2.

1. skupina

a) $y = 4x^2 - x + 1$

$$\boxed{\begin{aligned} y &= x^n \\ y' &= n \cdot x^{n-1} \end{aligned}}$$

$y' = 4 \cdot 2 \cdot x^{2-1} - 1$

$y' = 8x - 1$

$y'' = 8 \cdot 1 = 8$

b) $y = 2\sin x + 3\cos x$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$y' = 2\cos x - 3\sin x$

c) $y = \sqrt{x} + x^{-2} = x^{\frac{1}{2}} + x^{-2}$

$y' = \frac{1}{2}x^{\frac{1}{2}-1} + (-2)x^{-2-1}$

$y' = \frac{1}{2}\left(\frac{x}{1}\right)^{-\frac{1}{2}} - 2x^{-3}$

$\left(\frac{5}{3}\right)^{-2} = \left(\frac{3}{5}\right)^{2}$

$y' = \frac{1}{2\sqrt{x}} - \frac{2}{x^3}$

$5^{-1} = \left(\frac{5}{1}\right)^{-1} = \left(\frac{1}{5}\right)^{1}$

$\sqrt[2]{x^1} = x^{\frac{1}{2}}$

$\sqrt[4]{x^5} = x^{\frac{5}{4}}$

$y' = \frac{1}{2} \cdot \left(\frac{1}{x}\right)^{\frac{1}{2}} - 2 \cdot \left(\frac{1}{x}\right)^{3}$

$y' = \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}} - 2 \cdot \frac{1}{x^3}$

$y' = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} - \frac{2}{x^3}$

$y' = \frac{1}{2\sqrt{x}} - \frac{2}{x^3}$

d) $y = 6\sqrt[3]{x} - 5$

$y' = \underline{\underline{2x^{-\frac{2}{3}}}} = \frac{2}{\sqrt[3]{x^2}}$

e) $y = 3\ln x - 9\log x$

$y' = \frac{3}{x}, -\frac{1}{x\ln 9}$

$(\ln x)' = \frac{1}{x}$

$(\log_a x)' = \frac{1}{x\ln a}$

f) $y = \operatorname{tg} x + 11\operatorname{cotg} x$

$y = \frac{1}{\cos^2 x} - \frac{11}{\sin^2 y}$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{cotg} x)' = -\frac{1}{\sin^2 x}$

$(e^x)' = e^x$

$\boxed{(a^x)' = a^x \cdot \ln a}$

g) $y = 3^x + 2e^x (+2)$

$f'(x) = y' = 3^x \ln 3 + 2e^x$

$(2x^2)' = 4x$

1.

a) $f(x) = 2x^2 - x - 5 =$

$f'(x) = 4x - 1$

$f'(3) = 4 \cdot 3 - 1 = \underline{11}$

$x^2 \cdot x^3 = x^5$

3. b) $y = \dfrac{\sqrt{x} \cdot \left(\sqrt[3]{x} - 5\sqrt{x}\right)}{x} = \dfrac{x^{\frac{1}{2}}\left(x^{\frac{1}{3}} - 5x^{\frac{1}{2}}\right)}{x} = \dfrac{x^{\frac{1}{2}+\frac{1}{3}} - 5x^{\frac{1}{2}+\frac{1}{2}}}{x} = \dfrac{x^{\frac{5}{6}} - 5x}{x} = \dfrac{x^{\frac{5}{6}}}{x^1} - \dfrac{5\cancel{x}}{\cancel{x}} =$

$x^{\frac{5}{6}-1} - 5 = x^{-\frac{1}{6}} - 5$

$y' = -\frac{1}{6} \cdot x^{-\frac{1}{6}-1} = \underline{\underline{-\frac{1}{6} \cdot x^{-\frac{7}{6}}}} = \underline{\underline{\dfrac{1}{6\sqrt[6]{x^7}}}}$

$A^2 - B^2 = (A - B) \cdot (A + B)$

c) $y = \dfrac{\cos 2x}{\cos x - \sin x} = \dfrac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \dfrac{\cancel{(\cos x - \sin x)} \cdot (\cos x + \sin x)}{\cancel{\cos x - \sin x}} = \cos x + \sin x$

$y' = -\sin x + \cos x$

$f(x) \cdot g(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

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4. b) $y = \underbrace{(x^2+1)}_{f(x)} \cdot \underbrace{\sin x}_{g(x)}$

$y' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$y' = 2x \cdot \sin x + (x^2+1) \cos x$

d) $y = \underbrace{e^x}_{f(x)} \cdot \underbrace{\lg x}_{g(x)}$

$y' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$y' = e^x \cdot \ln x + e^x \cdot \frac{1}{x} = e^x \ln x + \dfrac{e^x}{x}$

$\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}$

e) $y = \dfrac{2x - 1}{x + 3}$ $f(x)$ $g(x)$

$y' = \dfrac{2 \cdot (x+3) - (2x-1) \cdot 1}{(x+3)^2} = \dfrac{\cancel{2x} + 6 - \cancel{2x} + 1}{(x+3)^2} = \underline{\underline{\dfrac{7}{(x+3)^2}}}$

2.)

