Ay 7b – Spring 2012
Week 2
Special Relativity

1. Minkowski Diagram

Suppose that a light bulb is suspended from the back wall of a freight car of length L, which is moving at a constant velocity u along a straight track. A clock on board the train and a clock at the station the train is passing synchronize as the light bulb turns on and the origins of the two frames of reference align($t = t' = 0$ when $x = x' = 0$). The hobo standing at the front of the freight car turns on his green laser pointer at time $t' = 0$ in his rest frame. (Let primed indicate the rest frame of the freight car and hobo, and unprimed indicate the frame of the station - let the light bulb be at $x' = 0$, and the hobo is standing at $x' = L$)

(a) Draw the Minkowski diagram for hobos rest frame, including the worldlines of the light from the bulb at the back of the train , the hobos green laser pointer, and the front and back walls of the freight car. Label the following events:
A - the light bulb turns on
B - the hobo turns on his laser pointer
C - the light from the bulb hits the front wall
D - the light from the laser pointer hits the back wall
Are events C and D simultaneous in this frame?

(b) Now draw the Minkowski diagram for frame of the station. Include the worldlines and events listed above. (HINT: be careful about placing event B – you dont need to be quantitative though, just get the right idea.)
Are events A and B simultaneous in the stations frame? Are C and D?

(c) Sketch the axes of the S frame (rest frame of the hobo) on the Minkowski diagram of the S frame (station's frame).

2. Energy & Momentum Conservation

(a) How much energy would it take to accelerate an electron to the speed of light according to classical physics (that is, before introducing any concepts from special relativity)?

With this energy, what would its actual velocity be, now using special relativity?

(b) An isolated photon cannot be converted into an electron-positron pair, $\gamma \rightarrow e^- + e^+$. (The conservation laws allow this to happen only near another object.) Why is that?

(c) A neutral pion π^0 moving at speed $v = 0.98c$ decays in flight into two photons. If the two photons emerge on each side of the pions direction with equal angles, θ, then find the angle θ and energies of the photons. The rest energy of π^0 is 135 MeV. (Hint: Use energy conservation and momentum conservation.)

3. Doppler Effect

A star is known to be moving away from the Earth at a speed of 4×10^4 m/s. This speed is determined by measuring the shift of the H_α line ($\lambda = 656.3$nm). By how much and in what direction is the shift of the wavelength of the H_α line? Use both classical physics and special relativity and compare the results.

4. Orbit of a Satellite (*Following C&O, although there are some integral nuances that are glossed over here, and are treated properly in the C&O discussion.*)

In General Relativity, we've told you that particles move along worldlines that are *geodesics* of the spacetime they inhabit (that is to say, their path is an extremum of $\int ds$). It's not intuative to see how more familiar Newtonian mechanics solutions come out of the weak field limit for GR, so let's look at the case of a satellite orbiting the Earth.

For a satellite orbiting the Earth, we should use the Schwarzschild metric (which describes the warped spacetime around a central point mass M). We'll ignore any effects of the satellite's mass on spacetime (since those effects would be very small, as $m_{satellite} << M_{\oplus}$). The Schwarzschild metric is

$$(ds)^2 = \left(c\,dt\sqrt{1 - 2GM/rc^2}\right)^2 - \left(\frac{dr}{\sqrt{1 - 2GM/rc^2}}\right)^2 - (r\,d\theta)^2 - (r\sin\theta\,d\phi)^2 \tag{1}$$

(a) Let's make this problem more simple: assume the satellite has a constant angular speed $\omega = v/r$, goes in a circular orbit, and orbits around the equator. How do these simplifications change the metric? Write out the modified metric.

(b) Using the simplified metric, what is the spacetime interval Δs over one orbit?

(c) What is the value of r such that Δs is an extremum?
(Recall extrema are found where $d/dx[f(x)] = 0$.)
(Aside: this is where we've been a bit sloppy as far as rigor.)

(d) What is the orbital velocity $v = \omega r$ corresponding to this value of r? How does this GR-determined orbital velocity compare with the result from Newtonian mechanics?