρ, where m_f is some average value of the fermion masses in that charge sector, assuming that the Yukawa couplings and VEVs of different Higgs multiplets are of the same order of magnitude. In some models one can make stronger statements. In particular, in models where a discrete symmetry is imposed to get natural flavour conservation (NFC) often the Higgs particle coupling to any fermion is proportional to the fermion mass. Since we are studying the phenomenology of flavour violation, models with NFC will not be considered. Stated another way, the Higgs-fermion couplings in any charged-fermion sector are of order $\sqrt{G_F}\, m_f$. In most models m_f will turn out to be the heaviest fermion mass of the given charge times a mixing angle which we will assume to be of order one. The comparison of different processes will not be affected if the mixing angles are assumed to be of order, say, 0.1–0.2.

We now consider the mass bounds on Higgs particles from different flavour-violating processes. Table 1 presents a summary of the mass bounds. First consider the process $\mu \to e\gamma$. The form of the μe current coupling to the photon can be written down using Lorentz invariance and the electromagnetic gauge invariance:

$$\begin{aligned}\langle e(P_e)|J_\lambda(0)|\mu(P_\mu)\rangle = \sqrt{\tfrac{1}{2}}\, G_F \bar{U}_e(P_e)\big[& f_{ML}(1+\gamma_5) i m_\mu \sigma_{\lambda\nu} q^\nu \\ & + f_{EL}(1+\gamma_5)\big(\gamma_\lambda q^2 - q_\lambda \gamma\cdot q\big)\big] U_\mu(P_\mu) \\ & + \text{right-handed terms}, \end{aligned} \tag{3}$$

where $q = P_\mu - P_e$ and m_μ is the muon mass. Only f_{ML} and f_{MR} contribute to $\mu \to e\gamma$,

TABLE 1
Mass bounds (in TeV) from different processes

Process	Higgs scalars	Pseudoscalar leptoquarks[a)] (A)	Pseudoscalar leptoquarks[b)] (B)	Vector leptoquarks	Experimental limit
$\frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to \text{all})}$	0.2				1.9×10^{-10}
$\frac{\Gamma(\mu \to ee\bar{e})}{\Gamma(\mu \to \text{all})}$	0.4				1.9×10^{-9}
$\frac{\Gamma(\mu A \to eA)}{\Gamma(\mu A \to \nu A')}$	11	1.5	11	60	7×10^{-11}(for S)
$\frac{\Gamma(K_L \to \mu\bar{e})}{\Gamma(K_L \to \text{all})}$	7	1.8	5	93	2×10^{-9}
$\frac{\Gamma(K_L \to \mu\bar{\mu})}{\Gamma(K_L \to \text{all})}$	4.7	1.2	3.6	62	9×10^{-9}
$\frac{\Gamma(K_L \to e\bar{e})}{\Gamma(K_L \to \text{all})}$	7	1.8	5	95	2×10^{-9}
$\frac{\Gamma(K^+ \to \pi^+ \mu\bar{e})}{\Gamma(K^+ \to \text{all})}$	0.7	0.1	0.3	3.5	7×10^{-9}
$\Delta m(K_L - K_S)$	150				3.5×10^{-15} GeV

[a)] The average fermion mass was taken to be 1 GeV.
[b)] The average fermion mass was taken to be of order $\sqrt{m_\tau m_t} = 7.8$ GeV for uμ couplings and $\sqrt{m_\tau m_b} = 3.1$ GeV for dμ couplings.

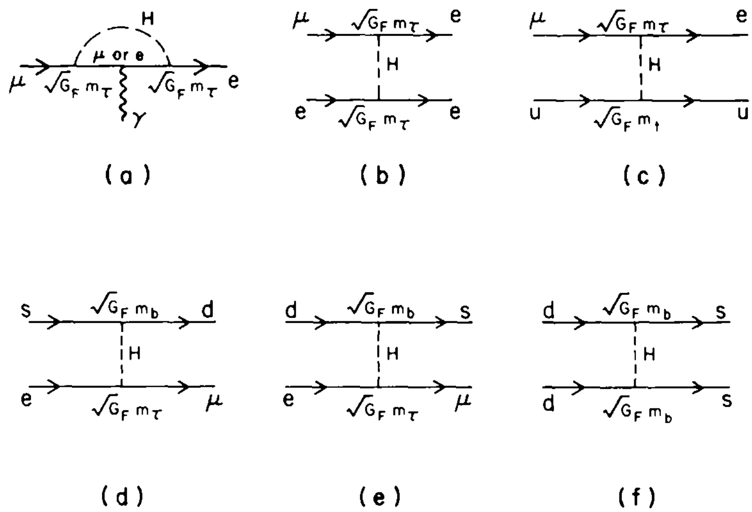

Fig. 1. Feynman diagrams for flavour-violating processes mediated by neutral scalar particles.

and the branching ratio is

$$
\begin{aligned}
\mathrm{B}(\mu \to \mathrm{e}\gamma) &= \frac{\Gamma(\mu \to \mathrm{e}\gamma)}{\Gamma(\mu \to \mathrm{e}\nu_{\mathrm{e}}\nu_{\mu})}, \\
&= 24\pi^2\left(|f_{\mathrm{ML}}|^2 + |f_{\mathrm{MR}}|^2\right).
\end{aligned}
$$

A typical diagram contributing to $\mu \to \mathrm{e}\gamma$ is shown in fig. 1a (H is a Higgs particle). This diagram has a logarithmic divergence, which should cancel between different graphs in a renormalisable theory. However, the contribution of this diagram to $G_{\mathrm{F}} f_{\mathrm{ML}}$ and $G_{\mathrm{F}} f_{\mathrm{MR}}$ is finite, and of order $\sqrt{\alpha}\, G_{\mathrm{F}} m_\tau^2/(\pi^2 m_{\mathrm{H}}^2)$. Thus, the order of magnitude for $\mathrm{B}(\mu \to \mathrm{e}\gamma)$ is

$$
\mathrm{B}(\mu \to \mathrm{e}\gamma) = \frac{24\alpha}{\pi^2}\left(\frac{m_\tau}{m_{\mathrm{H}}}\right)^4.
$$

The experimental limit of 1.9×10^{-10} [8] implies that $m_{\mathrm{H}} > 200$ GeV.

