CS 591: Computational Game Theory **Spring 2018**

Problem Set 3

Lecturer: Prof. Peter Chin *Due: March 29, 2018*

- ◇ Please email the written portion (either type up your answer or scan your handwritten solution) & code and report to `kenzhou@bu.edu` by 23:59PM on the due date.
- ◇ Late policy: there will be a penalty of 10% per day, up to three days late. After that no credit will be given.

1. **2-person zero sum games, via program**

   (a) Recall I covered in class, to find equilibrium points

      i. check for saddle points
      ii. use the formula for 2×2 case.
      iii. use the formula for $2 \times n$ case or $m \times 2$ case.
      iv. use the principle of indifference
      v. use the principle of invariance
      vi. use the formula for $n \times n$ case, if the payoff matrix is non degenerate
      vii. use the linear programming formulation

   Using these 7 principles above, write a program, using a language of your choice, whose input is a payoff matrix, and whose output is $(\vec{p}, \vec{q}, V)$, where $\vec{p} = (p_1, p_2, ..., p_m)$ and $\vec{q} = (q_1, q_2, ..., q_n)$ are equilibrium strategies for player 1 and player 2, respectively, and V is the the value of the game. Generate at random 100 payoff matrices with random value of $1 \leq m \leq 100$ and $1 \leq n \leq 100$ and $-1000 \leq a_{ij} \leq 1000$. Test your program on these 100 payoff matrices and compare your answers with the answer you can get from the gambit tool (http://www.gambit-project.org/gambit15/ideas.html). Self score how many you got it right and submit your program, the input, your result, gambit's result and your score in a printout.

   (b) Test your program on the following payoff matrices with the answer you get by solving it by hand (must show work).

      i.
$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 & 7 \end{bmatrix}$$

      ii.
$$A = \begin{bmatrix} 1 & -2 & 3 & -4 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iii.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 4 \\ -1 & 4 & -3 \end{bmatrix}$$

iv.

$$A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -4 \\ -1 & 4 & 0 \end{bmatrix}$$

v.

$$A = \begin{bmatrix} 10 & 0 & 7 & 0 \\ 0 & 6 & 4 & 0 \\ 0 & 0 & 3 & 3 \end{bmatrix}$$

2. **Mathematical Proof**

Prove the following:

(a) Show that every symmetric game must be fair (i.e. has a value 0).

(b) If a game $G = (X, Y, A)$ is invariant under $f : X \mapsto X$ and $g : Y \mapsto Y$ where $A(x, y) = (f(x), g(y)), \forall x \in X, \forall y \in Y$ and f, g are 1-1 and onto, then the optimal strategy $\vec{p}$ and $\vec{q}$ are also invariant. I.e. $f(\vec{p}) = \vec{p}$ and $g(\vec{q}) = \vec{q}$.