Lecture 7
Hypothesis Tests for a Population Mean

One sample z-test for a population mean (σ known)

One sample t-test for a population mean (σ unknown)

In the Last Lecture

- A **95% confidence interval** is an interval which we believe, with 95% confidence, will contain the value of the population parameter of interest.
- The general form for a confidence interval is:

$$sample\ estimate\ \pm critical\ value\ \times standard\ error$$

- We calculated confidence intervals for μ and for π.
- In order to **decrease the width** of a confidence interval but at the same time retain the level of confidence (we have used 95%), then we would need to **increase the sample size**.

In the Last Lecture

	Validity (assumption checks)	**95% CI**
95% CI for μ (σ known):	Check sample drawn from a normal distribution or sample size large enough for CLT to apply	$\bar{y} \pm 1.96 \times {}^{\sigma}/_{\sqrt{n}}$
95%CI for μ (σ unknown)	Check sample drawn from normal distribution or, (provided distribution not highly skewed), sample size large enough for distribution to be robust against non normality	$\bar{y} \pm t_{crit} \times {}^{s}/_{\sqrt{n}}$
95% CI for π (approx):	Check sample large enough for CLT to apply: $np \geq 5$ and $n(1-p) \geq 5$	$p \pm 1.96 \times \sqrt{\frac{p(1-p)}{n}}$

Review Quiz 1

A random sample of 40 airline passengers at Sydney airport were each asked how long they had to wait in line at the ticket counter. The information was used to calculate the following 95% confidence interval to estimate the population mean: (30.17, 34.27)

a. What was the mean waiting time for the 40 passengers who were sampled?

b. How could you produce a narrower 95% confidence interval for the population mean?

c. If you were asked to provide a 99% confidence interval for the population mean waiting time, what would need to change: $\bar{y} \pm t_{crit} \times {}^{s}/_{\sqrt{n}}$

Solution to Review Quiz 1

Review Quiz 2

The Melbourne Institute of Applied Economic and Social Research's Household, Income and Labour Dynamics in Australia (HILDA) report conducts annual surveys. The latest report found that 3162 people out of a sample of 17000 Australians aged 18 to 64 years, were receiving weekly welfare payments in 2014.

a. Calculate a 95% confidence interval to estimate the proportion of 18 to 64 year olds in Australia receiving weekly welfare payments in 2014.

b. In 2001, 23% of Australians aged 18 to 64 years were receiving weekly welfare payments. Do you think there has been a change since 2001?

Solution to Review Quiz 2

z – test for a population mean

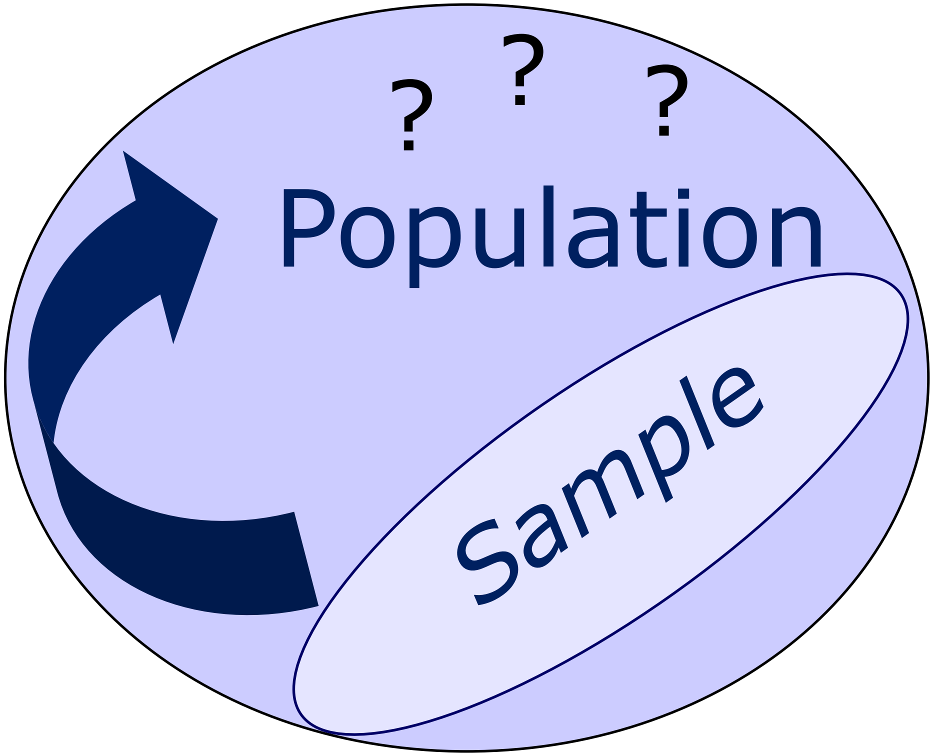
Research Questions

- Recall that we often want to answer a research question pertaining to some target population of interest.
- We design a study carefully, and obtain a representative sample, random if possible, from the target population.
- We use this sample to make inferences about the population and, thus, to answer the research question.

Answering Research Questions

Research Questions

We use a SAMPLE to answer questions about a target POPULATION



Answering a Research Question

Research Question:
Is the mean IQ of children who attend country schools the same as that of the general population?

We will perform a statistical test to answer this research question.

Let Y = variable of interest.

Here Y = IQ scores of country students.

Remember last week's exercise - we used a confidence interval to estimate the mean IQ score among country school children.

We know that in the general population, IQ scores follow a normal distribution with standard deviation, $\sigma = 15$. We will assume again here that this also applies to the target population - country school students in NSW. So the sample mean will also be from a normal distribution, regardless of sample size. We had a random sample of 36 students from country schools in NSW with sample mean, $\bar{y} = 103$.

