Please run the code in sections.

Question 1

In this question, we want to solve the equation $\mathbf{M}_q = -q^2\mathbf{D} - \mathbf{d} + \mathbf{A}$, where **D** includes submatrices $\mathbf{D}_u$ and $\mathbf{D}_v$ and **d** includes submatrices $\mathbf{d}_u$ and $\mathbf{d}_v$. The square matrix **A** is made up of Gaussian random variables with a mean of 0 and in the example case, a standard deviation of 0.1.

As we have the values $\mathbf{D}_u = 1$, $\mathbf{D}_v = 5$, $\mathbf{d}_u = \mathbf{d}_v = 1$, N= 8 and q=1, we make the submatrices into 4x4 diagonal matrices, then combine the $\mathbf{D}_u$ and $\mathbf{D}_v$ matrices with two 4x4 zero matrices to create the diagonal **D** matrix, and we do the same with the $\mathbf{d}_u$ and $\mathbf{d}_v$ matrices to create the diagonal **d** matrix.

We then use the normrnd function to generate a matrix of normally distributed random numbers according to the given mean and standard deviation. The rng(4) function is used to keep the original **A** matrix used in this example.

As a result, we get the following matrices as given in the Figure 1 below:

1	2	3	4	5	6	7	8
1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	5	0	0	0
0	0	0	0	0	5	0	0
0	0	0	0	0	0	5	0
0	0	0	0	0	0	0	5

1	2	3	4	5	6	7	8
1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0
0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	1

1	2	3	4	5	6	7	8
0.2346	-0.1250	-0.1708	-0.0938	0.0049	-0.0366	0.0121	0.2335
0.0089	-0.0233	-0.0257	-0.0550	-0.0415	0.0260	0.0146	-0.0166
0.2210	0.0160	-0.1640	0.0192	-0.0723	0.0462	0.2101	0.1592
0.0744	-0.1008	0.1999	-0.1571	0.0093	-0.1075	-0.0294	-0.0318
0.0676	0.0944	-0.0052	0.1066	0.0047	-0.0320	0.0602	0.1047
-0.0496	0.1667	0.0493	-0.0482	0.0335	7.5049e-04	-0.0515	-0.0482
0.1001	-0.0910	0.1581	0.0133	-0.0213	0.0483	-3.0343e-04	-0.0448
-0.1887	0.0426	0.1061	0.1978	0.0398	-0.0351	-0.1198	0.0332

1	2	3	4	5	6	7	8
-1.7654	-0.1250	-0.1708	-0.0938	0.0049	-0.0366	0.0121	0.2335
0.0089	-2.0233	-0.0257	-0.0550	-0.0415	0.0260	0.0146	-0.0166
0.2210	0.0160	-2.1640	0.0192	-0.0723	0.0462	0.2101	0.1592
0.0744	-0.1008	0.1999	-2.1571	0.0093	-0.1075	-0.0294	-0.0318
0.0676	0.0944	-0.0052	0.1066	-5.9953	-0.0320	0.0602	0.1047
-0.0496	0.1667	0.0493	-0.0482	0.0335	-5.9992	-0.0515	-0.0482
0.1001	-0.0910	0.1581	0.0133	-0.0213	0.0483	-6.0003	-0.0448
-0.1887	0.0426	0.1061	0.1978	0.0398	-0.0351	-0.1198	-5.9668

Figure 1: Tables representing each of the 8x8 matrices, with the top left image representing the **D** matrix, the top right image representing the **d** matrix, the bottom left representing the **A** matrix and the bottom right representing the $\mathbf{M}_q$ matrix.

Question 1

We now look at plotting the eigenvalues of the $\mathbf{M}_q$ matrix, similar to the **a, b** and **c** panels as shown in Figure 2 of Baron and Galla (2020). We separate the real and imaginary parts of the eigenvalues obtained after using the eig function on the $\mathbf{M}_q$ matrix, then plot the real values against the imaginary ones. The equilibrium is considered to be stable with respect to q if all the eigenvalues have negative real parts.