We now consider the process $\mu \to \mathrm{ee\bar{e}}$. This process occurs due to Feynman diagrams similar to the one shown in fig. 1b. The $\mu \to \mathrm{ee\bar{e}}$ amplitude contains terms of the type

$$
\begin{aligned}
m(k_1, k_2) = {} & \frac{G_{\mathrm{F}} m_\tau^2}{m_{\mathrm{H}}^2}\left(\bar{U}_{\mathrm{e}}(k_1)(1 - a\gamma_5)U_\mu(p)\right) \\
& \times \left(\bar{U}_{\mathrm{e}}(k_2)(1 - b\gamma_5)V_{\mathrm{e}}(k_3)\right) - (k_1 \leftrightarrow k_2),
\end{aligned}
\tag{4}
$$

where p is the four-momentum of the muon, k_1, k_2 and k_3 are the four-momenta of the two electrons and positron, respectively, and the amplitude is antisymmetric in k_1 and k_2 because there are two identical particles in the final state. With this amplitude the rate for $\mu \to ee\bar{e}$ is

$$\Gamma(\mu \to ee\bar{e}) = \left(G_F \frac{m_\tau^2}{m_H^2}\right)^2 \frac{m_\mu^5}{3(2^9)\pi^3}(2A' + B'),$$

where

$$A' = 3(1 + a^2)(1 + b^2) + 4ab,$$

$$B' = (1 + a^2)(1 + b^2) + 4ab. \tag{5}$$

The current limit on the branching ratio (compared to the ordinary decay rate) of 1.9×10^{-9} [9] implies that $m_H > 400$ GeV when $a = b = 1$.

A diagram contributing to μe conversion is shown in fig. 1c. The amplitude is of the form

$$\frac{G_F m_\tau m_t}{m_H^2} \bar{U}_e(1 + \gamma_5) U_\mu \bar{U}_u U_u. \tag{6}$$

There is a term involving the d quarks, and terms involving right-handed μe currents. The ratios of the different couplings depends on details of the mixing angles. Even if mixing angles conspire to suppress the μe conversion process the mass bound should not change by more than a factor of 2 or 3, as the process involves couplings to both u and d quarks. Because the Cabibbo angle is not zero, it seems difficult to suppress the couplings to both u and d quarks simultaneously. Ref. [10] gives a general discussion of μe conversion. Using the formula of that reference, eq. (6) gives the bound $M_H > 11$ TeV for a branching ratio limit of 7×10^{-11} for sulphur [11]. (This assumes a top quark mass of 30 GeV.) Among the muon number violating processes this is the most stringent bound. This process gives a good bound because the different quarks in the nucleus contribute coherently, and hence the rate is proportional to A^2, where A is the atomic number.

Two diagrams contributing to $K_L \to \mu\bar{e}$ are shown in fig. 1d, e. The amplitude is of the form

$$\frac{G_F m_b m_\tau}{m_H^2} \bar{\mu}(1 + \gamma_5) e(\bar{s}\gamma_5 d + \bar{d}\gamma_5 s). \tag{7}$$

I will discuss the process $K_L \to \ell_1 \bar{\ell}_2$ in general as the results will be needed later. Since the kaon has negative parity, the most general four-fermion hamiltonian contributing to $K_L \to \ell_1 \bar{\ell}_2$ is

$$H_{\rm eff} = \sqrt{\tfrac{1}{2}}\, G_F \Big[\bar{l}_1 \gamma^5 (f_P + f'_P \gamma_5) l_2 (\bar{s}\gamma_5 d + \bar{d}\gamma_5 s) + \bar{l}_1 \gamma^\mu \gamma^5 (f_A + f'_A \gamma_5) l_2 \times \left(\bar{s}\gamma_\mu \gamma^5 d + \bar{d}\gamma_\mu \gamma^5 s \right) \Big]. \tag{8}$$

In order to calculate the rate, one needs the K_L to vacuum matrix elements:

$$\langle 0 | \bar{s}\gamma_\lambda \gamma_5 d + \bar{d}\gamma_\lambda \gamma_5 s | K_L(p) \rangle = p_\lambda m_K a_A / \sqrt{2 m_K}\,,$$

$$\langle 0 | \bar{s}\gamma_5 d + \bar{d}\gamma_5 s | K_L(p) \rangle = -m_K^2 a_P / \sqrt{2 m_K}\,. \tag{9}$$

Herczeg [6] estimates that $a_A = 0.48$ and $a_P = 1.5$. The amplitude for the process takes the form

$$m = A \bar{U}_{l_1} \gamma_5 V_{l_2} + B \bar{U}_{l_1} V_{l_2},$$

where

$$A = \sqrt{\tfrac{1}{2}}\, G_F \left(m_K (m_{l_1} + m_{l_2}) f_A a_A - m_K^2 a_P f_P \right),$$

$$B = \sqrt{\tfrac{1}{2}}\, G_F \left(m_K (m_{l_1} - m_{l_2}) f'_A a_A - m_K^2 a_P f'_P \right). \tag{10}$$

The lepton masses enter because the Dirac equation has been used to eliminate γ matrices. Their presence indicates the fact that for the axial vector current, one of the emitted leptons has the wrong helicity, and hence a helicity flip mechanism is required (in this case, a mass insertion). The rate for $K_L \to l_1 \bar{l}_2$ is

$$\Gamma(K_L \to l_1 \bar{l}_2) = \frac{k}{2\pi m_K^2} \left[\left(|A|^2 + |B|^2 \right) \left(E_{l_1} E_{l_2} + k^2 \right) + \left(|A|^2 - |B|^2 \right) m_{l_1} m_{l_2} \right], \tag{11}$$

where E_{l_1} and E_{l_2} are the lepton energies in the rest frame of the kaon, and k is the magnitude of the three-momentum carried by either lepton. For the process $K_L \to \mu\bar{e}$, $m_{l_2} (= m_e)$ can be neglected. One then finds

$$\Gamma(K_L \to \mu\bar{e}) = \frac{m_K}{8\pi} \left(1 - \frac{m_\mu^2}{m_K^2} \right)^2 \left(|A|^2 + |B|^2 \right). \tag{12}$$

