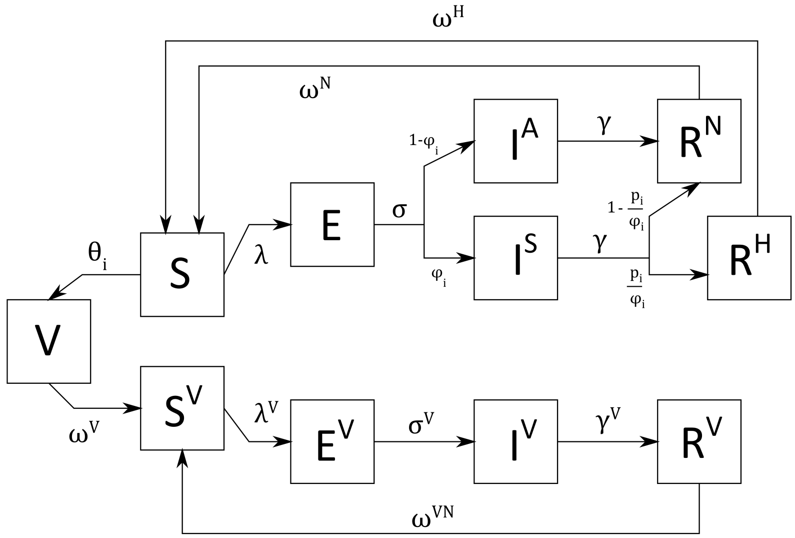

Susceptible

$$S_{t+1} = S_t(1 - \theta_i - \lambda_{t+1}) + f_t(R^N; o, \omega^N) + f_t(R^H; o, \omega^H)$$

$$S^V_{t+1} = S^V_t(1 - \lambda^V_{t+1}) + f_t(R^V; s, \omega^V) + f_t(V; s, \omega^V)$$

Vaccinated

$$V_{t+1} = V_t + S_t\theta_i - f_t(V; s, \omega^V)$$

Exposed/infected

$$E_{t+1} = E_t + S_t\lambda_{t+1} - f_t(E; m, \sigma)$$

$$E^V_{t+1} = E^V_t + S_t\lambda^V_{t+1} - f_t(E^V; q, \sigma^V)$$

Infectious

$$I^A_{t+1} = I^A_t + (1 - \phi_i)f_t(E; m, \sigma) - f_t(I^A; n, \gamma)$$

$$I^S_{t+1} = I^S_t + \phi_i f_t(E; m, \sigma) - f_t(I^S; n, \gamma)$$

$$I^V_{t+1} = I^V_t + f_t(E^V; q, \sigma^V) - f_t(I^V; r, \gamma^V)$$

Recovered

$$R^N_{t+1} = R^N_t + f_t(I^A; n, \gamma) + (1 - \tfrac{p_i}{\phi_i})f_t(I^S; n, \gamma) - f_t(R^N; o, \omega^N)$$

$$R^H_{t+1} = R^H_t + \tfrac{p_i}{\phi_i}f_t(I^S; n, \gamma) - f_t(R^H; o, \omega^H)$$

$$R^{VH}_{t+1} = R^{VH}_t + f_t(I^V; r, \gamma^V) - f_t(R^V; s, \omega^V)$$

Parameters

Table 1 summarises all parameters.

Table 1: Summary of parameters.

Symbol	Parameter
R_0	basic reproduction number
R_c	reproduction number under control measures for an entirely susceptible population
R_t	effective reporduction number in the presence of immunity
$\theta_{i,t}$	time-specificprobability of vaccination
σ	latency period mean
m	latency period shape non-vaccinated
q	latency period shape vaccinated
γ	infectious period mean
n	infectious period shape non-vaccinated
r	infectious period shape vaccinated
ω^N	immune duration mean non-hospitalized non-vaccinated case
ω^H	immune duration mean hospitalized non-vaccinated case
o	immune duration shape non-vaccinated
ω^{VN}	immune duration mean non-hospitalized vaccinated case
ω^{VH}	immune duration mean hospitalized vaccinated case
ω^{VH}	immune duration mean vaccinated individual
s	immune duration shape vaccinated
ϕ_i	probability of asymptomatic case given infection (non-vaccinated)
τ_i	probability of asymptomatic case given infection (vaccinated)
p_i	probability of hospitalisaion given infection (non-vaccinated)
p_i^V	probability of hostpitalisation given infection (vaccinated)
$C = c_{i,j}$	contact matrix
η_i	relative susceptibility of age group i
υ	relative infectiousness of asymptomatic cases
μ	relative infectiousness of vaccinated cases (symptomatic and asymptomatic??)