A Null Hypothesis

- A null hypothesis is a statement (or claim) that a population parameter has a specified value:

  $$H_0: \mu = \mu_0$$

  That is, the null hypothesis states that a population parameter (in this case μ) is **equal** to a specific value (which we are calling μ_0 here).
- For our research question, the **null** hypothesis would claim that the mean IQ score of country school students is the same as the general population ie. equal to 100. This is because the population mean is known to be 100, and we want to determine if this is the same for country school students. ('null' means 'no effect' or 'no change')
- So we express this null hypothesis as: $H_0: \mu = 100$

An Alternative Hypothesis

- We also need to specify an **alternative hypothesis**, $\mathbf{H_1}$, in case we find evidence against the null hypothesis, H_0.
  - If we were **only** prepared to reject a null hypothesis when we found evidence to indicate the population mean was significantly lower than the null value, μ_0, the alternative hypothesis would be: $H_1: \mu < \mu_0$ (known as a one sided test)
  - If we were **only** prepared to reject a null hypothesis when we found evidence to indicate the population mean was significantly higher than μ_0, the alternative hypothesis would be: $H_1: \mu > \mu_0$ (also a one sided test)
- ***In STAT170 we will <u>not</u> use one sided tests – we will only use two sided tests****. These are tests where we are prepared to reject a null hypothesis when we find evidence indicating the population parameter is* ***either*** *higher* ***or*** *lower than μ_0. Therefore, the alternative hypothesis for this test of a population mean will be:*

$$\boldsymbol{H_1: \mu \neq \mu_0}$$

Country School Students

- We now have a null and an alternative hypotheses for a test to answer the research question.

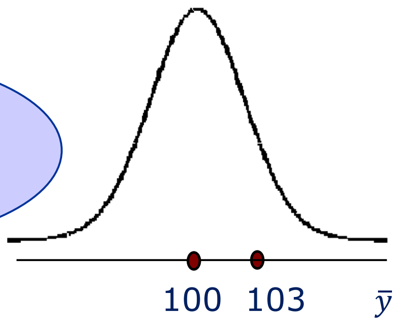
Research Question:
Is the mean IQ of children who attend country schools the same as that of the general population?

$$H_0: \mu = 100$$
$$H_1: \mu \neq 100$$

Checking Test Assumptions

- We need to ensure that a z-test for a population mean will be valid here.
- **The assumption for a z-test for a population mean is that the sample mean is drawn <u>from</u> a normal distribution.**

Distribution of sample means (n = 36)



- This will be a reasonable assumption if the sample is large enough for the Central Limit theorem to apply (which is the case here, although normality had been given). If the sample is not large, we should graph the sample data and if it appears that the sample may be drawn **from** a normal distribution, then we can assume the sample mean should also be drawn **from** a normal distribution.
- *Regardless of sample size, it is always good practice to start your analysis by graphing the data.*

Testing a Null Hypothesis

- To test a null hypothesis for a population mean, we compare the sample value, $\bar{y}$, with the corresponding null value, μ_0.

  For the research question about the IQ scores of country school students, the null value, $\mu_0 = 100$, and the sample mean $\bar{y} = 103$.
- Testing this null hypothesis is the same as asking the question *'if the null hypothesis is true (ie. country students' have IQ scores the same as the general population), how likely is a sample mean to be 3 points (or more) away from the population mean, in **either** direction?"*

A Test Statistic

- A test statistic is a scaled measure of the discrepancy between the observed sample statistic and the value specified by H_0.
- Dividing the discrepancy by the standard error gives a result that is independent of the measurement unit.
- For a one sample test of a mean where Y is the variable of interest, the test statistic takes the form:

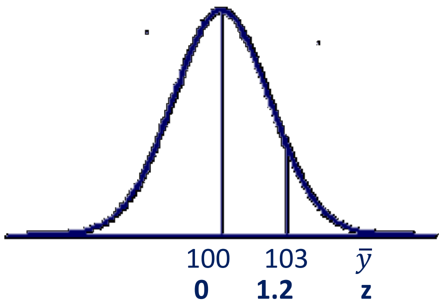
$$\boldsymbol{test\ statistic = \frac{\bar{y} - \mu_0}{st.\,error\ of\ \bar{y}}}$$

So the test statistic tells us how many standard errors the sample mean, $\bar{y}$, is away from the null value, μ_0.

Test Statistic for a One Sample z-test of a Population Mean

So if σ is known, we can use a one sample z-test for a population mean. The test statistic is:

$$z = \frac{\bar{y} - \mu_0}{\sigma / \sqrt{n}}$$



For our example we know the population standard deviation, σ = 15 and we are testing μ_0 = 100 vs $\mu_0 \neq$ 100 . With the sample size n = 36 and the sample mean, $\bar{y}$ =103, the test statistic is:

$$z = \frac{103 - 100}{15 / \sqrt{36}} = 1.2$$

ie. a sample mean IQ of 103 is 1.2 standard errors away from μ_0.

Size of a Test Statistic

- If the absolute value of the test statistic is ***large***, it means that there is a large discrepancy between the null value and the sample value.

- If the test statistic is ***small***, the null value is consistent with the data in the sample. (That is, the discrepancy could be attributable to chance ie. sampling variation)

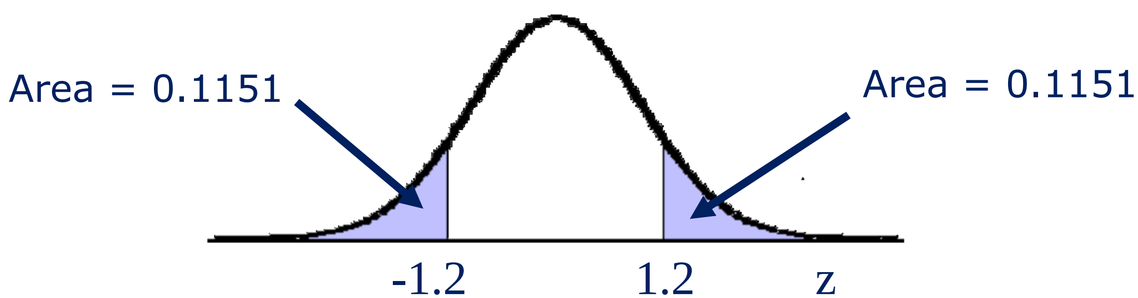
- A more precise measure of the discrepancy is given by a ***p-value***.

