INTERSEMESTER MEDIUM: THE DARK AGES!!!

Midterm 2 Review

Ay 7b 2012

1. **Caroll and Ostlie ♡ Magnitudes (C & O 3.9)**

   Consider a model of the star Dschubba (δ Sco), the center star in the head of the constellation Scorpius. Assume that Dschubba is a spherical blackbody with a surface temperature of 28,000 K and radius of 5.16×10^9 m. Let this model star be located at a distance of 123 pc from Earth. Determine the following for the star:

   (a) Luminosity.
   (b) Absolute bolometric magnitude.
   (c) Apparent bolometric magnitude.
   (d) Distance modulus.
   (e) Radiant flux at the star's surface.
   (f) Radiant flux at the Earth's surface (compare this with the solar irradiance).
   (g) Peak wavelength λ_{max}

2. **Dinosaurs are Extinct**

   A cloud of dust permeates the space between an observer and a star. For this problem, consider no other source of extinction besides this cloud of dust. The observer and the star is separated by a distance $d = 10\ pc$.

   (a) Treat the dust grains as spheres with radius $a = 0.5\mu m$, what is the cross section of these dust grains?
   (b) If the number density of the cloud is n, what is the optical depth, τ, between the observer and the star?
   (c) What is the extinction, A, between the observer and the star?
   (d) What if the number density profile of the cloud is $n(r) = n_0 e^{-r}$, what is the optical depth between the observer and the star?

3. **Bound Bound Unbound Velocity Mass Zwicky.**

   Let's use the Virial theorem to replicate Zwicky's argument for the necessity of dark matter in galaxy clusters.

   Assume we've observed the the Stahler galaxy cluster, which contains $N = 50$ galaxies, and the average mass of each galaxy is $M_{gal} = 10^{12}\ M_\odot$. The cluster has a radius of approximately $R_{cluster} = 1$ Mpc. We observe the average velocity of the galaxies in the cluster is $v_{obs} = 1500$ km/s.

   Useful constants:

$$M_\odot = 2 \times 10^{33}\ \text{g}$$
$$1\ \text{pc} = 3 \times 10^{18}\ \text{cm}$$
$$G = 6.67 \times 10^{-8}\ \text{cm}^3\ \text{s}^{-2}\ \text{g}^{-1}$$

(a) Assuming that all the galaxies in the Stahler cluster have mass $M = M_{gal}$, and the total mass in the cluster is just due to the mass in the galaxies, is this cluster bound?

(b) If there isn't enough matter in the cluster for it to be bound (*spoiler: there isn't!*), how much extra "dark matter" mass is necessary for the cluster to be bound?

4. **Up and Down on a Horsey on a Merry-Go-Round**

   You have a mass density profile for part of a galaxy that goes as $\rho = A \propto const.$

   (a) What is the mass $M(r)$ enclosed within some radius r?

   (b) What is the rotation curve $v(r) = v_{circ}(r)$ vs r for this density profile? Sketch this rotation curve. (Does this remind you of a familiar type of rotation?)

   Now consider a mass density profile for the outer part of the galaxy where $\rho = 0$, but $\rho = A$ for $r < R$.

   (c) What is the mass $M(r)$ enclosed within an arbitrary radius r (where $r > R$)?

   (d) What is the rotation curve $v(r)$ vs r (for $r > R$) for this density profile? Sketch this rotation curve. (Does this remind you of a familiar type of rotation?)

5. **The Impact of Dark Matter (MT 2, 2010)**

   Based on a number of observations, we believe that the mass density of dark matter in the solar neighborhood is about $\rho = 0.01\ M_\odot\ \mathrm{pc}^{-3}$ and is streaming through the Solar System with a velocity $v = 220$ km/s.

   (a) If all the dark matter was in the form of small black holes left over from the Big Bang each with mass $M_{DM} = 10^{12}$ kg, what is the number density n of these dark matter objects in the solar neighborhood in AU^{-3}? Explain your reasoning for full credit; don't just write down the answer.

   (b) Derive a general expression for the number of dark matter particles per second that pass through one side of a planet (with radius R) in the Solar System. Write your answer in terms of the given variables.

   (c) Evaluate your expression from (b) for the Earth to determine the number of such dark matter objects hitting the Earth per century. Recall that a century is 100 years. (*Hint:* $R_\oplus = 6.38 \times 10^6$ m)