Ay 7b – Spring 2012
Section Worksheet 12
Expanding Universe

1. Imagine a flat universe which expanded such that the scale factor evolved with time as

$$\frac{dR}{dt} = \frac{A}{R},$$

where A is some constant. Note that this is not true for our universe now (this describes the universe in the past when radiation was a dominant component of the energy of the universe), but everything below would be computed in exactly the same way for our universe. You'd just use the real expression for dR/dt (which we will do later in the class). Consider also light rays traveling radially in the universe ($d\theta = d\phi = 0$). The metric for this universe is

$$ds^2 = c^2dt^2 - R^2(d\varpi^2 + \varpi^2 d\theta^2 + \varpi^2 \sin^2\theta d\phi^2).$$

(a) Consider a light ray emitted a comoving distance ϖ away at a time t_e which is then received by us at the origin at some later time t_r (not necessarily today, allow it to possibly have been way in the past). Using the metric, show that $d\varpi = -cdt/R$, including arguing why it is negative.

(b) Integrate this result to find a general expression for ϖ as a function of R_e and R_r , the scale factor at the time the photon is emitted and received, respectively.

(c) Now consider photons observed today which are observed to have a cosmological redshift of z. In this case, you are finding the comoving distance (ϖ) to the object which emitted the light. Since the proper distance (or, physical distance) is given by $\mathcal{L} = R\varpi$, and $R = 1$ today, the comoving distance today is also the proper distance to the object now. (But note that this is different from the proper distance the object was when the light was emitted, see below.) Thus either way in this case, ϖ is what we would normally think of as the distance. Give a general expression for this comoving distances as a function of redshift. Show also that if $z \ll 1$, $cz = A\varpi$. This is just Hubble's Law, and it means that $A = H_0$

(d) Given the object is at the above comoving distance today, what was its comoving distance when it emitted the light? What was the proper distance to the object when the light was emitted?

(e) What you have found is comoving and proper distances to an object with redshift z. This relation is how astronomers use redshift as a measure of distance at cosmological scales. Think about your expressions for $\varpi(z)$ and $\mathcal{L}(z)$. Consider them in the limit where the redshift is vary large and very small. Do they make sense?

(f) Go back to your original general formula in terms of R_e and R_r (the answer you got in(b)). Consider now light emitted at the time of the big bang, from $t = 0$, and which are received at some later time when the scale factor is R_r. This is the maximum distance light could have traveled up to that time, and thus you can't see beyond it. For this reason it is called the cosmic horizon, and it marks the edge of the visible universe. Give general expressions for both the comoving and proper distances to this cosmic horizon in terms of the scale factor. What is the value of the horizon today?

(g) Imagine a photon emitted at time te when the scale factor was R_e and received at a time t_r when the scale factor was R_r. Given our expression for how the scale factor evolves, find and expression for $\Delta t = t_r - t_e$ as a function of the scale factor at those times.

(h) Just as before, consider such photons received today and observed to have a redshift of z. Find and expression for $\Delta t(z)$ for this case. This is the lookback time for that object, i.e. how long ago its light was emitted.

(i) Now consider if it was emitted at the time of the big bang and observed at some later time when the scale factor was R such that photons emitted at this later time would be observed today to have a redshift of z. Find an expression for the received time, t, as a function of the redshift at that time. This is how old the universe was when light observed today to have a redshift of z was emitted.

(j) What you have found above are the lookback time to an object with redshift z and how old the universe was at that time. This relation is how astronomers use redshift as a measure of time at cosmological scales. Thus when we say that galaxy is at a redshift of "3", that is a statement about how far away it is, how long ago the light was emitted, and how old the universe was at the time. Finally, what do your results say the age of the universe is today, t_0? What is the distance to the cosmic horizon today, d_h ? Give an expression for the horizon distance in terms of the age and argue why the result doesn't seem to make sense. Then argue why it actually is ok.