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mathmetical proof

a)

if a $2P0\sum$ game is symmetic then we know that the matrix A is skew-symmestic, $A = -A^T$,$a_{ij} = -aji$

a symmestic game doesn't matter if 2 palyers change roles which means $\overrightarrow{p} = \overrightarrow{q}$. $(p_1, p_2....., p_m) = (q_1, q_2....., q_m)$ (m=n) $\overrightarrow{p} = \overrightarrow{q}$ are optimal strategies for P_1 and P_2

the value of the game $V = \overrightarrow{p}^T A \overrightarrow{q} = \sum_{i,j=0}^{n} p_i a_{ij} q_j$

then $V = \sum_{i=0}^{n} \sum_{j=0}^{n} p_i a_{ij} q_j$

$= \sum_{i=0}^{n} \sum_{j=0}^{n} p_i a_{ij} p_j$

$= \sum_{i=0}^{n} \sum_{j=0}^{n} p_i - a_{ji} p_j$

$= - \overrightarrow{p}^T A \overrightarrow{p}$

So V=0

b)

assume $\overrightarrow{p}$ is an optimal strategy for P_1 , $\overrightarrow{p} = (p_1, p_2...p_m)$. then the value of the game $V \leq \sum_{i=0}^{m} p_i a_{ij}$

Define a new mixed strategy $\overrightarrow{p}^*$ f is a bijection on X

$\overrightarrow{p}^*(x_i) = \frac{1}{2}p(x_i) + \frac{1}{2}p(f(x_i))$

then $\overrightarrow{p}^*(f(x_i)) = \frac{1}{2}p(f(x_i)) + \frac{1}{2}p(f(f(x_i)))$

$= \frac{1}{2}p(x_i) + \frac{1}{2}p(f(x_i))$

so $\overrightarrow{p}^*(x_i) = \overrightarrow{p}^*(f(x_i))$

thus $\sum_{i=0}^{m} \overrightarrow{p}^* a_{ij}$

$= \sum_{i=0}^{m} \frac{1}{2}(p(x_i) + p(f(x_i)))a_{ij}$

$= \frac{1}{2}\sum_{i=0}^{m} p_i a_{ij} + \frac{1}{2}\sum_{i=0}^{m} p(f(x_i))A(x_i, y_i)$

$= \frac{1}{2}\sum_{i=0}^{m} p_i a_{ij} + \frac{1}{2}\sum_{i=0}^{m} p(f(x_i))A(f(x_i), g(y_i))$

$\geq \frac{1}{2}V + \frac{1}{2}V = V$

So $f(\overrightarrow{p}) = \overrightarrow{p}$

same theory that $g(\overrightarrow{q}) = \overrightarrow{q}$