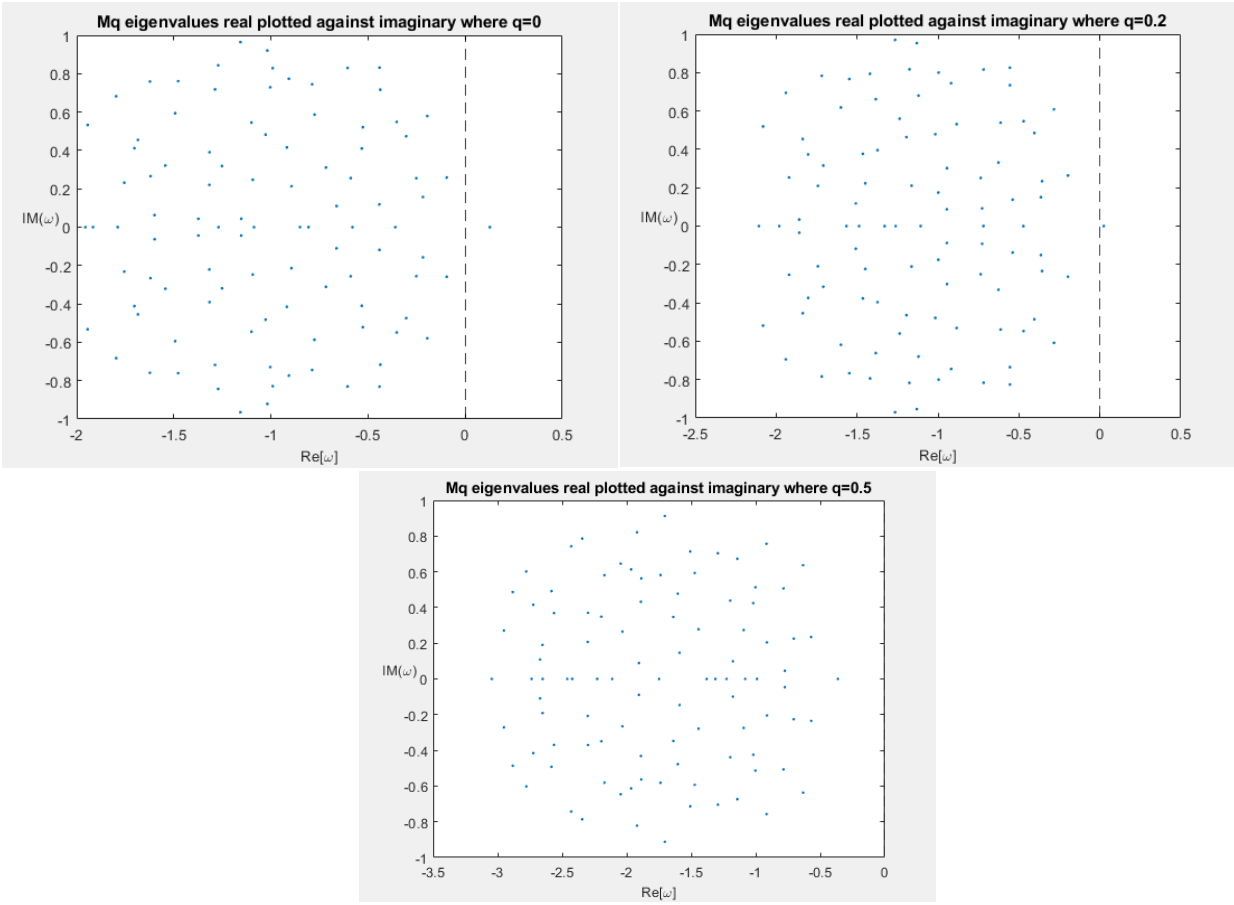

Figure 2: Graphs representing the eigenvalues plotted against each other, with q values of 0, 0.2 and 0.5

The graphs in Figure 2 show that the q values of 0 and 0.2 could cause the dynamical system to be unstable whereas in the third plot with a q value of 0.5, all the eigenvalues are below 0 and are negative, therefore they are all stable in that particular case.

<u>Question 3</u>

We now look to make a similar graph to the one seen in Figure 2**d** of Baron and Galla (2020), except with the matrices obtained from Question 2, using q values between 0 and 1 and any standard deviation of our choice. In our examples we will use standard deviations of 0.11, 0.19 and 0.24.