What is a p-value?

- **A p-value is the likelihood of getting such a large test statistic (or even larger), assuming that the null hypothesis is true.**
- The test statistic, z = 1.2, is a measure of the discrepancy between the sample mean, $\bar{y} = 103$, and the null value, $\mu_0 = 100$. The p-value will tell us **how likely** we are to select a sample of size 36 which has a sample mean at least 1.2 standard errors away from the null value, if H_0 is true.
- The p-value is the tail area of the distribution of the test statistic, assuming H_0 is true. For a two-sided z test of a population mean, the p-value is the area in ***both*** tails of the z distribution, because the measure of discrepancy could have been either side of the mean.

Using a z-table to Find a p-value

For this research question we have a **test statistic of z = 1.2**. Using the standard normal distribution, the total area in the two tails of the z-distribution to the right of z = 1.2 and to the left of z = −1.2 is 0.2302.

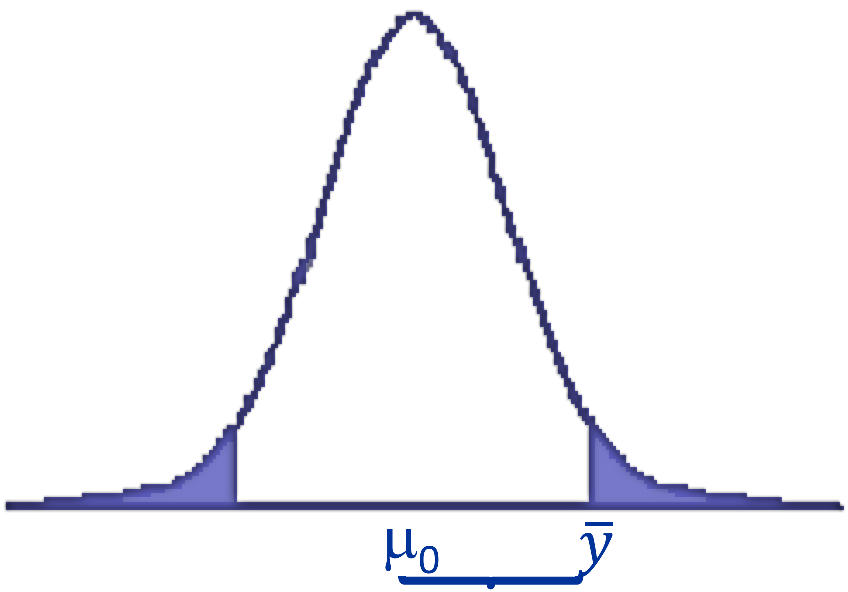


The p-value is 0.2302. It is the probability of getting a z-value with magnitude greater than 1.2 if H_0 is true.

Significance Level and Decisions

- If the ***p-value is small*** the correct decision would be to ***reject the null hypothesis***.
- By convention, a p-value is considered 'small' if it is less than 0.05. (This value of 0.05 we are using is called the ***significance level*** or α ***-level*** of the test.)
- If we reject a null hypothesis, the result is ***'statistically significant'***.
- If a p-value is not small (ie. ≥ 0.05) the correct decision is to ***not reject*** the null hypothesis. **However, this does <u>not</u> mean that H_0 is accepted!**
- ***So, for the country students, the correct decision is to <u>not</u> reject H_0 because the p-value (0.2302) is <u>not less</u> than the 5% significance level***.

When the p-value is Small



The discrepancy between $\bar{y}$ and μ_0 is large

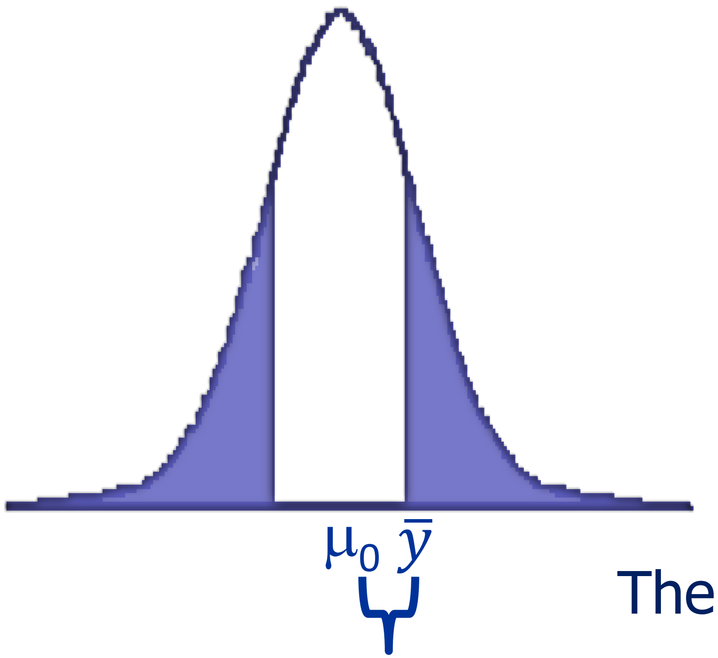
z is large

p-value is small

There is evidence against H_0

Reject H_0

When the p-value is Large



The discrepancy between $\bar{y}$ and μ_0 is small

z is small

p-value is large

There is no evidence against H_0

Do not reject H_0

A Significant Result

using a *significance level* of 5% (ie. $\alpha = 0.05$).

