

Ay 7B – Spring 2012
Section Worksheet 7
Rotation Curve and Density Profile

1. Model the Galaxy^{1 2}

By making some assumptions, we can model the mass distribution and the velocity curve of the inner Milky Way Galaxy. Observational evidence indicates that at the center of our Galaxy is a black hole of mass $M_0 = 4 \times 10^6 M_\odot$. For the rest of the Galaxy's mass, let's assume it has an isothermal density distribution that goes as r^{-2} (to see where this dependence comes from, refer to pages 917-918 in the book).

- (a) Write down the density profile, $\rho(r)$ in terms of r and a constant C .

- (b) Assuming the mass distribution in the Galaxy is spherically symmetric and integrating the density profile over the radius, show that the mass contained within a distance r from the center of the Galaxy can be written as $M_r = kr + M_0$, where k is a constant that we will calculate later on.

- (c) Now for another assumption: assume that masses are in perfectly circular orbits about the center of the Galaxy. Using Newtonian gravity, show that the orbital velocity curve is

$$v = \sqrt{G \left(k + \frac{M_0}{r} \right)}. \quad (1)$$

For small r , how does v scale with r ?

- (d) If a mass located 2 pc from the center of the Galaxy has an orbital velocity of 110 km/s, calculate the constant k .

- (e) Estimate at what radius does the contribution of the central black hole begin to become significant? (Consider when the central black hole and the rest of the matter distribution contribute equally. . .)

¹This problem is based on Carroll and Ostlie 24.36.

²This is not necessarily the *best* model of the MW. See problem 2.

2. Galaxy Rotation Curve Fitting

We've talked about rotation curves and that they imply density distributions and from those density distributions we can infer that more mass exists in a galaxy than is simply accounted for by luminous (stellar) material. Well, now we'll really drive this home by deriving a general form of the density distribution for a given rotation curve. Below is a plot of three rotation curves all generated from the functional form $v = \alpha r^\beta$ where r is in pc, v is in km/s, and α and β are constants.

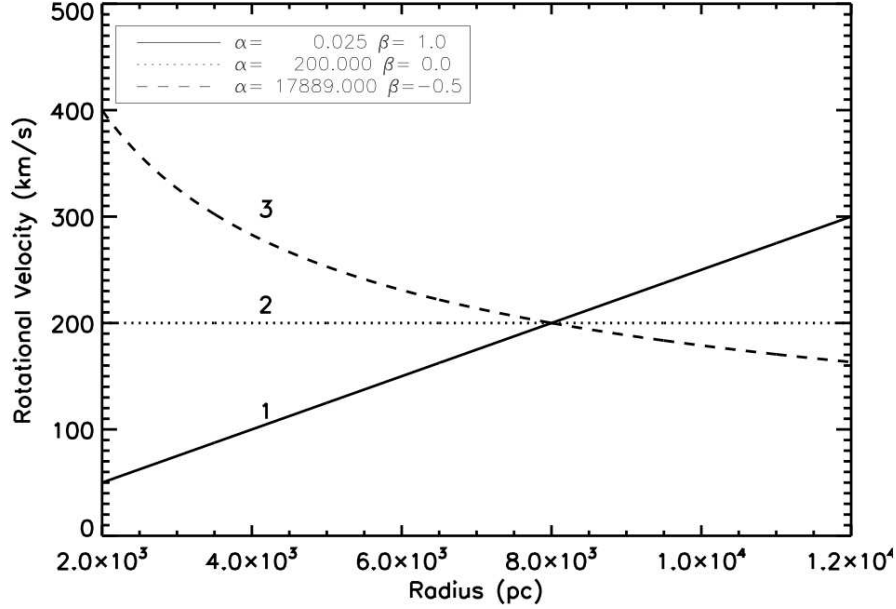


Figure 1: Different rotation curves

These rotation curves can be thought of as simple power law fits to observed rotating mass distributions. We want to use these rotation curves to generate their accompanying density distributions, $\rho(r)$. Before we get too involved in the math, let's recall that a Keplerian rotation curve goes like $r^{-0.5}$, a rigid body rotation curve goes like r^1 , and a flat rotation curve goes like r^0 (constant).

- Using the power law form for a rotation curve, $v = \alpha r^\beta$, solve for density $\rho(r)$ and enclosed mass $M(r)$ in terms of r , α , and β .
- How do density and enclosed mass scale with radius for each of the three rotation curves depicted in the plot above? Sketch density profiles.

- (c) Let's put this all together into a realistic (albeit simplified) view of the Milky Way's rotation curve. The Milky Way's rotation curve, when looked at over larger scales (ie not super close to the central SMBH), look roughly like the curve below.

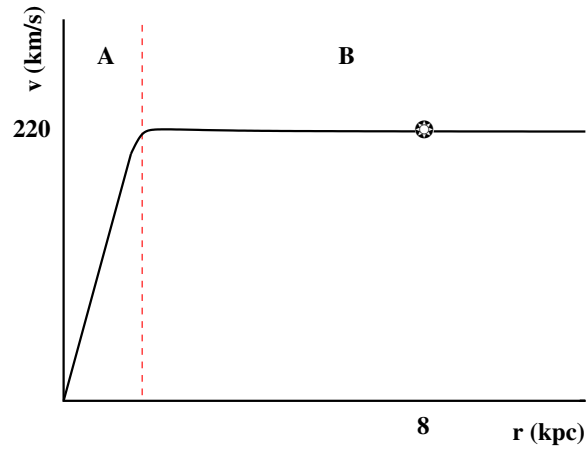


Figure 2: Simplified MW rotation curve

- i. In region (B), where the rotation curve is flat, how does v scale with r ? How do M and ρ scale with r ?
- ii. In region (A), where the rotation curve is rising, roughly how does v appear to be scaling with r ? What does this v say about M and ρ at these small radii?
- iii. We've said that the NFW profile,

$$\rho_{NFW}(r) = \frac{\rho_0}{(r/a)(1 + r/a)^2} \quad (2)$$

describes the dark matter halo of galaxies well. How does the NFW profile predict that ρ will scale with r for $r \ll a$?

- iv. Does the NFW ρ for small r agree with your answer to (ii)? If they don't, what might be different between the cases?

- (d) **(Bonus)**³ The real Milky Way rotation curve looks less flat than Figure 2 shows. Rather, it looks more like the figure below.

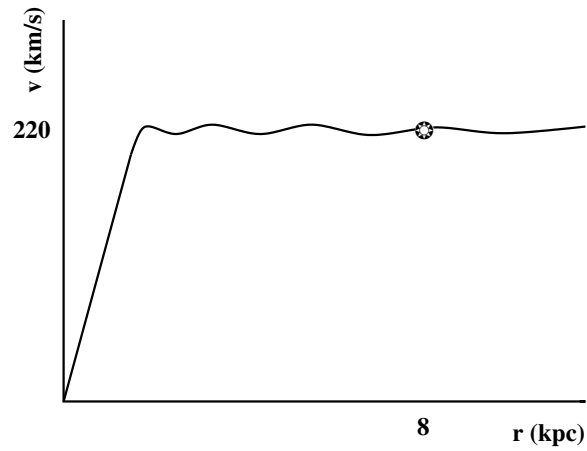


Figure 3: More realistic MW rotation curve

It is true that for large radii, the rotation curve is mostly flat but it definitely has waves in it. Can you think of a reason for why these waves exist?

³No really, this is outside the scope of Ay7b. I just think it's kinda neat.