With the above result, the amplitude eq. (7) leads to the branching ratio

$$\frac{\Gamma(K_L \to \mu\bar{e})}{\Gamma(K \to \text{all})} = 5 \times 10^4 \left(\frac{m_b m_\tau}{m_H^2} \right)^2. \tag{13}$$

The experimental limit 2×10^{-9} [12] implies that $m_{\mathrm{H}}>7$ TeV. For the processes $\mathrm{K_L}\to\mu\bar{\mu}$ and $\mathrm{K_L}\to\mathrm{e\bar{e}}$ the Feynman diagrams are similar. The amplitudes are given by eq. (7) with the $\bar{\mu}\mathrm{e}$ current replaced by $\bar{\mu}\mu$ and $\mathrm{\bar{e}e}$ currents respectively. One finds that

$$\frac{\Gamma(\mathrm{K_L}\to\mu\bar{\mu})}{\Gamma(\mathrm{K_L}\to\mathrm{all})}=4.6\times10^{4}\left(\frac{m_{\mathrm{b}}m_{\tau}}{m_{\mathrm{H}}^{2}}\right)^{2}. \tag{14}$$

Assuming that the Higgs particle contribution to the $\mathrm{K_L}\to\mu\bar{\mu}$ branching ratio is at most of the same order as the experimental ratio 9×10^{-9}, we find the bound $M_{\mathrm{H}}>4.7$ TeV. The branching ratio for $\mathrm{K_L}\to\mathrm{e\bar{e}}$ is

$$\frac{\Gamma(\mathrm{K_L}\to\mathrm{e\bar{e}})}{\Gamma(\mathrm{K_L}\to\mathrm{all})}=5.4\times10^{4}\left(\frac{m_{\mathrm{b}}m_{\tau}}{m_{\mathrm{H}}^{2}}\right)^{2}. \tag{15}$$

and the experimental upper limit 2×10^{-9} implies that $M_{\mathrm{H}}>7$ TeV.

The process $\mathrm{K}^{+}\to\pi^{+}\mu\bar{\mathrm{e}}$ has a smaller phase space available to it than the $\mathrm{K_L}$ decays. Hence the constraint from this process should be poorer. Even if the $\mathrm{K_L}\to\mu\mathrm{e}$ decays are suppressed because of cancellations in the pseudoscalar currents, $\mu\mathrm{e}$ conversion should provide a more stringent limit than $\mathrm{K}^{+}\to\pi^{-}\mu\bar{\mathrm{e}}$. A diagram contributing to $\mathrm{K}^{+}\to\pi^{+}\mu\bar{\mathrm{e}}$ is shown in fig. 1e. The effective hamiltonian for this process is

$$\frac{G_{\mathrm{F}}m_{\mathrm{b}}m_{\tau}}{M_{\mathrm{H}}^{2}}\bar{\mu}e\,\bar{s}d. \tag{16}$$

The branching ratio would then be [6]

$$\frac{\Gamma(\mathrm{K}^{+}\to\pi^{+}\mu\bar{\mathrm{e}})}{\Gamma(\mathrm{K}^{+}\to\mathrm{all})}=16\left(\frac{m_{\mathrm{b}}m_{\tau}}{M_{\mathrm{H}}^{2}}\right)^{2}. \tag{17}$$

The experimental limit of 7×10^{-9} [13] implies that $M_{\mathrm{H}}>700$ GeV, thus showing that constraints from this process are indeed poorer.

A diagram contributing to the $\mathrm{K_L}-\mathrm{K_S}$ mass difference is shown in fig. 1(f). The effective hamiltonian is

$$G_{\mathrm{F}}\frac{m_{\mathrm{b}}^{2}}{M_{\mathrm{H}}^{2}}\bar{s}\gamma_5 d\bar{s}\gamma_5 d. \tag{18}$$

The contribution of this hamiltonian to the $\mathrm{K_L}-\mathrm{K_S}$ mass difference is

$$\Delta m=2\frac{G_{\mathrm{F}}m_{\mathrm{b}}^{2}}{m_{\mathrm{H}}^{2}}\langle\bar{K}^{0}|\bar{s}\gamma_5 d\bar{s}\gamma_5 d|K^{0}\rangle. \tag{19}$$

Two methods of comparable reliability exist to calculate the hadronic matrix element occurring in this equation. They are the bag model calculation and the vacuum insertion method. The vacuum insertion method does not merely consist of inserting a complete set of states between the two currents and truncating the set with the vacuum state. The method involves breaking up the wave functions into creation and destruction parts, and Fierz transforming the different terms so that the quark anti-quark pair in the K^0 are destroyed before the pair in the $\overline{K}^0$ are created. When a complete set of states is inserted after the Fierz transformations, only the vacuum state contributes in the limit that the kaon is strictly a two body object. The Fierz transformations make the vacuum insertion technique a much more reliable tool than is generally believed (the large opposite sign contributions of higher terms that are commonly believed to make this method unreliable are really valid only when a complete set of states is introduced and truncated without making the Fierz transformations. These contributions have no relevance when the Fierz transformations have been made!). The bag model and vacuum insertion methods have been used to calculate the $K^0 - \overline{K}^0$ transition matrix elements for a general combination of four quark currents in ref. [14]. Ref. [14] finds that the two methods give similar results. The bag model predictions are about 0.7 times the vacuum insertion predictions. (For the familiar $(V - A)$ case the bag model prediction is 0.4 times the vacuum insertion prediction because a cancellation occurs between the V and A terms, and hence the results are more sensitive to the bag model parameters.) For the matrix element in eq. (19) ref. [14] finds the value of 0.123 GeV^3 with the vacuum insertion method and 0.085 GeV^3 with the bag model calculation. I use the former value. Assuming that the contribution of eq. (19) is comparable to the experimental $K_L - K_S$ mass difference of 3.5×10^{-15} GeV, we find that $M_H > 150$ TeV. This large lower bound illustrates the sensitivity of this process, and the fact that the $K_L - K_S$ mass difference is proportional to the flavour-changing amplitude, unlike the rates which are proportional to the square of the amplitude. Lahanas and Vayonakis have found a bound of about 1 TeV for the Higgs boson mass from the $K_L - K_S$ mass difference. The main reason for the different bounds is that the estimate of ref. [4] assumes two generations, in which case the relevant Yukawa couplings are proportional to m_s times mixing angles. In the realistic case of three generations these couplings are proportional to m_b times mixing angles.