"if p-value < 0.05 , Reject H_0"!

statistically significant result!!!

Writing a Conclusion

- For the country students, we tested $H_0: \mu = 100$ vs $H_1: \mu \neq 100$. With a test statistic of z = 1.2 we obtained a p-value of 0.2302. Since this p-value was ≥ 0.05 we ***did not reject H_0*** ie. we did not get a significant result.
- So what is our conclusion/answer to the question: *Is the mean IQ of children who attend country schools the same as that of the general population?*
- Since we did not reject the null hypothesis we do not have evidence to claim that country school students have a different average IQ score to that of the general population. **We conclude that, *on average, IQ scores could be the same among country school students as the general population.***

Steps in Hypothesis Testing

Hypothesis Test

H

Hypotheses:

State the *null* and the *alternative* hypotheses in terms of the *parameter* of interest.

A

Assumptions:

Check the underlying assumptions of the test.

T

Test Statistic:

Calculate the test statistic.

P

p-value:

Obtain the p-value for the test from the distribution of the test statistic.

D

Decision:

If the p-value is less than 0.05 (the significance level), reject the null hypothesis. If the p-value is not less than 0.05, do not reject the null hypothesis.

C

Conclusion:

Write a conclusion to the original research question in terms of the target population.

Example: One Sample z-test for Population Mean

Research Question:
Is the average stopping distance of school buses travelling at 75 km per hour equal to 80 metres as claimed by public transport authorities?

Studies by public transport authorities have indicated that the average stopping distance of a bus travelling at 75 km per hours is 80 metres. The standard deviation of stopping distances at that speed is known to be 1.2 metres.

Automotive engineers tested a randomly selected sample of 20 school buses and found that the average stopping distance for this sample, when travelling at 75 km per hour, was 80.74 metres.

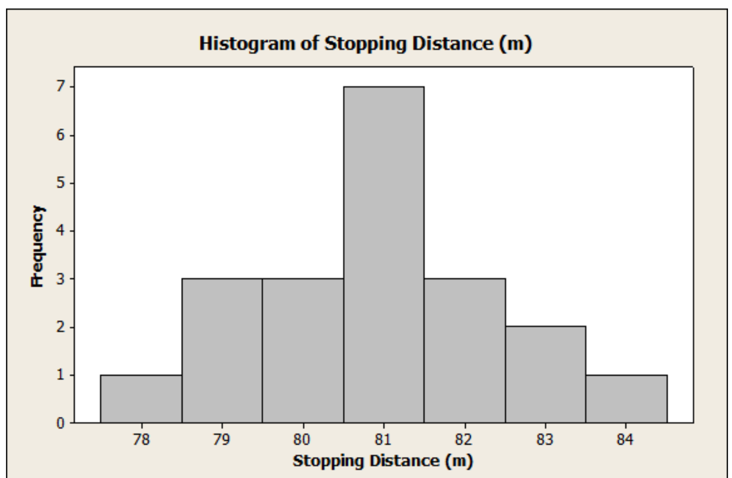
Example: Stopping Distances

Some Sample Information:

Variable of interest:

Y = stopping distance in metres

n = 20, $\bar{y}$ = 80.74 metres



Stopping Distance (m)
78.4
78.5
78.7
79.4
79.8
80
80.4
80.5
80.5
80.5
80.6
80.7
81
81
81.5
81.9
82
82.5
83.3
83.6

Example: Stopping Distances

Hypothesis Test

H H_0: $\mu = 80$

H_1: $\mu \neq 80$

A From the histogram, the normality assumption appears reasonable: the sample appears to be drawn from a normally distributed population.

T $$z = \frac{\bar{y} - \mu}{\sigma / \sqrt{n}} = \frac{80.74 - 80}{1.2 / \sqrt{20}} = 2.76$$

P p-value = 0.0058 (=0.0029×2)

D Since p-value < 0.05, reject H_0

C There is evidence to suggest that the average stopping distance for all school buses is <u>more</u> than 80 metres.

Minitab Output: One Sample z-test for Population Mean

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One-Sample Z: Stopping Distance (m)
Test of mu = 80 vs not = 80
The assumed standard deviation = 1.2

Variable                N    Mean   StDev  SE Mean        95% CI
Stopping Distance (m)  20  80.740   1.445    0.268  (80.214, 81.266)