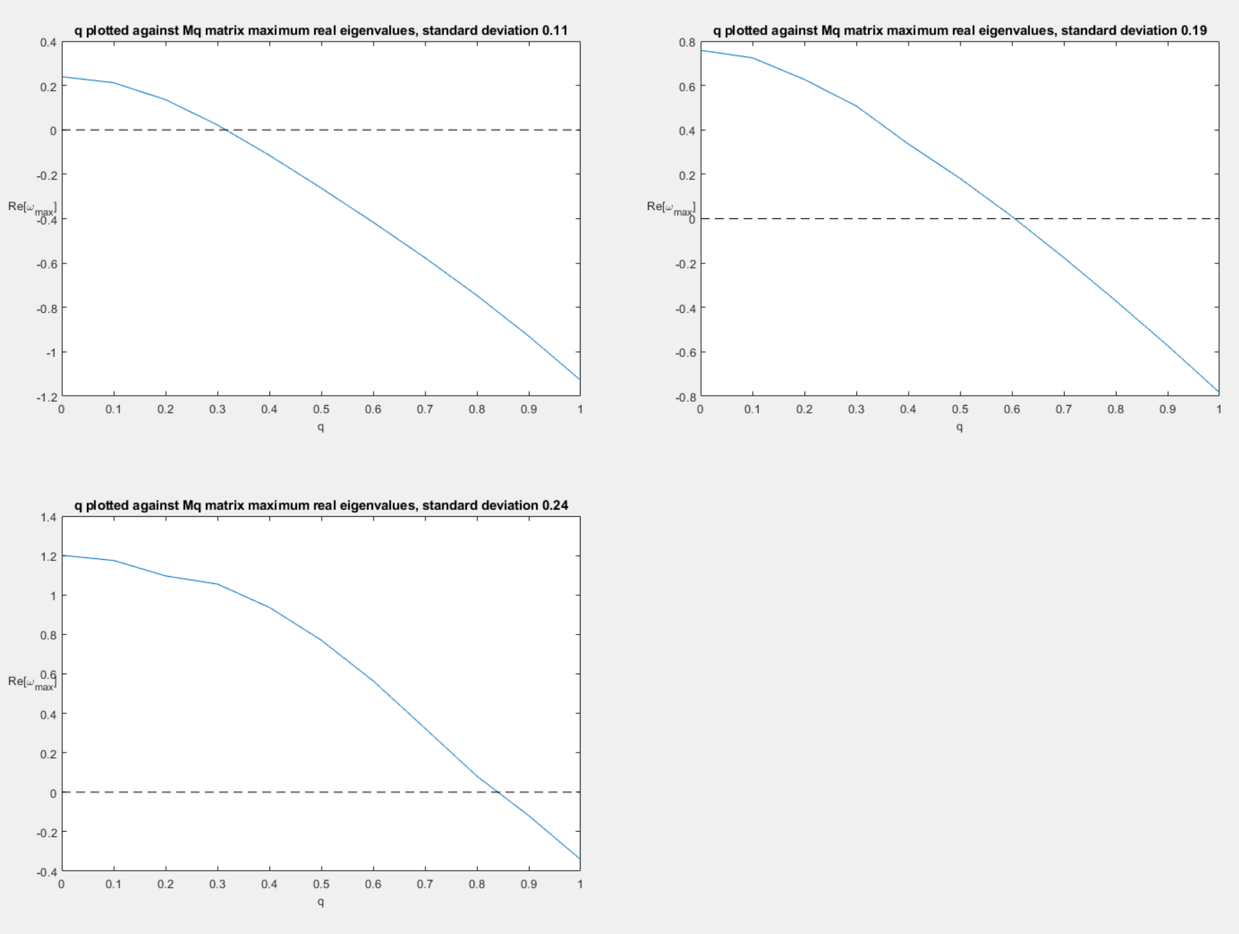

Figure 3: Graphs representing the largest eigenvalue for each q value used, using values from 0 to 1.0, increasing by 0.1 each time. These values are plotted against each of those q values to find where the equilibrium is unstable against perturbations of wavenumber q when $Re[\omega_{max}] > 0$

We can see in Figure 3 that the increase of the standard deviation causes for a much larger period of instability between q values of 0 and 1, especially with a standard deviation of 0.24. The inclusion of trophic structure in the model allows us to promote instability, however in the results shown, this is not necessarily essential to increase the amount of instability with our model, therefore dispersal on its own may be enough.

