General Problems

Midterm 1 Review

Ay 7b 2012

1. **It's All Relative (Modified from PMT 2011)**
   A signal is launched from the surface of the Earth ($M_\oplus, R_\oplus$) and is detected by someone at great distance.

   (a) By how much does the frequency, ν_0, of the signal change as measured by the distant observer if the source is stationary? Why?

   (b) If the lamp is in circular orbit at radius of the Earth, what is the frequency that an observer on the Earth would measure? (Assume the object comes straight at the observer.)

   (c) What is the frequency the distant observer sees facing the orbiting lamp?

   (d) The orbiting rocket synchronizes its clock with the distant observer (yes, with magical insta-info transfer). Then after an orbit (at height $R_\oplus$), the ship compares its clock to the stationary clock. What happened? How much time has the rocket clock been advanced/retarded?

2. **Star, Disrupted (MT 2011)**
   We think that at the center of all massive galaxies resides a massive black hole with mass M_{BH} and Schwarzschild radius (ie. event horizon) $R_s = 2GM_{BH}/c^2$. Every now and then a star with mass M_* and radius R_* passes close enough that it gets torn apart due to the tidal forces exerted by the black hole.

   (a) First consider the self-gravity of the poor star, and consider it composed of two halves (one facing the BH and the other facing away). Write an expression for the force of gravity, $F_{binding}$, exerted by one half on the other half of the star. Treat both halves as point masses, separated by a distance R_*. This is a reasonable estimate for the force that keeps the star together.

   (b) *(Not done in review – 2012)* Assume that the center of mass of the star passes (at closest approach) a distance l from the BH. Write (but do not yet simplify) an expression for the tidal force, ΔF, which is the difference of the gravitational force exerted by the BH on the two halves of the star.

   (c) *(Began this part assuming you knew the formula for ΔF – 2012)* Estimate the disruption radius, r_{dis}, inside of which the star will be torn apart ("disrupted").

   (d) If $M_{BH} = 2 \times 10^6\, M_\odot$ and a star with mass $M_* = 1\, M_\odot$ and $R_* = 1\, R_\odot$ passes nearby, what is r_{dis} (in meters and in AU)?

   (e) *(Not done in review – 2012)* Typically, about half the material from the tidally disrupted star ends up accreting onto the black hole. Assume this material forms a ring at the tidal disruption radius and takes one month to fall onto the black hole, accreting at a uniform rate. How does the luminosity of this accretion flow compare with the $1\, L_\odot$ luminosity of the distrupted star itself? Use the numbers given in part (d).

   (f) If the star is disrupted with $r_{dis} > R_s$, then we expect to see a bright flash of light. If the star is disrupted inside the BH ($r_{dis} < R_s$) then there is no event that can be recorded. By setting $R_s = r_{dis}$, estimate the maximum mass of a black hole for which a disruption event might be seen. In particular, show that

   $$\frac{M_{BH,max}}{M_*} = 2\left(\frac{R_*}{R_s(*)}\right)^{3/2}$$

   where $R_s(*) = 2GM_*/c^2$ is the Schwarzschild radius of the star. For the parameters given above, show that $M_{BH,max} \approx 2.3 \times 10^8\, M_\odot$.