Variable                Z      P
Stopping Distance (m)  2.76  0.006
```

Quiz 3

Research Question:
Is the average rent for an apartment in Paris \$4292 per month?

It is reported that Paris is the most expensive European city in which to live. A recent report stated that the average rental cost of an apartment in Paris is \$4292 per month. To test this claim, a researcher collected information on 55 randomly selected rental units in Paris. The sample mean was \$4008 per month. Previous studies indicate the population standard deviation is \$386 per month. Carry out an appropriate test to determine whether the population mean could be \$4292 as reported.

$\mu = ?$
$\sigma = \$386$

Source: Black et al (2012) *Australian Business Statistics, Wiley (adapted)*

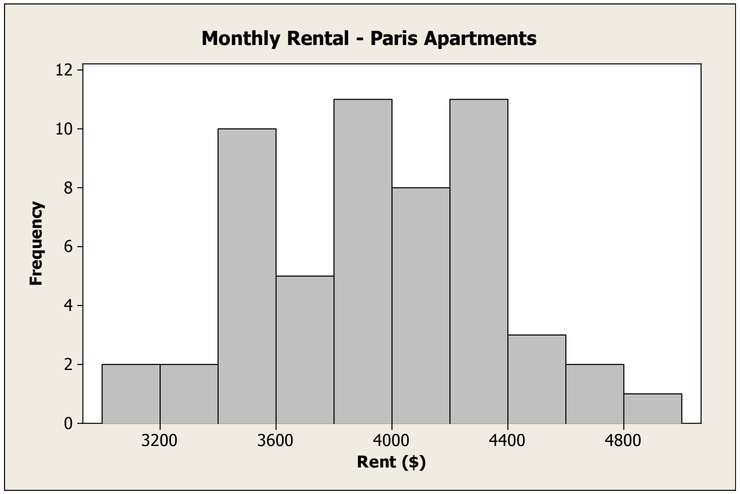
Monthly Rental – Paris Apartments

Some Sample Information:

Variable of interest:

Y = rent of a Paris apartment in dollars

$n = 55$, $\bar{y} = \$4008$



Solution to Quiz 3

Hypothesis Test

H

A

T

P

D

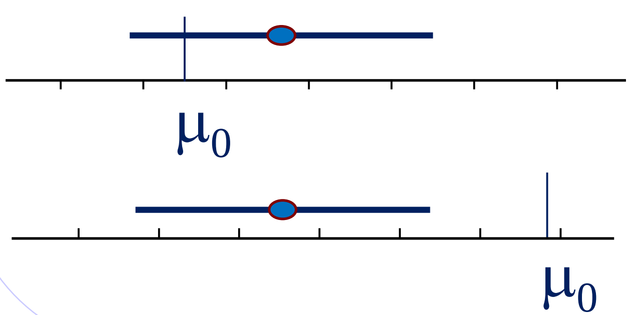
C

p-values and 95% Confidence Intervals

A confidence interval can be used to test a null hypothesis:

- If the **confidence interval does not include the null value**, the decision is '**reject H_0**'.
- If the confidence interval does include the null value, the decision is '**do not reject H_0**'.

A p-value is equivalent to this rule: If the p-value is less than 0.05, H_0 is rejected. If the p-value is greater than 0.05, H_0 is not rejected.



p-value ≥ 0.05 ***do not reject*** **H_0**

p-value < 0.05 ***reject*** **H_0**

p-values and 95% Confidence Intervals

Let's look at the confidence intervals for the population mean in each of the three hypothesis tests we have seen:

- IQ scores of country school students:
  H_0: $\mu = 100$ vs H_1: $\mu \neq 100$ We did not reject H_0. We concluded that the average IQ could be 100
  95% CI for μ = (98.1, 107.9)
- Stopping distances of school buses:
  H_0: $\mu = 80$ vs H_1: $\mu \neq 80$ We did reject H_0. We concluded that the average stopping distance was more than 80m.
  95% CI for μ = (80.2m, 81.3m)
- Monthly rental of Paris apartments:
  H_0: $\mu = 4292$ vs H_1: $\mu \neq 4292$ We did reject H_0. We concluded that the average rental was less than \$4292.
  95% CI for μ = (\$3906, \$4110)

t-test for a population mean

t-test for a Population Mean

- A z-test for a population mean is only used when the population standard deviation (σ) is known. More often, σ is unknown. When σ is unknown, it can be estimated using the sample standard deviation, s.

- When σ is unknown, the z-statistic should be replaced by a t-statistic with ν = n – 1 degrees of freedom (df):

  $t = \frac{\bar{y}-\mu}{s/\sqrt{n}}$ **this is called the *Student's t statistic***

- The p-value for a t-test is obtained from the t-distribution.

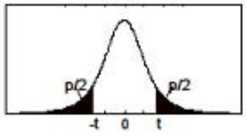
Finding a p-value from a t-table

The top row of the table shows the area in the two tails of the t-distribution. The p-value for a two-sided test is read from the top row. Since the table is very condensed you will usually not be able to find the exact p-value but will only be able to give a range.

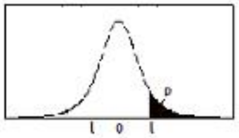
Each row of the table is a separate t-distribution with ν = degrees of freedom given in the left hand column.

Using the appropriate row for ν (round df down if you do not find the exact df), find where the test statistic lies and read the appropriate range for the p-value from the top row.

Two-tailed



One-tailed



Two-tailed	0.0005	0.001	0.002	0.005	0.01	0.02	0.05	0.1	0.2	0.5
One-tailed	0.00025	0.0005	0.001	0.0025	0.005	0.01	0.025	0.05	0.1	0.25
ν										
1	1273	636.6	318.3	127.3	63.66	31.82	12.71	6.314	3.078	1.000
2	44.70	31.60	22.33	14.09	9.925	6.965	4.303	2.920	1.886	0.816
3	16.33	12.92	10.22	7.453	5.841	4.541	3.182	2.353	1.638	0.765
4	10.31	8.610	7.173	5.598	4.604	3.747	2.776	2.132	1.533	0.741
5	7.976	6.869	5.893	4.773	4.032	3.365	2.571	2.015	1.476	0.727
6	6.788	5.959	5.208	4.317	3.707	3.143	2.447	1.943	1.440	0.718