Question 4

We firstly find the equilibrium point and by running fsolve on the dynamical system and repeating it for a large number of attempts allows us to confirm that the point (1.111,1.111) is a fixed point. We then use the ode function in a loop 5 times to obtain 5 graphs, each with different graphs starting at a different point to show that they each reach the stabled fixed point (1.111,1.111)

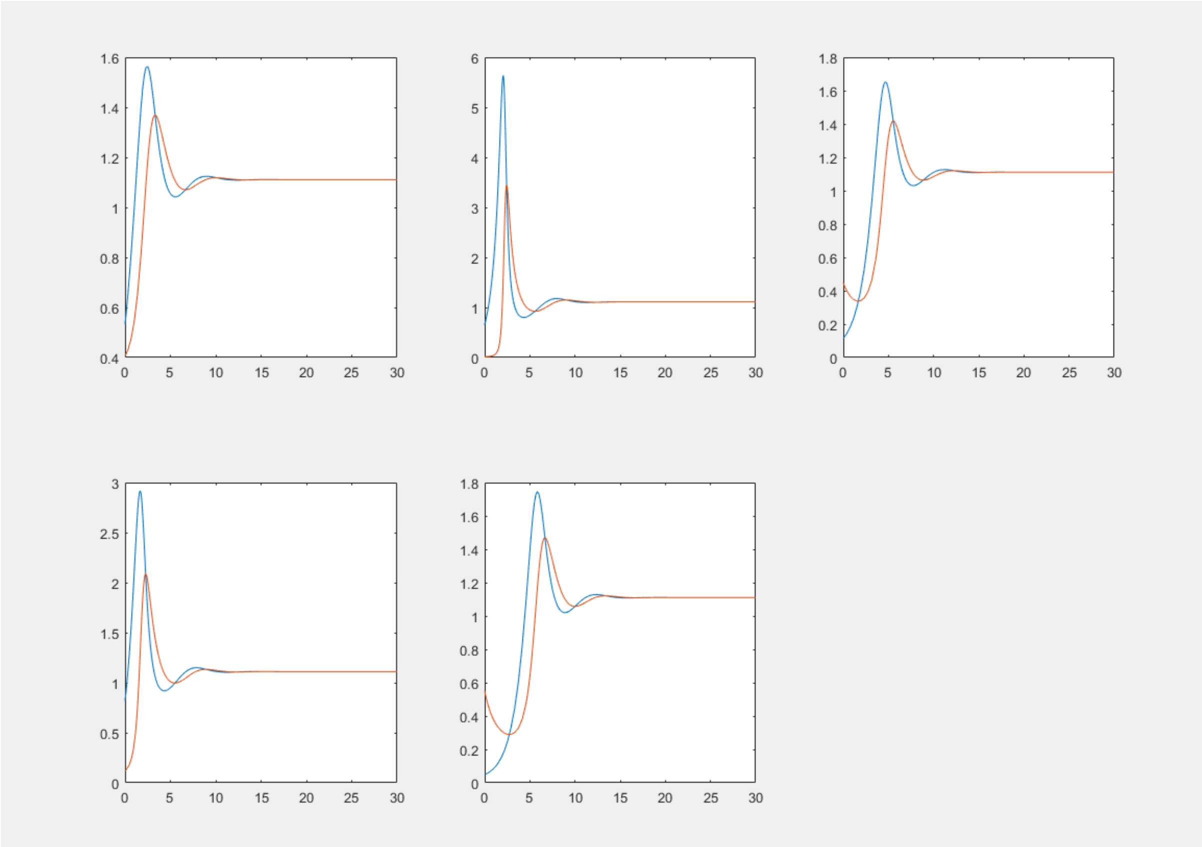

Figure 4: Graphs representing the stable fixed point for the system as the amount of time that passes increases. Each graph begins at a random point but ends at the same fixed point

As each graph ends after a certain point consistently goes to (1.111,1.111), this shows that the point is a working stable fixed point for the system.