In this section we have looked at some aspects of Higgs-fermion couplings in an extension of the standard model where two Higgs doublets are introduced. After discussing the general orders of magnitude for Higgs-fermion Yukawa couplings, the bounds on the masses of flavour-violating Higgs particles from different processes were calculated. The $K_L - K_S$ process gives by far the best bound of 150 TeV. Since the contribution of Higgs particles to this process could be suppressed in some models, the bounds from the other processes are also of interest. Among the other processes μe conversion gives the best bound of 11 TeV. The K_L decays give comparable bounds, $M_H > 7$ TeV. Table 1 summarises the mass bounds from different processes. McWilliams and Li [5] have calculated Higgs boson mass bounds in terms of an average fermion mass, for all the processes listed in table 1 except the $\mu \to e\gamma$ process. In this paper I have estimated what the average fermion mass to be used in the Yukawa couplings should be. If these estimates are used in their mass bounds, the results are similar to table 1.

3. Pseudoscalar leptoquarks

A modification of the standard model that is often considered [15–17] is the extended technicolour scheme where elementary scalar fields are absent and the $SU(2)_L \times U(1)$ symmetry is dynamically broken. In this scheme new gauge interactions (technicolour interactions) and new exotic fermions (technifermions) are introduced. Ref. [15] gives an introduction to these theories. The technicolour interactions are analogous to the colour SU(3) interactions. When the technicolour interactions become strong, the chiral flavour symmetry due to the technifermions is spontaneously broken (analogous to the chiral symmetry breaking of strong interactions), and the technifermions pick up a vacuum expectation value (form technifermion condensates) [16]:

$$\langle \bar{F}F \rangle_0 = \mu^3$$

where μ is of order two to three hundred GeV. As a result of chiral symmetry breaking, one gets (pseudo-) Goldstone bosons which are bound technifermion-antitechnifermion pairs (technipions). These exotic bound states take the place of the ordinary Higgs particle, and give masses to the gauge bosons (except to the technicolour gauge bosons that bind them). The appeal of this scheme is that all interactions are gauge interactions, and hence it has far fewer parameters (in principle) than the standard model. However, no satisfactory model has been constructed as yet. This class of theories has two sources of flavour violation: horizontal gauge bosons and pseudoscalar bosons (exotic fermion-antifermion bound states). The former are necessary to generate effective Yukawa couplings between technibosons and ordinary fermions, and thus to give masses to the ordinary fermions. The dynamical symmetry breaking scheme runs into problems because it predicts relatively large flavour-violating currents. In the present work we will study the phenomenology of flavour violation due to the pseudoscalar bosons (technipions).

The generation of fermion masses requires that ordinary fermions of a given charge are put together with technifermions in a multiplet of yet another gauge group (the extended technicolour (ETC) group). The diagram contributing to fermion masses and mixing angles is shown in fig. 2. The cross represents the vacuum expectation value picked up by the technifermion pair (technifermion condensation). The elements of the fermion mass matrix are thus of order

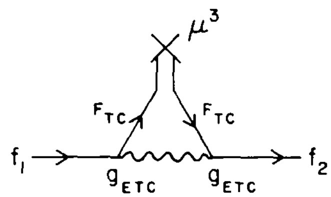

Fig. 2. The mass and mixing angle generating mechanism for ordinary fermions in technicolour theories. f_1 and f_2 are ordinary fermions and the wavy line represents an ETC gauge boson. F_{TC} is a technifermion, and $\langle \bar{F}_{TC} F_{TC} \rangle_0 = \mu^3$.

$(g_{ETC}/M_{ETC})^2\mu^3$. The large hierarchy of fermion masses is supposed to be due to ETC gauge bosons of different masses. One cannot narrow down the order of magnitude of flavour-changing couplings in this scheme to the same extent that one could in the elementary Higgs particle models, because of the hierarchy of ETC gauge boson masses. For example, M_{ETC} estimated using an up quark mass of 5 MeV gives 5.5 TeV (assuming that $g_{ETC}^2 \approx \alpha$) while the value is 70 GeV assuming a top quark mass of 30 GeV. Since the Kobayashi-Maskawa mixing angles are not zero, one would expect that a diagram like fig. 2 should exist connecting the top and up quarks. (Such a diagram would give the mixing between the two quarks.) It is not clear what the mass of the ETC gauge boson mediating the process should be, but it does appear that ETC gauge bosons of very different masses couple to the up quark and it seems too restrictive to assume that all processes involving the up quark are determined by an ETC mass scale of 5.5 TeV, for example*. In view of these uncertainties, the most reasonable course is probably to assume some average value for m_f, say 1 GeV, and use that to estimate the order of magnitude of M_{ETC}. The scale of M_{ETC} is found to be about 400 GeV.

