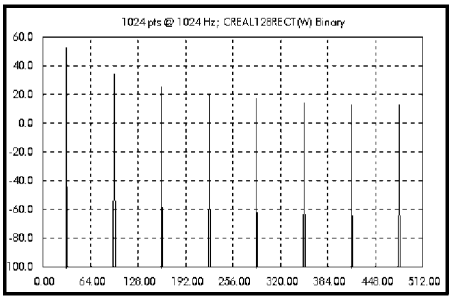
Matched filters

Since filtering is a frequency selective operation, if we know the frequency content of the desired signal then we might be able to be a bit more clever in defining the filter's frequency response.

Take the case of a triangle wave. The frequency response is distinctive: only the odd harmonics are present.

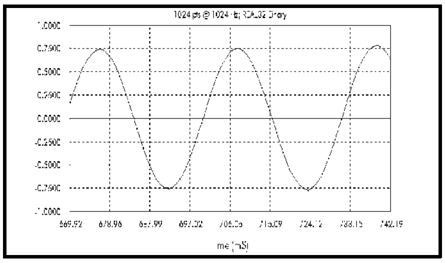


A triangle wave has only odd harmonics

If we applied a band pass filter centred on the frequency of the triangle wave, the signal to noise would be improved because all the noise power outside the band would be eliminated. But there is significant signal power at the first few odd harmonics, and this has also been eliminated.

In fact, to make the best of the signal to noise we would want to include all frequency bands where the signal is large compared with the noise, and eliminate all frequency bands where the noise is larger than the signal.

Also, by accepting only the primary frequency component we are eliminating useful information about the shape of the signal - so the signal will be distorted.


Filtering a triangle wave by passing only the primary component distorts the signal to look more like a sine wave

Actually, since a narrow bandpass filter forces the signal to have a narrow spectrum, the filtered waveform will look very much like a sine wave.

The first thing to try would be a filter that passed only frequencies at the odd harmonics of the triangle wave. Such a filter would have to have several passbands, and several stopbands, and is called a 'multiband' filter.

The multiband filter passes all the frequency components that make up the signal, but still eliminates the noise that lies between these signal frequencies. So it correctly represents the signal. But the signal to noise is worse.

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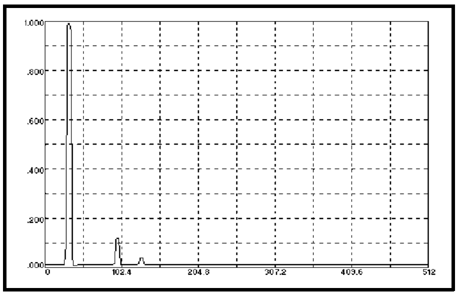
The amount of noise passed by a filter is proportional to the noise under each passband. The amount of signal passed is similarly proportional to the signal under each passband. Although the multiband filter passes all the signal frequency components, the higher harmonics have lower amplitude while the noise remains constant. So the signal to noise ratio is reduced.

The multiband filter represents the signal more correctly, but at the expense of signal to noise.

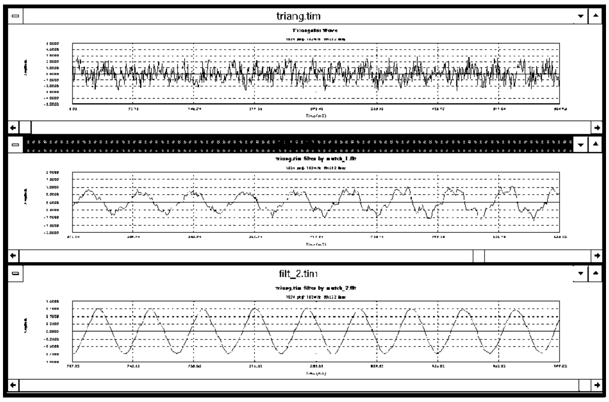
One way to tackle this problem is to weight the passbands to reflect the relative signal amplitude within each band. In other words, to accept that most of the information about a triangle wave is in its primary, and progressively less in the higher harmonics.



A matched filter assigns weights to the various signal frequency components

The filter can be matched to the signal frequencies, and their relative importance, by matching the passband gain to the relative amplitude of the signal harmonic within the band. Such a filter is called a 'matched' filter.

Because the matched filter gives different weight to different signal frequency components, it does actually still distort the signal. But in doing so it gains about as much signal to noise as there is to be had. So it represents a useful compromise in may instances.



Matched filtering.
Top: a very noisy triangle wave
Middle: filtered with only odd harmonics
Bottom: filtered with a matched filter

As usual, there is a trade off between signal to noise and signal fidelity.