Question 4

In this case, the dde23 function is used to find out whether adding a delay in the system causes instability to be added. The values for the delays range from 0.5 to 1.3 After adding a delay to the function in the loadderivs file and increasing the delay for each graph, we get the following in Figure 5:

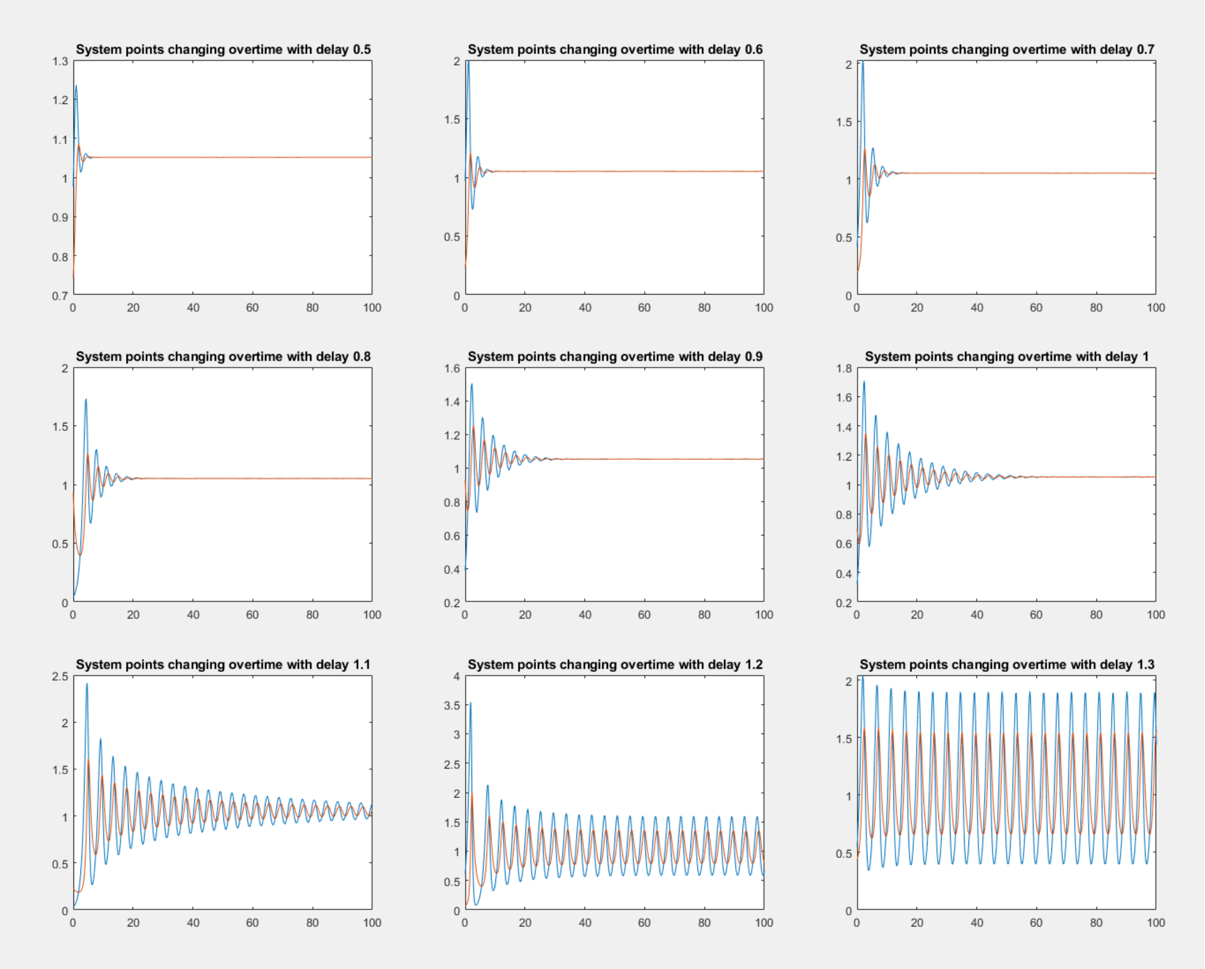

Figure 5: Graphs representing the delays affecting each of the graphs overtime as the delays in each example increase

This shows that after the delay value of 1, the graph gets significantly affected and beyond this the system starts becoming unstable. This tells us that a significant amount of delay being added to the system causes for instability within the system.