In technicolour theories flavour violating scalar particles are of two kinds: neutral scalars and coloured leptoquarks (particles taking a lepton into a quark or vice versa). If the neutral particle couplings are flavour-non-conserving, bounds similar to those of the previous section apply, and these bounds run into serious conflict with the masses expected for the scalars in technicolour theories. Ellis et al. [16] found that if all fermions of a given charge get their masses from the same technifermion condensate, the neutral scalar particle couplings become flavour conserving (they call this monophagy). However, there seems to be no way to avoid flavour violation due to leptoquarks. They find typical flavour violating couplings of the form:

$$\left(-i2\sqrt{2}\,\frac{\mu^3}{F_P}\right)P\left[\bar{e}\left(\Gamma_1\left(\frac{1+\gamma_5}{2}\right)+\Gamma_2\left(\frac{1-\gamma_5}{2}\right)\right)d\right] \tag{20}$$

* If this is true, then all mixing angles will be small. However, in the absence of a model the masses of the gauge bosons mediating different processes have large uncertainties.

where e and d are vectors (e, μ, τ) and (d, s, b), P is a pseudoscalar particle, and Γ is a matrix with entries of order $g_{\mathrm{ETC}}^2/M_{\mathrm{ETC}}^2$ (times ETC gauge boson mixing angles and Clebsch-Gordan factors). The masses of the pseudoscalar particles are estimated to be of order 160 GeV. F_{P} is the analogue of F_π in QCD, and is estimated to be of order 250 GeV [16]. The effective Yukawa couplings which we denote by f in what follows are thus of order $m_{\mathrm{f}}/(250\ \mathrm{GeV}) \approx \frac{1}{250}$. Because of quark mixing, it does not seem possible to be more precise without a model. However, one should keep in mind the fact that these Yukawa couplings could vary over a large range. If any future model gives more definite information on the effective Yukawa couplings it is straightforward to change the bounds correspondingly.

Leptoquark particles mediate only those flavour-changing processes which involve both quarks and leptons (to leading order). Therefore the stringent limit on the $\mathrm{K_L} - \mathrm{K_S}$ mass difference does not force other flavour-violating rates to be far below experimental limits. The $\mu \to \mathrm{e}\gamma$ and $\mu \to \mathrm{ee\bar{e}}$ processes do not occur at the tree level. So the first process we consider is μe conversion. The leptoquark contribution to μe conversion has not been discussed previously. The Feynman diagram is shown in fig. 3a. The effective hamiltonian is typically of the form

$$H_{\mathrm{eff}} = \left[a_1\bar{e}(1+\gamma_5)u\bar{u}\mu + a_2\bar{e}\gamma_5(1+\gamma_5)u\bar{u}\gamma_5\mu + a_3\bar{e}(1+\gamma_5)d\bar{d}\mu + a_4\bar{e}\gamma_5(1+\gamma_5)d\bar{d}\gamma_5\mu\right]\left(\frac{f^2}{M_{\mathrm{P}}^2}\right). \tag{21}$$

After a Fierz transformation the hamiltonian becomes

$$\left(\frac{f^2}{M_{\mathrm{P}}^2}\right)\left[\frac{a_1+a_2}{4}\bar{e}(1+\gamma_5)\mu\bar{u}u + \frac{a_1-a_2}{4}\bar{e}\gamma_\mu(1+\gamma_5)\mu\bar{u}\gamma^\mu u + \cdots + \frac{a_3+a_4}{4}\bar{e}(1+\gamma_5)\mu\bar{d}d + \frac{a_3-a_4}{4}\bar{e}\gamma_\mu(1+\gamma_5)\mu\bar{d}\gamma^\mu d + \cdots\right] \tag{22}$$

where the dots represent tensor, pseudoscalar and axial vector quark currents, which do not contribute to the coherent μe conversion rate. The point to be noted is that

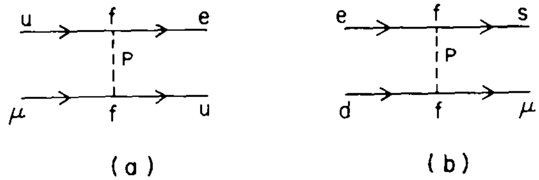

Fig. 3. Feynman diagrams for flavour-violating processes mediated by leptoquarks.

both scalar and vector currents are present. One can rearrange the hamiltonian into two parts, leading to left- and right-handed electrons respectively. The rates for the two parts can be calculated separately and added. Interference terms will be absent because of the different final states. Also, we re-express the hamiltonian in terms of isoscalar and isovector currents:

$$\begin{aligned}
V_\mu^{(0)} &= \tfrac{1}{2}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right),\\
V_\mu^{(1)} &= \tfrac{1}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right),\\
S^{(0)} &= \tfrac{1}{2}(\bar{u}u + \bar{d}d),\\
S^{(1)} &= \tfrac{1}{2}(\bar{u}u - \bar{d}d).
\end{aligned} \tag{23}$$

The general hamiltonian can be written

$$\begin{aligned}
H_{\text{eff}} = \sqrt{\tfrac{1}{2}}\, G_{\mathrm{F}}\big[& g_{\mathrm{VL}}^{(0)} V_\lambda^{(0)} \bar{e}\gamma^\lambda(1-\gamma_5)\mu + g_{\mathrm{VL}}^{(1)} V_\lambda^{(1)} \bar{e}\gamma^\lambda(1-\gamma_5)\mu\\
&+ g_{\mathrm{SL}}^{(0)} S^{(0)} \bar{e}(1+\gamma_5)\mu + g_{\mathrm{SL}}^{(1)} S^{(1)} \bar{e}(1+\gamma_5)\mu\big]\\
&+ \text{right-handed terms}.
\end{aligned} \tag{24}$$

For example, the relation between eq. (24) and eq. (22) is

$$\begin{aligned}
g_{\mathrm{SL}}^{(0)} &= \left[\left(\frac{a_1+a_2}{4}\right) + \left(\frac{a_3+a_4}{4}\right)\right]\left(\frac{f^2}{M_{\mathrm{P}}^2}\right)\frac{\sqrt{2}}{G_{\mathrm{F}}},\\
g_{\mathrm{SL}}^{(1)} &= \left[\left(\frac{a_1+a_2}{4}\right) - \left(\frac{a_3+a_4}{4}\right)\right]\left(\frac{f^2}{M_{\mathrm{P}}^2}\right)\frac{\sqrt{2}}{G_{\mathrm{F}}},\\
g_{\mathrm{VR}}^{(0)} &= \left[\left(\frac{a_1-a_2}{4}\right) + \left(\frac{a_3-a_4}{4}\right)\right]\left(\frac{f^2}{M_{\mathrm{P}}^2}\right)\frac{\sqrt{2}}{G_{\mathrm{F}}},\\
g_{\mathrm{VR}}^{(1)} &= \left[\left(\frac{a_1-a_2}{4}\right) - \left(\frac{a_3-a_4}{4}\right)\right]\left(\frac{f^2}{m_{\mathrm{P}}^2}\right)\frac{\sqrt{2}}{G_{\mathrm{F}}}.
\end{aligned} \tag{25}$$

