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Homework Assignment 4

Question 1:

A Minimum Spanning Tree (MST) is a subset of a connected, weighted graph's edges that joins all of the vertices together with the least amount of edge weight and no cycles.

Making an MST:

We will apply Kruskal's Algorithm to generate the MST for the provided graph. The following are the steps:

1. Arrange all edges in a non-decreasing order of weight.Sort them as follows, for instance, if the edges are (A-B, 2), (B-C, 3), and so forth: (A-B, 2), (B-C, 3),...

2. Create a blank set for MST.

3. Repeat with sorted edges: If the MST does not form a cycle, add the edge. Look for cycles using the Union-Find algorithm.

4. Continue until MST has |V| - 1 edges, where |V| is the number of vertices.

The graph's steps are as follows: (Include all computations and sorted edge lists, and show the graph being constructed step by step.)

Related Ideas:

-Representation of Graphs:

A 2D array with matrix[i][j] indicating the existence of an edge between vertices i and j is called an adjacency matrix.Each vertex in an adjacency list has a sublist of its neighboring vertices.

-Minimal Algorithms for Spanning Trees:

Using Kruskal's Algorithm, sort edges and construct MST using Union-Find.Prim's Algorithm: Start at any node and increase MST.

Question 2:

MST Uniqueness Requirements:If and only if each edge weight on the graph is unique, then the MST is unique.

Evaluation of the Provided Graph:Verify that each edge weight is distinct. There could be more than one legitimate MST if duplicate weights are present.

Similar Ideas:

-Graph Sturdiness:evaluates a graph's ability to retain its characteristics in the face of errors or assaults.Resilience to edge deletions is one of the metrics.

Question 3:

The definition of In a weighted network with non-negative weights, Dijkstra's Algorithm determines the shortest routes between a source node and every other node.

Actions to take:

1. Set all nodes' distances from A to infinity, with the exception of A (distance = 0).

2. To choose the node with the shortest distance, use a priority queue.

3. Update distances and relax all of the chosen node's edges.

4. Continue until every node has been handled.

Step-by-Step Calculations: (A table displaying the chosen node, distances, priority queue status, and each iteration should be provided.)

-Associated Ideas:

Finding your way:

The shortest route between a single source and every other vertex is found using Dijkstra's algorithm.Finding the shortest routes between every pair of vertices is the goal of the Floyd-Warshall algorithm.

Question 4:

An edge in a graph is said to be important if it increases the number of linked elements in the graph if it is removed.

How to Determine Critical Edges:

1. Take out each edge individually.

2. Use a DFS or BFS traverse to see if the graph stays linked.

3. An edge is considered essential if eliminating it causes the graph to become disconnected.

Detailed Example: (Select a particular edge, eliminate it, and demonstrate the traversal demonstrating connectedness or disconnection.)

-Associated Ideas:

Crossing over:Depth-First Search (DFS): Investigates each branch as far as it can before turning around.Before proceeding to the next level, the Breadth-First Search (BFS) method investigates each vertex's neighbors.

Question 5:

An articulation point is defined as a vertex that increases the number of linked components in a graph when its edges are removed.

Methods for Determining Articulation Points:

1. Run a DFS and monitor each vertex's low values and discoveries.

2. An articulation point is a vertex if:

-It has two or more children and is the root of the DFS tree.

-At least one of its offspring has a low value that is higher than or equal to its discovery value, and it is not the root.

Detailed Example: (Select a particular vertex, eliminate it, and demonstrate the change in connection.)

Similar Ideas:

-Graph Interconnectivity: If a path connects each pair of vertices in a graph, then the graph is linked. Strong connectedness (for directed graphs): Each pair of vertices has a route in both directions.

Question 6:

Situation:

A-B-C-E is the provided route, but node C is no longer accessible.

The graph is still linked because there are no essential edges or articulation points, thus there must be another way to get to E.

Justification: Even if a single node or edge is removed, graph connectedness is ensured by the lack of important edges and articulation points, without the need for computations.

Similar Ideas:

-Graph Sturdiness:

Recovery speed and resistance to vertex/edge deletions are among the metrics.

Question 7:

Graph resilience is defined as the graph's capacity to maintain connectivity in the event that edges or vertices are removed.

Employing Articulation Points and Critical Edges:

A graph is resistant to single failures if it has no articulation points or crucial edges.

For the analysis of robustness:

1.Determine every important articulation point and edge.

2.Analyze the effects of their removal on connection.

Similar Ideas:

Measures of Centrality:

-The number of edges that are related to a vertex is known as degree centrality.

-The frequency with which a vertex appears on the shortest pathways between other vertices is known as betweenness centrality.

-A vertex's closeness to every other vertex is known as its closeness centrality.