Provably secure compilation of side-channel countermeasures: the case of cryptographic “constant-time”

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Side channels

Running a program of physical devices leak information through side channels.

- Light
- Heat
- Sound
- Power
- Time
- ...

- Memory cache
- Branch predictor
- ...

Constant-time programming

Software-based countermeasure against **timing** attacks and **cache** attacks.

Guideline: control-flow and memory accesses should not depend on sensitive data.

Rationale: crypto implementations without this property are vulnerable.

Caveat: wide range of attacker models.

Secure compilation

- Can we reason about “constant-time” at the source level?
- Do compilers preserve “constant-time”-ness?

Counter-example A: emulation of conditional-move

Before

```
int cmove(int x, int y, bool b) {
    return x + (y − x) * b;
}
```

Counter-example A: emulation of conditional-move

Before

```
int cmove(int x, int y, bool b) {
  return x + (y − x) * b;
}
```

After

```
int cmove(int x, int y, bool b) {
  if (b) {
    return y;
  } else {
    return x;
  }
}
```

Counter-example B: double-word multiplication

Before

```
long long llmul(long long x, long long y) {
  return x * y;
}
```

Counter-example B: double-word multiplication

Before

```
long long llmul(long long x, long long y) {
  return x * y;
}
```

After

```
long long llmul(long long x, long long y) {
  long a = High(x);
  long c = High(y);
  if (a | c) {
    /* ... */
  } else {
    return Low(x) * Low(y);
  }
}
```

Counter-example Γ: tabulation

Before

```
char rot13(char x) {
    return 'a' + ((x - 'a' + 13) % 26);
}
```

Counter-example Γ: tabulation

Before

```
char rot13(char x) {
  return 'a' + ((x - 'a' + 13) % 26);
}
```

After

```
char rot13(char x) {
  static char table[26] = "nopqrstuvwxyzabcdefghijklm";
  return table[x - 'a'];
}
```

Counter-example Δ: speculative load introduction

Before

```
if (false) {
  let x = *ptr;
  ... x ...
}
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```

Good news…

Some compilers do preserve "constant-time"-ness.

Let's prove it (very formally)!

Case studies:

- Constant folding
- Constant propagation
- Variable spilling
- Expression flattening
- Loop peeling
- Pull common instructions out of branches
- Swap independent instructions
- Linearization

A non-interference property

Decorate the small-step relation with a *leakage*: $\mathrm{a} \xrightarrow{\ell} \mathrm{b}$

A non-interference property

Decorate the small-step relation with a *leakage*: $\mathrm{a} \xrightarrow{\ell} \mathrm{b}$

Definition (Constant-time)

For every two execution prefixes

$$i \xrightarrow{\ell_0} s_0 \xrightarrow{\ell_1} s_1 \xrightarrow{\ell_2} s_2 \cdots$$

$$i' \xrightarrow{\ell'_0} s'_0 \xrightarrow{\ell'_1} s'_1 \xrightarrow{\ell'_2} s'_2 \cdots$$

the leakages agree whenever the inputs agree:

$$\varphi(i, i') \Longrightarrow \ell_0 \cdot \ell_1 \cdot \ell_2 = \ell'_0 \cdot \ell'_1 \cdot \ell'_2$$

Leakage?

Any combination of:

- tick per step
- branching conditions
- dereferenced addresses
- arguments of arithmetic operators (division, shift, etc.)
- content of freed memory
- …

Compiler correctness & simulation diagrams

Given a relation $\approx$ between source and target execution states,

if initial states (for the same input values) are in relation

if related final states yield the same result

If the following diagram holds

$$\begin{tikzcd} a \arrow[r] \arrow[d, dash, dotted, "\approx"'] & b \arrow[d, dash, dotted, "\approx"] \\ \alpha \arrow[r] & \beta \end{tikzcd}$$

then the compiler is correct

(moreover, the $\approx$ relation is a relational invariant of any two related executions).

Lockstep 2-simulation

- Each target step is related by the simulation proof to a source step.
- Use this relation to justify that the target leakage is benign.
- Take two instances of the simulation diagram with equal source leakage; and prove that target leakages are equal:

$$
\begin{tikzcd}
& a' \arrow[rr, "t"] \arrow[dd, dotted, "\approx"'] & & b' \arrow[dd, dotted, "\approx"] \\
a \arrow[rr, "t"] \arrow[dd, dotted, "\approx"'] & & b \arrow[dd, dotted, "\approx"'] & \\
& \alpha' \arrow[rr, "\color{red}\tau"] & & \beta' \\
\alpha \arrow[rr, "\color{red}\tau"] & & \beta &
\end{tikzcd}
$$

Lockstep 2-simulation

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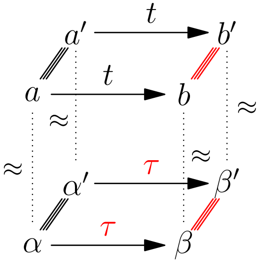


Use relations $\equiv$ between states to link the two executions.

Many-steps simulation

- Some compilation passes require a more general simulation diagram

$$\begin{tikzcd} a \arrow[r] \arrow[d, dotted, "\approx"'] & b \arrow[d, dotted, "\approx"] \\ \alpha \arrow[r, "+"] & \beta \end{tikzcd}$$

Many-steps simulation

- Some compilation passes require a more general simulation diagram

$$
\begin{tikzcd}
a \arrow[r] \arrow[d, dash, dotted, "\approx"'] & b \arrow[d, dash, dotted, "\approx"] \\
\alpha \arrow[r, "+"] & \beta
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
a \arrow[r] \arrow[d, dash, dotted, "\approx"'] & b \arrow[d, dash, dotted, "\approx"'] \arrow[dr, dash, dotted, "\approx"] & \\
\alpha \arrow[r, "+"] & \beta \arrow[r, "+"] & \beta'
\end{tikzcd}
$$

- **Issue**: how to (universally) quantify over instances of this diagram?
- Complying with hypotheses and conclusions is not enough

Many-steps simulation

- Some compilation passes require a more general simulation diagram

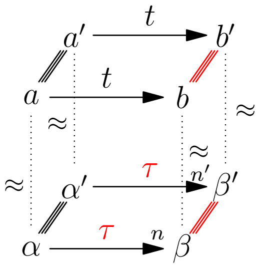
$$
\begin{tikzcd}
a \arrow[r] \arrow[d, dash, dotted, "\approx"'] & b \arrow[d, dash, dotted, "\approx"] \\
\alpha \arrow[r, "n"] & \beta
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
a \arrow[r] \arrow[d, dash, dotted, "\approx"'] & b \arrow[d, dash, dotted, "\approx"'] \arrow[dr, dash, dotted, "\approx"] & \\
\alpha \arrow[r, "+"] & \beta \arrow[r, "+"] & \beta'
\end{tikzcd}
$$

- **Issue**: how to (universally) quantify over instances of this diagram?
- Complying with hypotheses and conclusions is not enough
- Explicitly state the number of target steps: use a function “$n = \text{num-steps}(a, \alpha)$”

  and prove the simulation diagram for this number of steps

Many-steps 2-simulation

- The 2-diagram then generalizes to many-steps:



- **NB**: also works for $n, n' = 0$ (the *size* of the source state needs to strictly decrease)

Take-away

- A general theorem to reduce constant-time preservation to one diagram.
- Builds atop correctness proofs.
- Constant-time preservation is usually (much) simpler to prove.
- Can be instantiated to several leakage/adversary models.
- Many transformations are actually secure.
- Direct proof vs. translation validation is irrelevant

  (we prove that all correct runs of the transformation are secure).