With the hamiltonian, eq. (24), the branching ratio for μe conversion can be written:

$$\begin{aligned}
R_{\mathrm{eN}} &= \frac{\omega(\mu\mathrm{N}\to\mathrm{eN})}{\omega(\mu\mathrm{N}\to\nu\mathrm{N}')}\\
&= \frac{\omega_{\mathrm{G}}(Z)}{\omega(\mu\mathrm{N}\to\nu\mathrm{N}')}\left[\left(g_{\mathrm{VL}}^{(0)} + g_{\mathrm{SL}}^{(0)}\sqrt{\frac{\omega_{\mathrm{H}}(Z)}{\omega_{\mathrm{G}}(Z)}}\right) + \left(g_{\mathrm{VL}}^{(1)} + g_{\mathrm{SL}}^{(1)}\sqrt{\frac{\omega_{\mathrm{H}}(Z)}{\omega_{\mathrm{G}}(Z)}}\right)\right.\\
&\qquad\left.\times\left(\frac{Z-N}{3A}\right)\right]^2 + \text{right-handed term},
\end{aligned} \tag{26}$$

where $\omega_G(Z)$ and $\omega_H(Z)$ involve integrals over the nuclear charge distribution of the electron and muon wave functions. They are defined in ref. [10], and numerical values are tabulated for a number of elements. If we take $a_1 = a_2 = a_3 = a_4 = 1$, we find the bound $(f/M_P)^2 < 7 \times 10^{-7} \sqrt{\frac{1}{2}} G_F$ from the limit $R_{eN} < 7 \times 10^{-11}$ for sulphur. (For sulphur, $\omega_G/\omega(\mu N \to \nu N') = 153$, and $\sqrt{\omega_H/\omega_G} = 0.95$.) Assuming that $f = \frac{1}{250}$ (average fermion mass ≈ 1 GeV) gives $M_P > 1.5$ TeV. If we assume that $m_f \approx \sqrt{m_\tau m_t}$ we find that $M_P > 11$ TeV. If the experiments can be done with sufficient accuracy, the Z dependence of the coherent μe conversion rate gives information on the nature of the flavour-violating current, and hence on the mechanism of flavour violation. Eq. (26) was derived assuming that the neutron and proton distributions are proportional. While this is a good assumption for the light elements, it is straightforward to do the calculations using neutron distributions calculated from nuclear many-body theory [10], and this should be done if a non-zero rate for μe conversion is found experimentally.

A Feynman diagram contributing to $K_L \to \mu \bar{e}$ is shown in fig. 3b. The effective interaction is of the form

$$\left(\frac{f^2}{M_P^2}\right)\left[a_1\bar{\mu}(1+\gamma_5)d\bar{s}e + a_2\bar{\mu}\gamma_5(1+\gamma_5)d\bar{s}\gamma_5 e + a_1\bar{\mu}(1+\gamma_5)s\bar{d}e \right.$$
$$\left. + a_2\bar{\mu}\gamma_5(1+\gamma_5)s\bar{d}\gamma_5 e\right]. \tag{27}$$

After a Fierz transformation the term contributing to $K_L \to \mu\bar{e}$ is

$$\left(\frac{f^2}{M_P^2}\right)\left[\bar{\mu}\gamma_5(1+\gamma_5)e(\bar{s}\gamma_5 d + \bar{d}\gamma_5 s)\left(\frac{a_1+a_2}{4}\right) - \left(\frac{a_1-a_2}{4}\right)\right.$$
$$\left. \times \bar{\mu}\gamma_\mu\gamma_5(1+\gamma_5)e\left(\bar{s}\gamma^\mu\gamma_5 d + \bar{d}\gamma\mu\gamma_5 s\right)\right]. \tag{28}$$

Assuming $a_1 = a_2$, eqs. (28) and (11) lead to the branching ratio

$$\frac{\Gamma(K_L \to \mu\bar{e})}{\Gamma(K_L \to \text{all})} = 1.25 \times 10^4 \left(\frac{f^2}{G_F M_P^2}\right)^2. \tag{29}$$

Assuming $f \approx \frac{1}{250}$ gives $M_P > 1.8$ TeV. If we assume that the average fermion mass is of order $\sqrt{m_b m_\tau}$ we get the bound $m_P > 5$ TeV. For $K_L \to \mu\bar{\mu}$ one finds from eq. (11) that

$$\frac{\Gamma(K_L \to \mu\bar{\mu})}{\Gamma(K_L \to \text{all})} = 1.1 \times 10^4 \left(\frac{f^2}{G_F M_P^2}\right)^2. \tag{30}$$

The experimental ratio is 9×10^{-9}, and assuming $f = \frac{1}{250}$ gives $M_P > 1.2$ TeV while an average fermion mass of $\sqrt{m_b m_\tau}$ gives 3.5 TeV. For $K_L \to e\bar{e}$ the experimental limit on the branching ratio is 2×10^{-9} and eq. (11) gives

$$\frac{\Gamma(K_L \to e\bar{e})}{\Gamma(K_L \to \text{all})} = 1.4 \times 10^4 \left(\frac{f^2}{G_F M_P^2} \right)^2 . \tag{31}$$