Next generation matrix

- Expected number of secondary infections in age group i resulting from contact with an index case in age group j:
  $k_{ij} = \frac{\beta}{\gamma}\eta_i c_{i,j}(\phi\nu + (1-\phi_i))$
  - η_i = relative susceptibility of of age group i;
  - ν = relative infectiousness of asymptomatic cases.
  - $c_{i,j}$ = average number of daily contacts between a single individual in age group j and all individuals in age group i;
- The basic reproduction number R_0 is given by the spectral radius $\rho(K)$ = the largest absolute eigenvalue of K.
- As R_0 is specified in the model, the transmission parameter β is left as a free parameter that is scaled to the correct value.

Force of infection

- $\lambda_{i,t+1}$ = force of infection acting on a single individual in age group i at time $t+1$:
  $\lambda_{i,t+1} = \beta\eta_i \sum_{j=1}^{M} \frac{c_{i,j}N_j}{N_i}(\frac{I_{j,t}^S + I_{j,t}^A\nu}{N_j})$ $\lambda_{i,t+1} = \frac{\beta\eta_i}{N_i}\sum_{j=1}^{M} c_{i,j}(I_{j,t}^S + I_{j,t}^A\nu)$
  - M = number of discrete age groups ($M = 15$);
  - N_i = population size of age group i.

Model Assumptions

- Model assumes that individuals are vaccinated and gain immunity at some rate (probability) constnant per age group θ_i.
  - Could make this $f_t(...)$?
- E^V, I^V, and R^V have their own F_t parameters which differ between vaccinated and non-vaccinated individuals.
- Vaccinated individuals who recover from infection "replenish" their vaccination immunity, *i.e.*, R^V flows into V.

No longer sure about these assumptions...

- E^V flows into own I^V compartment, allowing adjustment of infectiousness.
  - Implicitly, vaccination protects against symptomatic and/or asymptomatic disease.
- There end up being two different forces of infection for vaccinated and unvaccinated people: λ^V and λ^N. It would be better to formulate a more complex expression for λ_t which accounts for vaccination.

Questions to answer

- Are vaccinated individuals less susceptible to infection than non-vaccinated ones?
- Does vaccinatoin protect against symptomatic disease? (Rather what is the proportion of asymptomatic cases in the infections reported by vaccinated individuals?)
- Do infected vaccinated individuals "replenish" their immunity upon recovery from infection?
  - Is the effect the same for symptomatic and asymptomatic cases?
- Does vaccination protect against hospitalisation?
- Do the R boxes contribute to transmission?

To do

- Are vaccinated individuals more suscseptible to infection than non-vaccinated, *i.e.* $S^V = S$?
  - If $S^V \neq S$, then $\lambda^V \neq \lambda$.
  - Then we need to change η_i, the relative susceptibility by age group.
- Vaccine distribution (and therefore immunity) $\sim$ age group. Therefore, immunity will differ by age group.
  - If immunity differs by age group susceptibility might too.
- Find **probability** of *asymptomatic case* given *vaccinated infection/case*
- Find **probability** of *hospitalisation* given *symptoamtic vaccinated case/infection*
  - Oxford paper has plenty of data which might be useful. (Intercept, odds -> probabilities.)

Possible angles

- Future of the pandemic i.e. what the winter will look like.
- Booster shot vs no booster shot ~ impact on transmission
- Waning immunity vs immune evasion vs increased contact *i.e.* how much vaccinated people transmit Delta?
  - Implicaions: vaccine passports & booster shots
- Herd immunity with imperfect vaccination?