7	6.082	5.408	4.785	4.029	3.499	2.998	2.365	1.895	1.415	0.711
8	5.617	5.041	4.501	3.833	3.355	2.896	2.306	1.860	1.397	0.706
9	5.291	4.781	4.297	3.690	3.250	2.821	2.262	1.833	1.383	0.703
10	5.049	4.587	4.144	3.581	3.169	2.764	2.228	1.812	1.372	0.700
11	4.863	4.437	4.025	3.497	3.106	2.718	2.201	1.796	1.363	0.697
12	4.716	4.318	3.930	3.428	3.055	2.681	2.179	1.782	1.356	0.696
13	4.597	4.221	3.852	3.372	3.012	2.650	2.160	1.771	1.350	0.694
14	4.499	4.140	3.787	3.326	2.977	2.624	2.145	1.761	1.345	0.692
15	4.417	4.073	3.733	3.286	2.947	2.602	2.131	1.753	1.341	0.691
16	4.346	4.015	3.686	3.252	2.921	2.583	2.120	1.746	1.337	0.690
17	4.286	3.965	3.646	3.222	2.898	2.567	2.110	1.740	1.333	0.689
18	4.233	3.922	3.610	3.197	2.878	2.552	2.101	1.734	1.330	0.688
19	4.187	3.883	3.579	3.174	2.861	2.539	2.093	1.729	1.328	0.688
20	4.146	3.850	3.552	3.153	2.845	2.528	2.086	1.725	1.325	0.687
21	4.110	3.819	3.527	3.135	2.831	2.518	2.080	1.721	1.323	0.686
22	4.077	3.792	3.505	3.119	2.819	2.508	2.074	1.717	1.321	0.686
23	4.047	3.768	3.485	3.104	2.807	2.500	2.069	1.714	1.319	0.685
24	4.021	3.745	3.467	3.091	2.797	2.492	2.064	1.711	1.318	0.685
25	3.996	3.725	3.450	3.078	2.787	2.485	2.060	1.708	1.316	0.684
26	3.974	3.707	3.435	3.067	2.779	2.479	2.056	1.706	1.315	0.684
27	3.954	3.690	3.421	3.057	2.771	2.473	2.052	1.703	1.314	0.684
28	3.935	3.674	3.408	3.047	2.763	2.467	2.048	1.701	1.313	0.683
29	3.918	3.659	3.396	3.038	2.756	2.462	2.045	1.699	1.311	0.683
30	3.902	3.646	3.385	3.030	2.750	2.457	2.042	1.697	1.310	0.683
35	3.836	3.591	3.340	2.996	2.724	2.438	2.030	1.690	1.306	0.682
40	3.788	3.551	3.307	2.971	2.704	2.423	2.021	1.684	1.303	0.681
45	3.752	3.520	3.281	2.952	2.690	2.412	2.014	1.679	1.301	0.680
50	3.723	3.496	3.261	2.937	2.678	2.403	2.009	1.676	1.299	0.679
60	3.681	3.460	3.232	2.915	2.660	2.390	2.000	1.671	1.296	0.679
70	3.651	3.435	3.211	2.899	2.648	2.381	1.994	1.667	1.294	0.678
80	3.629	3.416	3.195	2.887	2.639	2.374	1.990	1.664	1.292	0.678
90	3.612	3.402	3.183	2.879	2.632	2.368	1.987	1.662	1.291	0.677
100	3.598	3.390	3.174	2.871	2.626	2.364	1.984	1.660	1.290	0.676
∞	3.481	3.291	3.090	2.807	2.576	2.326	1.960	1.645	1.282	0.674

t-Distribution: Values of |t| corresponding to two-tailed and one-tailed p-values for Student's t- distribution

Example: Finding the range for a p-value from a t-table

Suppose you have conducted a t-test for a population mean and have **$|t| = 2.31$** (since we are only doing two-sided tests you will get the same p-value for $t = 2.31$ or $t = -2.31$). Say the sample size $n = 11$, then ν is 10 (ie. $n – 1$).

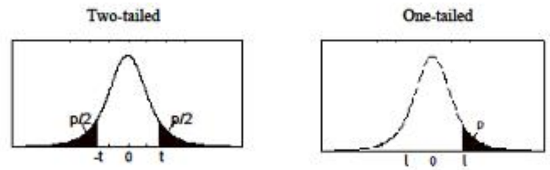
p two-tail	*0.0005*	*0.001*	*0.002*	*0.005*	*0.01*	*0.02*	*0.05*	*0.1*	*0.2*	*0.5*
p one-tail	*0.00025*	*0.0005*	*0.001*	*0.0025*	*0.005*	*0.01*	*0.025*	*0.05*	*0.1*	*0.25*
ν = 10	5.049	4.587	4.144	3.581	3.169	2.764	2.228	1.812	1.372	0.700

If the t-value had been 2.764 the p-value would have been 0.02. If the t-value had been 2.228 the p-value would have been 0.05. Since the t-value is between 2.764 and 2.228, the p-value must be between 0.02 and 0.05.

$$0.02 < \text{p-value} < 0.05$$

Most statistical packages calculate the exact p-value.

Example: Finding the range for a p-value from a t-table



Two-tailed	0.0005	0.001	0.002	0.005	0.0	0.02	0.05	1	0.2	0.5
One-tailed	0.00025	0.0005	0.001	0.0025	0.005	0.01	0.025	0.05	0.1	0.25
v										
1	1273	636.6	318.3	127.3	63.66	31.82	12.71	6.314	3.078	1.000
2	44.70	31.60	22.33	14.09	9.925	6.965	4.303	2.920	1.886	0.816
3	16.33	12.92	10.22	7.453	5.841	4.541	3.182	2.353	1.638	0.765
4	10.31	8.610	7.173	5.598	4.604	3.747	2.776	2.132	1.533	0.741
5	7.976	6.869	5.893	4.773	4.032	3.365	2.571	2.015	1.476	0.727
6	6.788	5.959	5.208	4.317	3.707	3.143	2.447	1.943	1.440	0.718
7	6.082	5.408	4.785	4.029	3.499	2.998	2.365	1.895	1.415	0.711
8	5.617	5.041	4.501	3.833	3.355	2.896	2.306	1.860	1.397	0.706
9	5.291	4.781	4.297	3.690	3.250	2.821	2.262	1.833	1.383	0.703
10	5.049	4.587	4.144	3.581	3.1	2.764	2.228	812	1.372	0.700
11	4.863	4.437	4.025	3.497	3.106	2.718	2.201	1.796	1.363	0.697
12	4.716	4.318	3.930	3.428	3.055	2.681	2.179	1.782	1.356	0.696
13	4.597	4.221	3.852	3.372	3.012	2.650	2.160	1.771	1.350	0.694
