Ay 7B – Spring 2012
Section Worksheet 3
Tidal Forces

1. **Tides on Earth**
   On February 1st, there was a full Moon over a beautiful beach in Pago Pago. Below is a plot of the water level over the course of that day for the beautiful beach.

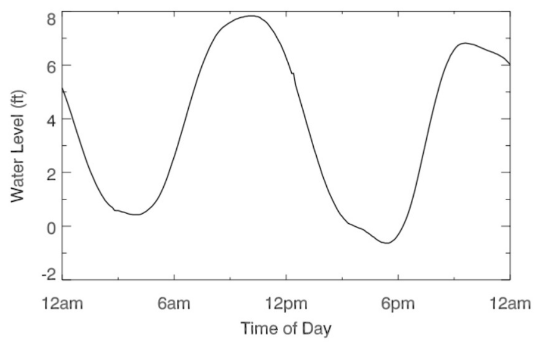

Figure 1: Phase of tides on the beach on February 1st

   (a) What is the main source of the tides on Earth? Why are there two peaks and two troughs over the course of one day?

   (b) Sketch a similar plot of the water level for February 15th. Nothing fancy here, but be sure to explain how you came up with the maximum and minimum heights and at what times they occur in your plot.

   (c) Imagine that the Moon suddenly disappeared. Then how would the water level change? How many times would tides happen? (For the level of water change, calculate the ratio of the height of tides between when moon exists and when moon doesn't exist.) Sketch a plot of the water level on the figure.

2. **Tidal Forces and Spaghettification**[1]

(a) Prof. Chiang is so sad that Ay 7A is over that he dives out of his spaceship head first and floats toward a black hole. What is the gravitational force on his head (F_h) due to a black hole of mass M when his head is a distance r from the center of the black hole? Assume Prof. Chiang has a mass m.

(b) Now let's write down an expression for the force on his feet (F_f), assuming that the professor's height is x.

(c) Subtract these two values to get the *difference* in forces across Prof. Chiang's body. This is called the "tidal force" on the professor. (You calculated this in class on Thursday, more or less.)

(d) Now assume that $r \gg x$ (i.e., the distance to the center of the black hole is much, much larger than the professor's height) and Taylor expand the above expression for the tidal force (as usual, anything $\ll 1$ to the second – or larger – power can be dropped).

(e) In this problem, let's assume Prof. Chiang is held together by his own *self-gravity*. Model him as two identical spheres of mass $0.5m$ gravitationally bound to each other. Go ahead. It's OK. He won't mind.[2]

What is the maximum tidal force Prof. Chiang can withstand?

[1] **Yes, that is a technical term.**

[2] If you didn't take 7A with Prof. Chiang, this probably doesn't seem very funny to you. Trust me, it is.

(f) If the tidal force outside a black hole (ΔF) is larger than the maximum force human bones can withstand (F_{max}) then we say that a person will get "spaghettified" by the black hole (i.e., they will be gravitationally ripped apart before plunging into the black hole). By "outside" we mean beyond the "event horizon" (aka the edge, or the "point of no return") of the black hole. Anything (including light) that gets closer than this distance to the center of a black hole *must* free-fall to the center of the black hole (known as the "singularity" — a single point of infinite density).

However, if the tidal force from a black hole is low enough, then a human can plunge directly into a black hole in one piece. For this to be true, the tidal force from a black hole at its event horizon must be smaller than the maximum force human bones can withstand. The event horizon is, by definition, one Schwarzschild radius away from the center of a black hole. The Schwarzschild radius is given by:

$$r_S = \frac{2GM}{c^2}$$

Using this equation, derive an expression for the tidal force of a black hole *at its event horizon.*

(g) Set the above tidal force equal to the maximum force Prof. "Two Spheres" Chiang can withstand and solve for M. This represents the *minimum* black hole mass that will not spaghettify Prof. Chiang and thus allow him to float right through the event horizon in one piece.

(h) To order of magnitude, let's say the professor is about 1 m tall and 100 kg.[3] Calculate an actual value for the M you just solved for, in solar masses.

(i) Calculate the tidal forces this gravitationally bound model of Prof. Chiang experiences every day on earth. Can people possibly be gravitationally bound?

[3] Actually, these numbers would be more accurate for an Oompa-Loompa version of Prof. Chiang. Now come on, that's funny.