Macquarie University

STAT 683: Introduction to Probability

Assignment 1 solution, First semester 2018

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|---------------|----------------|
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Game 1: Betting on red

A. Expected value: This is a Bernoulli trial therefore expected value can be given by the following expression

$$E(X) = \frac{18}{37} * (1) + \frac{19}{37} * (-1) = -\frac{1}{37}$$

Where 18/37 is the probability of winning, 19/37 is probability of losing, 1 and -1 are the amount gained in a scenario of a win/loss respectively.

Simulation results for 100000 trials: -0.02642

(percentage error =
$$\frac{Exact\ value\ - Estimate\ value}{Exact\ value} * 100$$
)

Percentage error=2.14%

B. Proportion of wins: Number wins for this game will be theoretically equal to

$$P(W) = \frac{18}{37} \approx 0.48648$$

Simulation results for 100000 trials: 0.48651

Percentage error = -0.0061%

- C. Expected playing time = 1 bet
- D. Maximum money player can lose = \$1

E. Maximum money player can earn = \$1

Game 2: Betting on a number

A. Expected value: This is a Bernoulli trial so the value of the winnings can be given by the following expression

$$E(X) = \frac{1}{37} * (35) + \frac{36}{37} * (-1) = -\frac{1}{37} \approx -0.0270$$

Where 1/37 is the probability of winning, \$35 amount won, 36/37 is the probability of losing,-1 is the money lost.

Simulation results for 100000 trials = -0.0265

(percentage error =
$$\frac{Exact\ value\ - Estimate\ value}{Exact\ value} * 100$$
)

Percentage error = 1.85%

B. Proportion of wins: Theoretically the proportion of winning can be given by

$$P(W) = \frac{1}{37} \approx 0.0270$$

Simulation results for 100000 trials = 0.02584

(percentage error =
$$\frac{Exact\ value\ - Estimate\ value}{Exact\ value} * 100$$
)

Percentage error=4.296%

- C. Expected playing time: 1 bet
- D. Maximum amount player can lose= \$1
- E. Maximum amount player can earn= \$35

Game 3: Martingale system of betting

- A. Expected winnings by simulation = -1.82907
- B. Proportion of wins by simulation = 0.9138
- C. Expected playing time by simulation = 16
- D. Maximum amount player can lose = \$127

Consecutive losses since start = -1 -2 -4 -8 -16 -32 -64 = -127

E. Maximum amount player can win= \$10

Game 4: Labouchere System

- A. Expected winnings by simulation = -3.32318
- B. Proportion of wins by simulation = 0.95636
- C. Expected playing time by simulation = 4
- D. Maximum amount of money player can lose = \$4940 Consecutive losses since start = -5 -6 -7 -8 ... -99
- E. Maximum amount of money player can earn =\$10

| Game ID | Exp winnings (min,max) | Prop wins (min,max) | Exp play time(min,max) |
|---------|------------------------|---------------------|------------------------|
| Game 1 | -0.02992,-0.023 | 0.48412,0.48795 | 1,1 |
| Game 2 | -0.02656,-0.012 | 0.02708,0.2748 | 1,1 |
| Game 3 | -2.08799,-1.8787 | 0.90861,0.9112 | 15,24 |
| Game 4 | -3.69058,-3.40586 | 0.95569,0.95768 | 2,20 |

| Game ID | Winnings (mean, std | Prop wins (mean, std | Play time (mean, std |
|---------|---------------------|----------------------|----------------------|
| | dev) | dev) | dev) |
| Game 1 | -0.02604, 0.9996659 | 0.48667, 0.499824 | 1, 0 |
| Game 2 | -0.019, 5.861227 | 0.02758, 0.164722 | 1, 0 |
| Game 3 | -1.98986, 38.03104 | 0.92482, 0.274563 | 17.46590,4.190035 |
| Game 4 | -3.897, 70.4128 | 0.95674, 0.204374 | 8.57188, 7.751844 |