14	4.499	4.140	3.787	3.326	2.977	2.624	2.145	1.761	1.345	0.692
15	4.417	4.073	3.733	3.286	2.947	2.602	2.131	1.753	1.341	0.691
16	4.346	4.015	3.686	3.252	2.921	2.583	2.120	1.746	1.337	0.690
17	4.286	3.965	3.646	3.222	2.898	2.567	2.110	1.740	1.333	0.689
18	4.233	3.922	3.610	3.197	2.878	2.552	2.101	1.734	1.330	0.688
19	4.187	3.883	3.579	3.174	2.861	2.539	2.093	1.729	1.328	0.688
20	4.146	3.850	3.552	3.153	2.845	2.528	2.086	1.725	1.325	0.687
21	4.110	3.819	3.527	3.135	2.831	2.518	2.080	1.721	1.323	0.686
22	4.077	3.792	3.505	3.119	2.819	2.508	2.074	1.717	1.321	0.686
23	4.047	3.768	3.485	3.104	2.807	2.500	2.069	1.714	1.319	0.685
24	4.021	3.745	3.467	3.091	2.797	2.492	2.064	1.711	1.318	0.685
25	3.996	3.725	3.450	3.078	2.787	2.485	2.060	1.708	1.316	0.684
26	3.974	3.707	3.435	3.067	2.779	2.479	2.056	1.706	1.315	0.684
27	3.954	3.690	3.421	3.057	2.771	2.473	2.052	1.703	1.314	0.684
28	3.935	3.674	3.408	3.047	2.763	2.467	2.048	1.701	1.313	0.683
29	3.918	3.659	3.396	3.038	2.756	2.462	2.045	1.699	1.311	0.683
30	3.902	3.646	3.385	3.030	2.750	2.457	2.042	1.697	1.310	0.683
35	3.836	3.591	3.340	2.996	2.724	2.438	2.030	1.690	1.306	0.682
40	3.788	3.551	3.307	2.971	2.704	2.423	2.021	1.684	1.303	0.681
45	3.752	3.520	3.281	2.952	2.690	2.412	2.014	1.679	1.301	0.680
50	3.723	3.496	3.261	2.937	2.678	2.403	2.009	1.676	1.299	0.679
60	3.681	3.460	3.232	2.915	2.660	2.390	2.000	1.671	1.296	0.679
70	3.651	3.435	3.211	2.899	2.648	2.381	1.994	1.667	1.294	0.678
80	3.629	3.416	3.195	2.887	2.639	2.374	1.990	1.664	1.292	0.678
90	3.612	3.402	3.183	2.879	2.632	2.368	1.987	1.662	1.291	0.677
100	3.598	3.390	3.174	2.871	2.626	2.364	1.984	1.660	1.290	0.676
∞	3.481	3.291	3.090	2.807	2.576	2.326	1.960	1.645	1.282	0.674

t-Distribution: Values of |t| corresponding to two-tailed and one-tailed p-values for Student's t- distribution

df = 10
t = 2.31
0.02<p-value<0.05

Quiz 4

Use the t-tables to determine a p-value or a range of p-values for the following test statistics:

a. $t = 2.492$ with sample size, $n = 25$

b. $t = 3.50$ with $n = 17$

c. $t = 0.55$ with $n = 30$

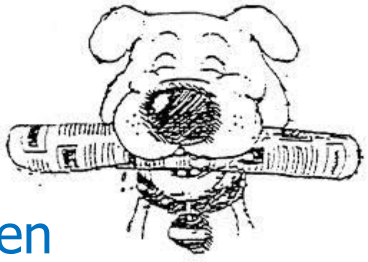
d. $t = 4.54$ with $n = 46$

e. $t = 1.75$ with $n = 88$

f. $t = 2.60$ with $n = 188$

Example: One Sample t-test for a Population Mean

Research Question: Is the average weekly weight of recycled newspaper equal to 2 pounds (ie. 0.91kg) per household , on average?



Unlike most products which can be recycled, newspaper recycling can be profitable. However, collection from households is the major expense in newspaper recycling. A feasibility study was undertaken to determine whether the average weight of recycled newspaper by households was 2 pounds (0.91kg per week), as suggested by a local council. A random sample of 148 households in the area was selected for the study and the weekly weight of newspapers discarded for recycling was recorded for each household . Use the results of the study which are shown (in kilograms) on the following slide to answer the research question.

Source: Keller (2014) *Statistics for Management and Economics, Cengage*

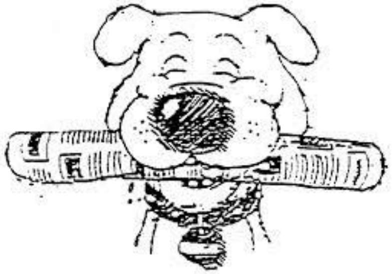
Newspaper Recycling

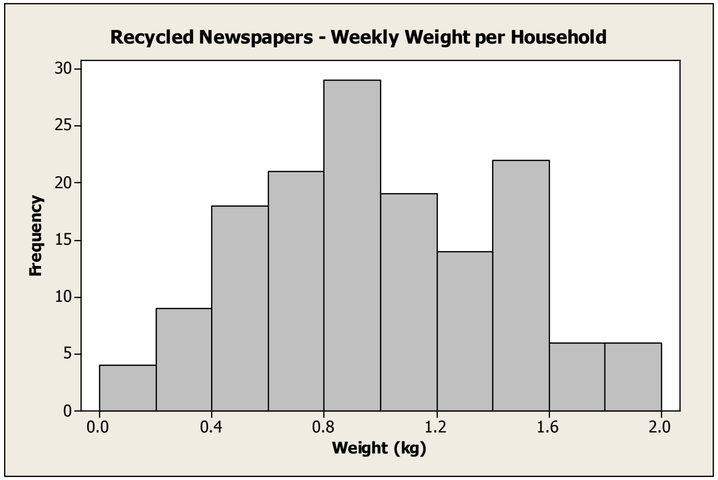
Some Sample Information:

Variable of interest:

Y = weight of newspaper collected from each household per week

$n = 148$, $\bar{y} = 0.989$ kg, $s = 0.445$ kg





Example: Newspaper Recycling

Hypothesis Test

H $H_0: \mu = 0.91$

$H_1: \mu \neq 0.91$

A From the histogram, the normality assumption appears reasonable. Also, the sample size is large so the t-distribution would be very robust here against non-normality.

T $t = \frac{\bar{y}-\mu}{s/\sqrt{n}} = \frac{0.989-0.91}{0.445/\sqrt{148}} = 2.16$ with 147 df

P $0.02 < \text{p-value} < 0.05$

D Since p-value < 0.05, reject H_0

C There is evidence to suggest that the average weight of recycled newspapers collected from all households would be higher than 0.91kg. This may affect whether it will be profitable to recycle newspapers.

Minitab Output for a One Sample t-test for a Population Mean

One-Sample T: Weight of Recycled Newspapers

