

## 2 Generalize the Take-Away Game

a) remove any number from 1 to 6 chips at each turn, then like this table

<i>residue</i>	<i>WhichwinPorN</i>
0	<i>P</i>
1	<i>N</i>
2	<i>N</i>
4	<i>N</i>
5	<i>N</i>
6	<i>N</i>
7	<i>P</i>
8	<i>N</i>
9	<i>N</i>
10	<i>N</i>
11	<i>N</i>
12	<i>N</i>
13	<i>N</i>

Using the total number of the chips  $N$ , then  $N \bmod 7 = 0$  if the result is 0 then P-position can win, if not then N-position can win

b) if there are 31 chips in the pile. we can compute  $31 \bmod 7 = 3$ , the first remove 3 chips. keeping the remaining chips  $M \bmod 7 = 0$

## 3 Find the set of P-positions for the subtraction games with subtraction sets

a)  $S = \{1, 3, 5, 7\}$

Set initial chips is  $N$ , if  $N \bmod 2 = 0$  then P-position win.

if  $N \bmod 2 \neq 0$  then N-position win

b)  $S = \{1, 3, 6\}$

Set initial chips is  $N$ , if  $N \bmod 9 = 0$  then P-position win.

if  $N \bmod 9 \neq 0$  then N-position win

c)  $S = \{1, 2, 4, 8, 16, \dots\}$  = all powers of 2

Set initial chips is  $N$ , if  $N \bmod 3 = 0$  then P-position win.

if  $N \bmod 3 \neq 0$  then N-position win

d) if play starts at 100 chips

(a)  $100 \bmod 2 = 0$  first-player

(b)  $100 \bmod 9 = 1$  second-player

(c)  $100 \bmod 3 = 1$  second-player