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Worksheet 7 - Binary Addition and Subtraction

Answers

1.

a) $1_2 + 0_2 = 1_2$ b) $0_2 + 0_2 = 0_2$ (0_2 - null element of addition)

c) $1_2 + 1_2 = 1_{10} + 1_{10} = 2_{10} = 10_2$

In practice, we take $1_2 + 1_2 = 0_2$ carry 1

d) $10_2 + 01_2 = 11_2$ Adding column by column ($0_2 + 1_2 = 1_2$, $1_2 + 0_2 = 1_2$)

e) $11_2 + 01_2 = 100_2$

Adding column by column, we have

$1_2 + 1_2 = 0_2$ carry 1

$1_2 + 1_2 = 0_2$ carry 1

f) $1101_2 + 11_2 = 10000_2$

Adding column by column from right to left, we have

$1_2 + 1_2 = 0_2$ carry 1

$(0_2 + 1_2) + 1_2 = 0_2$ carry 1

$(1_2 + 1_2) + 0_2 = 0_2$ carry 1

$(1_2 + 1_2) + 0_2 = 0_2$ carry 1

2. A CPU with a capacity of 8 bits has a capacity of up to 1111 1111 in binary. If one more bit was added there would be an overflow error

$128_{10} \div 2 = 64$, remainder 0
$64 \div 2 = 32$, remainder 0
$32 \div 2 = 16$, remainder 0
$16 \div 2 = 8$, remainder 0
$8 \div 2 = 4$, remainder 0
$4 \div 2 = 2$, remainder 0
$2 \div 2 = 1$, remainder 0

$128_{10} = 1000\,0000_2$

$$
\begin{array}{r}
128_{10} = 1000\ 0000 \\
+\,128_{10} = 1000\ 0000 \\
\hline
1\,0000\ 0000
\end{array}
$$

$1_2 + 1_2 = 0_2$ carry 1

R/ $128_{10} + 128_{10} \rightarrow$ overflow error

$(x > 128)$ $128_{10} + x$

3. ANSWERS

(B)-(A) using word length of 8 bits

a) $15_{10} = (8+4+2+1)_{10} = (2^3+2^2+2+1)_{10} = 00001111_2$

$10_{10} = (8+2)_{10} = (2^3+2^1)_{10} = 00001010_2$

Subtraction

$$\begin{array}{r} 00001111_2 \\ -\ 00001010_2 \\ \hline 00000101_2 \end{array}$$

From right to left

$1_2 - 0_2 = 1_2$
$1_2 - 1_2 = 0_2$
$1_2 - 0_2 = 1_2$
$1_2 - 1_2 = 0_2$

b) $69_{10} = (64+4+1)_{10} = (2^6+2^2+1)_{10} = 001000101_2$

$68_{10} = (64+4)_{10} = 001000100_2$

SUBTRACTION

$$\begin{array}{r} 001000101_2 \\ -\ 001000100_2 \\ \hline 000000001_2 \end{array}$$

From right to left

$1_2 - 0_2 = 1_2$
$0_2 - 0_2 = 0_2$
$1_2 - 1_2 = 0_2$
$0_2 - 0_2 = 0_2$
$1_2 - 1_2 = 0_2$
$0_2 - 0_2 = 0_2$
$0_2 - 0_2 = 0_2$

(c) $125_{10} = (64+32+16+8+4+1)_{10} = (2^6+2^5+2^4+2^3+2^2+2^0)_{10}$
$= 01111101_2$

15_{10} (from 3a)

SUBTRACTION

$$\begin{array}{r} 01111101_2 \\ -\ 00001111_2 \\ \hline 01101110_2 \end{array}$$

From right to left

$1_2 - 1_2 = 0_2$
$0_2 - 1_2 = 1_2$, borrow 1
$(1_2 - 1_2) - 1_2 = 1_2$, borrow 1
$(1_2 - 1_2) - 1_2 = 1_2$, borrow 1
$(1_2 - 1_2) - 0_2 = 0_2$
$1_2 - 0_2 = 1_2$
$1_2 - 0_2 = 1_2$
$0_2 - 0_2 = 0_2$

$01101110_2 = (64+32+8+4+2)_{10} = 110_{10}$

4. ANSWERS

a) $1_2 + 0_2 = 1_2$
$= 1_{10} + 0_{10} \equiv 1_{10}$

b) $0_2 + 0_2 = 0_2$
$= 0_{10} + 0_{10} \equiv 0_{10}$

c) $1_2 + 1_2 = 10_2$
$= 1_{10} + 1_{10} = (1 \cdot 2^1 + 0 \cdot 2^0)_{10}$
$2_{10} \equiv 2_{10}$

d) $10_2 + 01_2 = 11_2$
$= (2^1 + 0)_{10} + (1_{10}) = (1 \cdot 2^1 + 1 \cdot 2^0)_{10}$
$3_{10} \equiv 3_{10}$

(e) $11_2 + 01_2 = 100_2$
$= (1 \cdot 2^1 + 1 \cdot 2^0)_{10} + (1_{10}) = (1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0)_{10}$
$= 3_{10} + 1_{10} = 4_{10}$
$4_{10} \equiv 4_{10}$

ANSWERS

4 f)

$1.101_2 + 11_2 = 10000_2$

$(2^3 + 2^2 + 1)_{10} + (2^1 + 1)_{10} = (2^4)_{10}$

$13_{10} + 3_{10} = 16_{10}$

$16_{10} = 16_{10}$

5 (B) - (A) using word length of 8 bits

$25_{10} = (16 + 8 + 1)_{10} = (2^4 + 2^3 + 1)_{10} = 11001_2$

$67_{10} = (64 + 2 + 1)_{10} = (2^6 + 2 + 1)_{10} = 1000011_2$

$$
\begin{array}{rcl}
25_{10} & = & 00011001_2 \\
- \; 67_{10} & = & 01000011_2 \\
\hline
-42_{10} & & 11010110_2 = (128 + 64 + 16 + 4 + 2)_{10} = 212_{10}
\end{array}
$$

b) No. the answers are not equals, But the subtraction of unsigned numbers has this incompatibility is solved using two's complement.

Two's complement

$25_{10} - 67_{10}$

$= -(67_{10} - 25_{10}) = -42_{10}$

$= -(1000011_2 - 11001_2)$

Leading zeros

$0|1000011_2$

$0|0011001_2$

Two's complement of

0 0 0 1 1 0 0 1

1 1 1 0 0 1 1 0 + 1

11100111_2

Addition

$$
\begin{array}{r}
01000011_2 \\
+11100111_2 \\
\hline
100101010_2
\end{array}
$$

Remove the first one and the zeros adjacents on thee right

$-101010_2 = -42_{10}$

$32 + 8 + 2 = 42_{10}$