Labouchere System's winning is most variable with std dev 70.41.28

Labouchere System's play time is most variable with std dev 7.751844

Rcode: Github Link

https://github.com/sedhasukhdeep/Roulette- (Copy and paste the url in address bar of browser)

Simulation Code

```
play game1 <- function() {</pre>
  win amount = sample(c(1, -1),
                       1,
                       replace = TRUE,
                       prob = c(18 / 37, 19 / 37))
  #In game 1 the number of bets are constant for each game i.e 1
  number of bet <- 1
  return (c(win amount, number of bet))
}
#Calls the game1 function and checks for proportion of wins
proportion win game1 <- function() {</pre>
  wins < - \overline{0}
  gamecounter <- 1</pre>
  #looping and checking the first element of the game for 1
  while (gamecounter <= 100000) {
    if (play game1()[1] == 1) {
      wins = wins + 1
    gamecounter = gamecounter + 1
  }
  return (wins / 100000)
}
#This function calculates the expected winnings in game 1
expected game1 <- function() {</pre>
```

```
total win <- 0
  gamecounter <- 1
  while (gamecounter <= 100000) {
    total win = total_win + play_game1()[1]
    gamecounter = gamecounter + 1
  return (total win / 100000)
#This function plays the game 2 once and returns the winnings and numbers of
play game2 <- function() {</pre>
  win amount = sample(c(35, -1),
                       replace = TRUE,
                       prob = c(1 / 37, 36 / 37))
  #In game 1 the number of bets are constant for each game i.e 1
  number of bet <-1
  return (c(win amount, number of bet))
}
#Checking for proportion of wins for game 2
proportion win game2 <- function() {</pre>
  wins <-0
  gamecounter <- 1</pre>
  \#looping and checking the first element of the game for win amount == 35
  while (gamecounter <= 100000) {
    if (play game2()[1] == 35) {
      wins = wins + 1
    gamecounter = gamecounter + 1
  }
  return (wins / 100000)
}
\#This function checks the expected winnings in game 2
expected game2 <- function() {</pre>
  total win <- 0
  gamecounter <- 1
  while (gamecounter <= 100000) {
   total win = total win + play game2()[1]
    gamecounter = gamecounter + 1
  }
```

```
return (total_win / 100000)
#This game plays the martindale system game once and returns the winning
amount and number of bets played
play game3 <- function() {</pre>
 win amount <- 0
 number bets <- 0
 bet amount <-1
  while (win amount < 10 && bet amount < 100) {
    current winnings = play game1()[1]
    if (current_winnings == -1) {
      win amount = win_amount - bet_amount
      bet amount = bet amount * 2
    else{
      win amount = win amount + bet amount
      bet_amount = 1
    number bets = number bets + 1
  return(c(win amount, number bets))
}
#This function returns the expected amount of winnings for game 3 when it's
played for 100000 times
expected_game3 <- function() {</pre>
  counter <- 1
 win_amount <- 0</pre>
  while (counter <= 100000) {
    win amount = win amount + play game3()[1]
    counter = counter + 1
  return (win amount / 100000)
}
#This function calculates the proportion of wins for game 3
proportion win game3 <- function() {</pre>
  counter <- 1
  win amount <- 0
 number of wins <-0
  while (counter <= 100000) {
    win_amount = play_game3()[1]
    if (win amount == 10) {
     number_of_wins = number_of_wins + 1
    counter = counter + 1
  return (number of wins / 100000)
```

```
#This function plays the lebouchere system of game and returns the win amount
and number of bets
play game4 <- function() {</pre>
 list game = c(1, 2, 3, 4)
 win amount <- 0
 bet amount = list game[1] + list game[length(list game)]
  number bets <- 0</pre>
  while (length(list game) >= 1 && bet amount < 100) {
    current winnings <-
      sample(
        c(-bet amount, bet amount),
        replace = T,