2. skupina

a) $y = 4x^2 - x + 1$

$y' = 8x - 1$

$y'' = 8 \quad y''' = 0$

$y' \quad y'' \quad y^{(4)}$

$(x^n)' = n \cdot x^{n-1}$

$(x^1)' = 1 \cdot x^{1-1} = 1 \cdot x^0 = 1 \cdot 1 = \underline{\underline{1}}$

$(k)' = 0 \qquad 5' = 0$

$(3x^2)' = 3 \cdot 2x = 6x$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

b) $y = 2\sin x + 3\cos x$

$y' = 2\cos x - 3\sin x$

c) $y = \sqrt{x} + x^{-2}$

$\sqrt[2]{x^1} = x^{\frac{1}{2}}$

$y = x^{\frac{1}{2}} + x^{-2}$

$\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^{3}$

$y' = \frac{1}{2}x^{\frac{1}{2}-1} + (-2)x^{-3}$

$5^{-2} = \left(\frac{5}{1}\right)^{-2} = \left(\frac{1}{5}\right)^{2} = \frac{1}{5^2}$

$$\boxed{\begin{aligned} y' &= \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-3} \\ y' &= \frac{1}{2\sqrt{x}} - \frac{2}{x^3} \end{aligned}}$$

↓

$\frac{1}{2\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{2x^1}$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{2}}$

d) $y = 6\sqrt[3]{x} - 5$

$y = 6x^{\frac{1}{3}} - 5$

$y' = 6 \cdot \frac{1}{3} \cdot x^{\frac{1}{3}-1}$

$\underline{\underline{y' = 2x^{-\frac{2}{3}} = \frac{2}{\sqrt[3]{x^2}}}} = \frac{2}{x^{\frac{2}{3}}} \cdot \frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}} = \frac{2\sqrt[3]{x}}{x^1}$

e) $y = 3\ln x - 9\log x$

$y' = \frac{3}{x} - \frac{9}{x \cdot \ln 10}$

$(\ln x)' = \frac{1}{x}$

$(\log_a x)' = \frac{1}{x \cdot \ln a}$

$\frac{3}{1} \cdot \frac{1}{x} = \frac{3}{x}$

f) $y = \operatorname{tg} x + 11\operatorname{cotg} x$

$y' = \frac{1}{\cos^2 x} - \frac{11}{\sin^2 x}$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{cotg} x)' = -\frac{1}{\sin^2 x}$

g) $y = 3^x + 2e^x$

$y' = 3^x \cdot \ln 3 + 2e^x$

$(a^x)' = a^x \cdot \ln a$

$(e^x)' = e^x$

1.) d) $f(x) = \cos x$

$f'(x) = -\sin x$

$f'\left(\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = \underline{\underline{-\frac{1}{2}}}$

3.) b) $y = \dfrac{\sqrt{x}\cdot\left(\sqrt[3]{x} - 5\sqrt{x}\right)}{x} = \dfrac{x^{\frac{1}{2}}\cdot\left(x^{\frac{1}{3}} - 5x^{\frac{1}{2}}\right)}{x} = \dfrac{x^{\frac{1}{2}+\frac{1}{3}} - 5x^{\frac{1}{2}+\frac{1}{2}}}{x} = \dfrac{x^{\frac{5}{6}} - 5x}{x} = \dfrac{x^{\frac{5}{6}}}{x^{1}} - \dfrac{5\cancel{x}}{\cancel{x^{1}}} = x^{\frac{5}{6}-\frac{6}{6}} - 5 = x^{-\frac{1}{6}} - 5$

$y' = -\frac{1}{6}x^{-\frac{7}{6}} = \dfrac{-1}{6\cdot\sqrt[6]{x^{7}}}$

d) $y = \dfrac{\sin 2x + 1}{\sin x + \cos x} = \dfrac{2\sin x\cos x + 1}{\sin x + \cos x} = \dfrac{\overset{2AB}{2\sin x\cos x} + \overset{A^2}{\sin^2 x} + \overset{B^2}{\cos^2 x}}{\sin x + \cos x} = \dfrac{\overset{(A+B)^2}{(\sin x + \cos x)^{\cancel{2}}}}{\cancel{\sin x + \cos x}} = \sin x + \cos x$

$y' = \cos x - \sin x$

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$(f(x)\cdot g(x))' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

4.) b) $y = \underbrace{(x^2+1)}_{f(x)}\cdot\underbrace{\sin x}_{g(x)}$

$y' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

$y' = 2x\cdot\sin x + (x^2+1)\cdot\cos x = 2x\cdot\sin x + x^2\cdot\cos x + \cos x$

d) $y = e^x\cdot\ln x$

$y' = e^x\cdot\ln x + e^x\cdot\frac{1}{x} = e^x\ln x + \dfrac{e^x}{x}$

$\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{f(x)'\cdot g(x) - f(x)\cdot g'(x)}{g^2(x)}$ $\quad (A-B)^2$

$(A+B)^2$

f) $y = \dfrac{\sin x + \cos x}{\sin x - \cos x}$ $\begin{matrix} - f(x) \\ - g(x) \end{matrix}$

$y' = \dfrac{(\cos x - \sin x)\cdot(\sin x - \cos x) - (\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2} =$

$\dfrac{-(\sin x - \cos x)^2 - (\sin x + \cos x)^2}{(\sin x - \cos x)^2} = \dfrac{-(\sin^2 x - 2\cancel{\sin x}\cos x + \cos^2 x) - (\sin^2 x + 2\cancel{\sin x}\cos x + \cos^2 x)}{(\sin x - \cos x)^2}$

$= \dfrac{-2\sin^2 x - 2\cos^2 x}{(\sin x - \cos x)^2} = \dfrac{-2(\sin^2 x + \cos^2 x)}{(\sin x - \cos x)^2} =$

$= \dfrac{-2}{(\sin x - \cos x)^2}$