This gives the bound $M_P > 1.8$ TeV for an average fermion mass of 1 GeV and $M_P > 5$ TeV for an average fermion mass of $\sqrt{m_b m_\tau}$. It should be noted that Ellis et al. [16] estimate that the $K_L \to \mu\bar{e}$ rate occurs at the present experimental limit for $M_P \approx 160$ GeV with the average fermion mass of order $\sqrt{m_s m_d} = 110$ MeV. This illustrates the ambiguities that arise because of the hierarchy of fermion masses. Dimopoulos and Ellis [17] also find a bound of order $M_P > 450$ GeV times mixing angle factors from the $K_L \to \mu\bar{e}$ process, using an average fermion mass of order $\sqrt{m_\mu m_s}$. Dimopoulos et al. [18] find a bound of 310 TeV from the same process, assuming vector leptoquarks. These references do not study the other processes mediated by leptoquarks. If the pseudoscalar current term in eq. (28) cancels (i.e., if $a_1 = -a_2$) the limits from the K_L processes will be poorer because they will be suppressed by lepton masses. The $K_L \to e\bar{e}$ will get the most severe suppression. The ratios of the rates for $K_L \to e\bar{e}$, $\bar{\mu}e$ and $\bar{\mu}\mu$ contains information on the mechanism of flavour violation. For example, if the mechanism is horizontal gauge bosons, the ratio of $K_L \to e\bar{e}$ and processes containing muons is of order m_e^2/m_μ^2. In the mechanisms studied in this paper the ratio is of order one (unless the helicity structure of the currents causes a cancellation of the coefficient of the pseudoscalar current. If the cancellation occurs, the ratio is again of order m_e^2/m_μ^2).

Fig. 3b also contributes to $K^+ \to \pi^+ \mu\bar{e}$. The effective four-fermion interaction is of the form

$$\frac{f^2}{M_P^2} (\bar{\mu} d \bar{s} e - \bar{\mu}\gamma_5 d \bar{s}\gamma_5 e) . \tag{32}$$

After a Fierz transformation the above interaction becomes

$$\frac{f^2}{2M_P^2} (\bar{\mu}\gamma_\mu e \bar{s}\gamma^\mu d - \bar{\mu}\gamma_\mu\gamma_5 e \bar{s}\gamma^\mu\gamma_5 d) . \tag{33}$$

The axial vector current does not contribute to $K^+ \to \pi^+ \mu\bar{e}$. Using the calculation of Herczeg [6] gives the branching ratio

$$\frac{\Gamma(K^+ \to \pi^+ \mu\bar{e})}{\Gamma(K^+ \to \text{all})} = 0.32 \left(\frac{f^2}{G_F M_P^2} \right)^2 . \tag{34}$$

The experimental bound of 7×10^{-9} gives the bound $M_P>100$ GeV for $f\approx$ (1 GeV/250 GeV) and $M_P>300$ GeV for $f\approx(\sqrt{m_b m_\tau}/250$ GeV). The bounds from this process are poorer than the bounds from the other processes considered above, as expected. In general one gets contributions from scalar and tensor currents also. These contributions should not change the bounds by an order of magnitude. Since this process involves the measurement of three charged particles, their energy spectra should contain useful information on the structure of the muon number violating currents. However, if muon number is violated, it will probably be discovered in other processes first. The feasibility of getting information from $K^+\to\pi^-\mu\bar{e}$ would then have to be studied, based on the level at which muon number violation occurs.

In conclusion, we see that μe conversion and the three K_L decays give comparable bounds on the masses of flavour-violating scalar leptoquarks. The mass bounds are listed in table 1. In fact, the bounds are somewhat larger than the estimated masses of these particles in technicolour models, $\approx$ 160 GeV. However, the theoretical uncertainties in technicolour model predictions are large enough that this violation of the bounds is not significant. If dynamical symmetry breaking is indeed realised in nature, and if present ideas regarding the phenomenology of this breaking mechanism are correct, then it looks as if muon number violation should occur in μe conversion and $K_L\to\mu\bar{e}$ at levels close to present experimental limits.

4. Vector leptoquarks

In the Pati-Salam type of models [19] there exist vector gauge bosons that mediate transitions between quarks and leptons. The masses of these particles are expected to be of order $10-10^3$ TeV, and they mediate all the processes considered in the previous section. The Feynman diagrams are the same as in fig. 3, with the pseudoscalar P replaced by a vector particle G, and the Yukawa coupling f replaced by a gauge coupling g, with $g^2=O(\alpha)$. In this section I will consider the bounds on the masses of these vector leptoquarks. Nieves [20] has made a general classification of flavour violating currents in the standard schemes of grand unification. He finds that the leptoquarks in these schemes are ultraheavy, since they mediate proton decay. In the Pati-Salam type of theories, however, the contribution of leptoquarks to proton decay is severely suppressed [21], and hence the range of $10-10^3$ TeV for leptoquark gauge boson masses does not violate the bound from the proton decay lifetime limit.

For μe conversion, fig. 3a gives an effective interaction of the form:

$$\left[a_1\bar{e}\gamma_\mu(1-\gamma_5)u\bar{u}\gamma^\mu\mu+a_2\bar{e}\gamma_\mu\gamma_5(1-\gamma_5)u\bar{u}\gamma^\mu\gamma_5\mu+a_3\bar{e}\gamma_\mu(1-\gamma_5)d\bar{d}\gamma^\mu\mu\right.$$
$$\left.+a_4\bar{e}\gamma_\mu\gamma_5(1-\gamma_5)d\bar{d}\gamma^\mu\gamma_5\mu\right]g^2/M^2. \tag{35}$$

After a Fierz transformation, the above can be put into the form of eq. (24) with

$$g_{\mathrm{SR}}^{(0)} = [(a_1 - a_2) + (a_3 - a_4)]\left(\frac{g^2}{M^2}\right)\frac{\sqrt{2}}{G_{\mathrm{F}}},$$

$$g_{\mathrm{SR}}^{(1)} = [(a_1 - a_2) - (a_3 - a_4)]\left(\frac{g^2}{M^2}\right)\frac{\sqrt{2}}{G_{\mathrm{F}}},$$

$$g_{\mathrm{VL}}^{(0)} = -\,[(a_1 + a_2) + (a_3 + a_4)]\frac{1}{2}\left(\frac{g^2}{M^2}\right)\frac{\sqrt{2}}{G_{\mathrm{F}}},$$

$$g_{\mathrm{VL}}^{(1)} = -\,[(a_1 + a_2) - (a_3 + a_4)]\frac{1}{2}\left(\frac{g^2}{M^2}\right)\frac{\sqrt{2}}{G_{\mathrm{F}}}. \tag{36}$$

The branching ratio for μe conversion is given by eq. (26). With $a_1 = a_2 = a_3 = a_4 = 1$, and $g^2 = 0.01$, the branching ratio limit of 7×10^{-11} for sulphur gives the bound $M > 60$ TeV.