        prob = c(19 / 37, 18 / 37)
    if (current winnings > 0) {
      win amount = win amount + current winnings
      list game = list game[-c(1, length(list game))]
      if (length(list game) == 1) {
        bet_amount <- list_game[1]</pre>
      }
      else {
        bet_amount <- list_game[1] + list_game[length(list_game)]</pre>
    }
    else{
      win amount = win amount + current winnings
      list game = c(list game, abs(current winnings))
     bet amount = list game[1] + list game[length(list game)]
    number bets = number_bets + 1
  }
  return (c(win amount, number bets))
}
#This function returns the expected winnings of game 4
expected game4 <- function() {</pre>
 counter <- 1
 win amount <- 0
 while (counter <= 100000) {
   win amount = win amount + play game4()[1]
   counter = counter + 1
  return (win amount / 100000)
```

```
}
#This function returns the proportion of wins
proportion win game4 <- function() {</pre>
  counter <- 1
 win amount <- 0
 number of wins <- 0
  while (counter <= 100000) {
    win_amount = play_game4()[1]
    if (win amount == 10) {
     number of wins = number of wins + 1
    counter = counter + 1
  }
 return (number of wins / 100000)
#This function is calculating the required min-max values of expected
winnings, playtime and winning proportions of game 1,2,3 and 4
table 1 stuff<-function(){</pre>
game1 expected wins = replicate(5, expected game1()[1])
game2 expected wins = replicate(5, expected game2()[1])
game3 expected wins = replicate(5, expected game3()[1])
game4 expected wins = replicate(5, expected game4()[1])
game1 propwins = replicate(5, proportion win game1())
game2 propwins = replicate(5, proportion win game2())
game3 propwins = replicate(5, proportion win game3())
game4 propwins = replicate(5, proportion win game4())
game1 playtime <- 1
game2 playtime <- 1</pre>
game3 playtime = replicate(5, play game3()[2])
game4 playtime = replicate(5, play game4()[2])
```

```
return (list(min(game1 expected wins), max(game1 expected wins)
              , min(game2 expected wins), max(game2_expected_wins)
              , min(game3_expected_wins)
              , max(game3 expected wins), min(game4 expected wins)
              , max(game4 expected wins), min(game1 propwins)
              , max(game1 propwins), min(game2 propwins)
              , max(game2 propwins), min(game3 propwins), max(game3 propwins)
              , min(game4 propwins)
              , max(game4 propwins), min(game1 playtime)
              , max(game1 playtime), min(game2 playtime)
              , max(game2_playtime), min(game3_playtime)
              , max(game3 playtime), min(game4 playtime)
              , max(game4 playtime)))
}
game1 wins <- function() {</pre>
  g1 <- replicate(100000, play game1()[1])</pre>
  return(list(mean(g1), sd(g1)))
}
game2 wins <- function() {</pre>
  g2 <- replicate(100000, play game2()[1])</pre>
  return(list(mean(g2), sd(g2)))
game3 wins <- function() {</pre>
  g3 <- replicate(100000, play game3()[1])
  return(list(mean(g3), sd(g3)))
}
game4 wins <- function() {</pre>
  g4 <- replicate(100000, play game4()[1])
  return(list(mean(g4), sd(g4)))
}
prop1 wins <- function() {</pre>
  g1 <- replicate(100000, proportion win game1())</pre>
  return(list(mean(g1), sd(g1)))
}
prop2 wins <- function() {</pre>
  g2 <- replicate(100000, proportion win game2())
  return(list(mean(g2), sd(g2)))
prop3 wins <- function() {</pre>
  g3 <- replicate(100000, proportion win game3())
  return(list(mean(g3), sd(g3)))
```

```
prop4_wins <- function() {
    g4 <- replicate(100000, proportion_win_game4())

    return(list(mean(g4), sd(g4)))
}

playtime_Game3 <- function() {
    x = replicate(100000, play_game3()[2])
    return(list(mean(x), sd(x)))
}

playtime_Game4 <- function() {
    x = replicate(100000, play_game4()[2])
    return(list(mean(x), mean(y)))
}</pre>
```