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The DNL absorbing boundary condition: applications to wave problems

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Abstract

A general methodology for developing absorbing boundary conditions is presented. For planar surfaces, it is based on a straightforward solution of the system of block difference equations that arise from partial discretization in the directions transversal to the artificial boundary followed by discretization on a constant step 1D grid in the direction normal to the boundary. This leads to an eigenvalue problem of the size of the number of degrees of freedom of the lateral discretization. The eigenvalues are classified as right- or left-going and the absorbing boundary condition consists in imposing a null value for the ingoing modes, leaving free the outgoing ones. Whereas the classification is straightforward for operators with definite sign, like the Laplace operator, a *virtual dissipative* mechanism has to be added in the mixed case, usually associated with wave propagation phenomena, like the Helmholtz equation. The main advantage of the method is that it can be implemented as a black-box routine, taking as input the coefficients of the linear system, obtained from standard discretization (FEM or FDM) packages and giving on output the *absorption matrix*. We present the application of the DNL methodology to typical wave problems, like Helmholtz equations and potential flow with free surface (the *ship wave resistance* and *sea-keeping* problems).

Keywords: Potential flow; Finite element method; Wave resistance; Absorbing boundary condition; Free surface flow; Partial discretization

1. Introduction

When solving elliptic problems in unbounded domains with numerical methods, like the Finite Element Method (FEM) or Finite Difference Method (FDM) one faces the problem of truncating the domain at a certain artificial boundary. For positive definite operators, like the Laplace operator or the elasticity equations, imposing null Dirichlet conditions at an artificial boundary located far enough from the region of interest is enough, in the sense that pushing this boundary to infinity converges to the *unbounded solution* (i.e. the Cauchy problem). Essentially the same thing happens for Neumann or mixed boundary conditions. The convergence to the unbounded solution may depend on space dimension, and on the order of the perturbation (i.e. if it can be approximated by a single pole, dipole, or higher order term), and some numerical techniques, like coupling with an external Boundary Integral solution or infinite elements, have been developed in order to minimize the computational effort.

In contrast, for wave-like problems like the Helmholtz equation, the situation is much more complex. The process of pushing the boundary to infinity may be not convergent at all unless an appropriate boundary condition is imposed in the artificial boundary. For instance in 2D, the solution is known to

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