For the process $\mathrm{K_L} \to \mu\bar{\mathrm{e}}$ the effective interaction is of the form

$$\left(\frac{g^2}{M^2}\right)\Big[a_1\bar{\mu}\gamma_\mu(1-\gamma_5)d\bar{s}\gamma^\mu e + a_2\bar{\mu}\gamma_\mu\gamma_5(1-\gamma_5)d\bar{s}\gamma^\mu\gamma_5 e$$
$$+ a_1\bar{\mu}\gamma_\mu(1-\gamma_5)s\bar{d}\gamma^\mu e + a_2\bar{\mu}\gamma_\mu\gamma_5(1-\gamma_5)s\bar{d}\gamma^\mu\gamma_5 e\Big]. \tag{37}$$

After a Fierz transformation the above can be written in the form of eq. (8) with $l_1 = \mu$, $l_2 = \mathrm{e}$ and

$$f_{\mathrm{P}} = -f'_{\mathrm{P}} = (-a_1 + a_2)\frac{g^2}{M^2}\frac{\sqrt{2}}{G_{\mathrm{F}}},$$

$$f_{\mathrm{A}} = -f'_{\mathrm{A}} = -\left(\frac{a_1 + a_2}{2}\right)\frac{g^2}{M^2}\frac{\sqrt{2}}{G_{\mathrm{F}}}.$$

If we set $a_2 = -a_1 = 1$ and use eq. (11), we find for the $\mathrm{K_L} \to \mu\bar{\mathrm{e}}$ branching ratio:

$$\frac{\Gamma(\mathrm{K_L} \to \mu\bar{\mathrm{e}})}{\Gamma(\mathrm{K_L} \to \mathrm{all})} = \left(\frac{g^2}{M^2 G_{\mathrm{F}}}\right)^2 2 \times 10^5. \tag{38}$$

The experimental limit 2×10^{-9} implies that $M > 93$ TeV. The process $\mathrm{K_L} \to \mu\bar{\mu}$ gives the bound $M > 62$ TeV while $\mathrm{K_L} \to \mathrm{e}\bar{\mathrm{e}}$ gives a bound of 95 TeV. If $a_1 = a_2 = 1$, the processes $\mathrm{K_L} \to \bar{\mu}\mathrm{e}$, $\mathrm{K_L} \to \mu\bar{\mu}$ and $\mathrm{K_L} \to \mathrm{e}\bar{\mathrm{e}}$ give the bounds $M > 17$ TeV, 14 TeV and 1 TeV respectively. The bounds are poorer in this case because the coefficient of the pseudoscalar current becomes zero. The contribution of the axial vector current is suppressed by lepton masses as discussed below eq. (10).

The interaction leading to $K^+ \to \pi^+ \mu \bar{e}$ could take the form:

$$\frac{g^2}{M^2}\left(\bar{\mu}\gamma_\mu d\bar{s}\gamma^\mu e + \bar{\mu}\gamma_\mu\gamma_5 d\bar{s}\gamma^\mu\gamma_5 e\right).$$

For convenience I have assumed a Fierz invariant form, so the Fierz transformation gives

$$\frac{g^2}{M^2}\left(\bar{\mu}\gamma_\mu e\bar{s}\gamma^\mu d + \bar{\mu}\gamma_\mu\gamma_5 e\bar{s}\gamma^\mu\gamma_5 d\right).$$

In general one should calculate the contribution of scalar and tensor terms also, but the above should suffice for an order of magnitude estimate. Using Herczeg's formula [6] one finds

$$\frac{\Gamma(K^+ \to \pi^+ \mu \bar{e})}{\Gamma(K^+ \to \text{all})} = 1.3\left(\frac{g^2}{G_F M_P^2}\right)^2$$

and the experimental bound, 7×10^{-9}, gives $M_P > 3.5$ TeV for $g^2 \approx 0.01$.

For vector leptoquarks μe conversion and the K_L decays give comparable bounds. The bounds are listed in table 1. The bounds from the K_L decays become much poorer if a cancellation occurs for the pseudoscalar term in the interaction. The mass bounds lie within the range of mass expectations for the leptoquark gauge bosons. However, this range is so large that it could easily accomodate a factor of 10 increase in the phenomenological mass bounds (such an increase would require improving present experimental bounds by a factor of 10^4!).

5. Conclusions

General features of neutral Higgs particle-fermion Yukawa couplings, and the relation of these couplings to the charge of the fermion, were discussed. Some assumptions regarding the Higgs-fermion Yukawa couplings that were implicit in the work of Lahanas and Vayonakis [4] were pointed out. The bounds on the masses of flavour-changing Higgs particles from different processes were calculated, and the results are shown in table 1. The $K_L - K_S$ mass difference gives a very stringent bound. The other bounds would become relevant if the Higgs particle contribution to the $K_L - K_S$ mass difference is suppressed by a natural mechanism.

The $K_L - K_S$ mass difference does not severely constrain leptoquark couplings. The μe conversion process and flavour-changing K_L decays give comparable bounds. Technicolour theories require the existence of pseudoscalar leptoquarks whose contribution to these processes should be close to present experimental limits. Flavour violation in μe conversion and K_L decays at presently interesting levels is possible, but not required, by Pati-Salam models which contain vector leptoquarks.

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