```
Test of mu = 0.91 vs not = 0.91


Variable     N    Mean    StDev  SE Mean        95% CI            T      P
Weight     148  0.9890   0.4450   0.0366  (0.9167, 1.0613)  2.16  0.032
```

National geographic archive

Quiz 5

Research Question:
Is the average length of female green anacondas 3 metres?

A herpetologist is researching green anacondas which are among the largest snakes in the world. *Male green anacondas grow to an average length of 3 metres*. Green anacondas have been known to swallow live goats and even people. The herpetologist has measured 21 female green anacondas found in shallow water in the Llanos grasslands shared by Venezuela and Colombia. Their lengths are shown on the following slide. *Carry out an appropriate test to determine whether the average length of female green anacondas could also be 3 metres.*

Source: Johnson et al (2010) *Statistics Principles and Methods, Wiley*

7.42Q

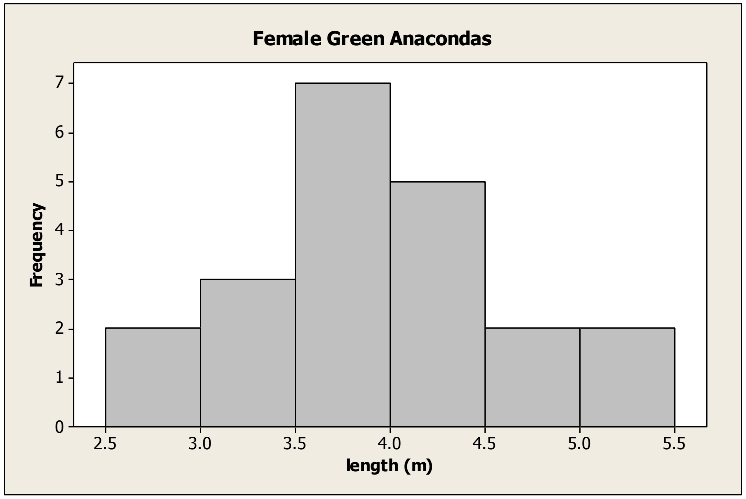
Female Green Anacondas

Some Sample Information:

Variable of interest:

Y = length of female green anacondas in metres										
3.11	3.47	4.15	5.21	5.03	3.60	4.75	3.60	3.44	3.63	2.93
4.39	4.02	4.11	3.78	3.69	3.54	2.62	3.96	4.97	4.39	

$n = 21,\ \bar{y} = 3.923,\ s = 0.690$



Solution to Quiz 6

Homework Questions

Homework Quiz 1

A 95% confidence interval for a mean is an interval containing:

a. the sample mean with 95% confidence
b. the population mean with 95% confidence
c. 95% of the data in the sample
d. 95% of the data in the population
e. the population mean in 95% of repeated samples

Homework Question 2a

a) The one-sample t statistic for testing

$$H_0: \mu = 40$$
$$H_1: \mu \neq 40$$

from a sample of n = 28 observations has the value t = 2.01

(i) What are the degrees of freedom?

(ii) Between what two values does the p-value of the test fall?

(iii) Based on the range for the p-value found in (ii), do you reject or not reject the null hypothesis?

Homework Question 2b

b) The one-sample t statistic for testing

$$H_0: \mu = 20$$
$$H_1: \mu \neq 20$$

based on n = 14 observations has the value t = -2.55

(i) What are the degrees of freedom?

(ii) Between what two values does the p-value of the test fall?

(iii) Based on the range for the p-value found in (ii), do you reject or not reject the null hypothesis?

Homework Question 3

Research Question:
Did the average length of long distance calls placed with Tell-Star change in the first three months of 2013?

Tell-Star telephone company provides long-distance telephone service in an area. The company's records indicate that the average length of all long-distance calls placed through this company in 2012 was 12.5 minutes. A sample of 125 calls placed in the first three months of 2013 had a mean length of 13.15 minutes with a standard deviation of 2.5 minutes. Carry out an appropriate hypothesis test to address the research question above.

Solution to Homework Question 3

Lecture 7 Summary

A null hypothesis (H_0) is a statement that a population parameter has a particular value, known as the null value.

A test statistic is a scaled measure of the discrepancy between the null value and the corresponding value obtained from a sample.

A p-value is the probability of getting such a test statistic or even more extreme in a study, assuming H_0 is true.

Lecture 7 Summary

Hypothesis testing involves:

answering the research question: H stating the null and the alternative hypothesis, A checking the assumptions of the test, T calculating the test statistic, P obtaining the p-value and D making a decision about the believability of H_0 based on this and C writing a conclusion that is framed in terms of the original research question.

Textbook References

Further information on the topics discussed in this lecture can be found in the prescribed textbook:

Modern Statistics: An Introduction
by Don McNeil and Jenny Middledorp
(ISBN 9781486007011).

- Chapter 7: Pages 134 – 149