CS 591: Computational Game Theory	**Spring 2018**
Problem Set 4	
Lecturer: Prof. Peter Chin	*Due: April 25, 2018*

- ◇ Please email the written portion (either type up your answer or scan your handwritten solution) & code and report to `kenzhou@bu.edu` by 23:59PM on the due date.
- ◇ Late policy: there will be a penalty of 10% per day, up to three days late. After that no credit will be given.

1. **Recursive games, via program**

   (a) Recall I covered in class the game of **Guess-Target-Card**. From a deck with $m+n+1$ distinct cards, m cards are dealt to player 1, n are dealt to player 2, and the remaining *target* card is placed on the table. The goal of the game is to correctly guess the target card as the players alternate. The payoff matrix, as we discussed in class is shown below:

$$G_{m,n} = \begin{bmatrix} \frac{n}{n+1}\bar{G}_{n-1,m} + \frac{1}{n+1} & \frac{n}{n+1}\bar{G}_{n-1,m} \\ \bar{G}_{n,m-1} & 1 \\ \frac{1}{n+1} & \frac{1}{n+1} \end{bmatrix}$$

   where $G_{m,n}$ is the payoff matrix of the game described above and $\bar{G}_{n-1,m}$ is the payoff matrix matrix from player II's perspective with $n-1$ cards in his hand and player 1 having m cards in his hand. The row player has three strategies = {honest, bluff, guess} and column player has two strategies {**ignore** the asked car, **call** the bluff by guessing the asked car}. Write a recursive program that computes the value of this recursive game. Submit your code as well as the printout of the value of this game and the optimal strategies for each player for $1 \leq m, n \leq 6$.

   (b) Write a program or solve by hand (must show work) of the following recursive games

      i.

$$G = \begin{bmatrix} 0 & G_1 \\ G_2 & G_3 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0 & 6 \\ 5 & 1 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$$

ii.

$$G_1 = \begin{bmatrix} G_2 & 1 \\ 1 & 0 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} G_3 & 0 \\ 0 & 2 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} G_1 & 1 \\ 1 & 0 \end{bmatrix}$$

and if the game continues for ever, suppose the payoff is Q.

2. **Coin Toss**

   Louis tosses a coin with the probability p of having heads. For each $k = 1, 2, ...$, if Louis tosses k heads in a row, he may stop and challenge Laura to toss the same number of heads. In that case, if Laura tosses the coin and wins if and only if she tosses k heads in a row. if Louis tosses tails before challenging Laura, then the game continues with the roles reversed. If neither player ever challenges, the game is a draw.

   (a) Solve the game when $p = 1/2$

   (b) For arbitrary p, what are the optimal strategies of the players? Find the limit as $p \to 1$ of the probability that Louis wins.