Ay 7B – Spring 2010
Section Worksheet 9
Propagating up the Distance Ladder

First let's refresh your memory a little bit on error propagation. Recall that if you have a function $f(x_1, x_2, ...)$ and errors in the x_n's $\delta x_1, \delta x_2, ...$, then the error in f is given by:

$$(\delta f)^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 (\delta x_1)^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 (\delta x_2)^2 + ...$$

1. So for $f(x) = x^3$, what is the error in f for a given error in x, δx? Or perhaps more useful would be what is the percent error in f, $\frac{\delta f}{f}$ in terms of the percent error in x, $\frac{\delta x}{x}$?

2. Now that you've got that down, let's look at how error might propagate up the distance ladder. The Tully-Fisher relation tells you that the luminosity of a galaxy goes as the fourth power of the maximum rotational velocity, v_{max}: i.e., $L = Cv_{max}^4$ where C is a constant (c.f. C&O page 955). Since this is an empirical relation, we need to determine the value of C from observations.

   (a) Good news! You observe a Cepheid variable star in Galaxy A! Using the Cepheid you calculate that Galaxy A is at a distance d_A. You also observe that Galaxy A has a flux F_A and a maximum rotational velocity $v_{max,A}$. Write an expression for the value of C.

   (b) Hooray! Now you can figure out the distance to super-mysterious Galaxy X, which you have observed to have flux F_X and $v_{max,X}$. Since you know C (from part 2), what is the distance to Galaxy X, d_X, in terms of C?

(c) Bummer! Your good luck has come to an end - it turns out we didn't understand Cepheids very well and the distance you calculated using the Cepheid in Galaxy A has a 10% error (i.e., $\frac{\delta d_a}{d_A} = 0.10$). What percent error does this mean for C?

(d) Ouch! What percent error does this mean for d_X?

(e) ¡Qué terriblé! Just when you thought your luck might change, you find out that Galaxy X was observed on a hazy night and there is a 5% error in your value for $v_{max,X}$. Now what is the percent error in d_X?