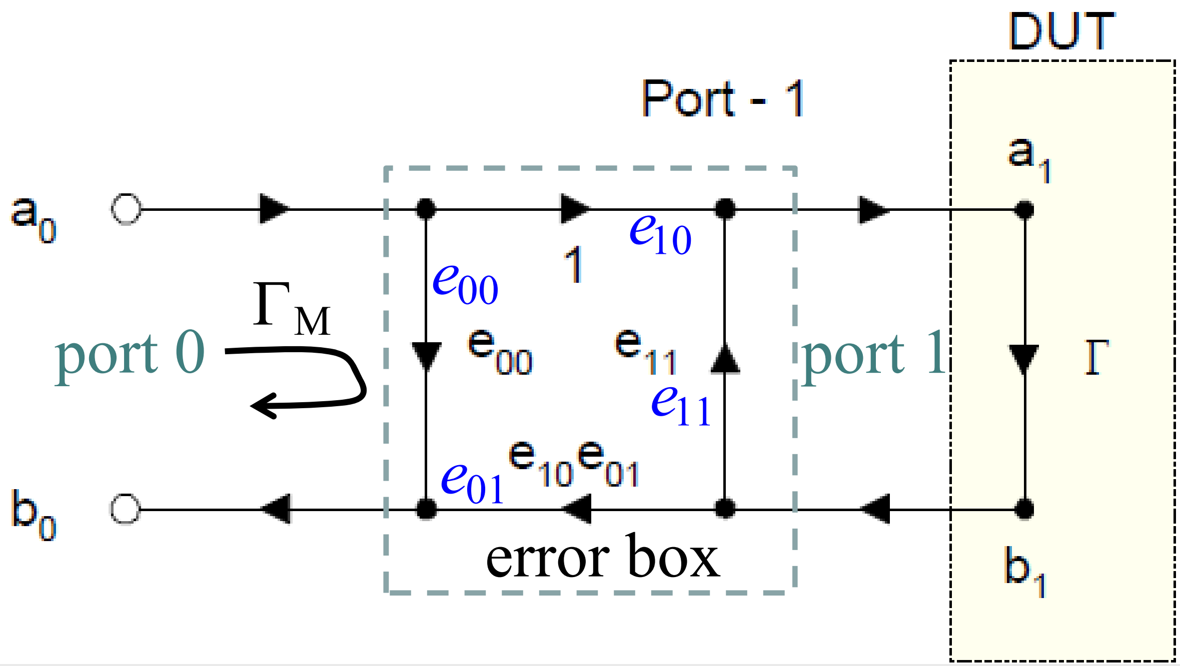
VNA Calibration for 1-port Measurements (3-term Error Model)

- the 3-term error model is known as the OSM (Open-Short-Matched) *cal* technique (*aka* OSL or SOL, Open-Short-Load)
- the *cal* procedure includes 3 measurements performed before the DUT is measured: 1) open circuit, 2) short circuit, 3) matched load
- used when $\Gamma = S_{11}$ of a single-port device is measured
- actual measurements include losses and phase delays in connectors and cables, leakage and parasitics inside the instrument – these are viewed as a 2-port ***error box***
- calibration aims at de-embedding these errors from the total measured *S*-parameters

3-term Error Model: Signal-flow Graph

[Rytting, *Network Analyzer Error Models and Calibration Methods*]



e_{00} = Directivity

e_{11} = Port Match

$(e_{10}e_{01})$ = Tracking

Note: SFG branches without a coefficient have a default coefficient of 1.

- the *S*-matrix of the error box contains in effect 3 unknowns

$$\boldsymbol{S}_E = \begin{bmatrix} e_{00} & 1 \\ e_{10}e_{01} & e_{11} \end{bmatrix} \overset{\text{equivalent}}{\Leftrightarrow} \boldsymbol{S}'_E = \begin{bmatrix} e_{00} & e_{01} \\ e_{10} & e_{11} \end{bmatrix}$$

3-term Error Model: Error-term Equations

Measured

$$\Gamma_M = \frac{b_0}{a_0} = \frac{e_{00} - \Delta_e \Gamma}{1 - e_{11}\Gamma}$$

Actual

$$\boxed{\Gamma = \frac{\Gamma_M - e_{00}}{\Gamma_M e_{11} - \Delta_e}}$$

error de-embedding formula

$$\Delta_e = e_{00}e_{11} - (e_{10}e_{01})$$

compare with sl. 10

Using the result from the example on sl. 10 and the signal flow graph in sl. 12, prove the formula

$$\Gamma_{\mathrm{M}} = \frac{e_{00} - \Delta_e \cdot \Gamma}{1 - e_{11}\Gamma}$$

Prove that the S-matrices of the error box in sl. 12, $\boldsymbol{S}_E$ and $\boldsymbol{S'}_E$, result in the same expression for Γ_{M}.

3-term Error Model

- the 3 calibration measurements with the 3 standard known loads (Γ_1, Γ_2, Γ_3) produce 3 equations for the 3 unknown error terms

$$\begin{cases} e_{00} + \Gamma_1 \Gamma_{M1} e_{11} - \Gamma_1 \Delta_e = \Gamma_{M1} \\ e_{00} + \Gamma_2 \Gamma_{M2} e_{11} - \Gamma_2 \Delta_e = \Gamma_{M2} \\ e_{00} + \Gamma_3 \Gamma_{M3} e_{11} - \Gamma_3 \Delta_e = \Gamma_{M3} \end{cases}$$

linear system for $\boldsymbol{x}^T = [e_{00}, e_{11}, \Delta_e]$

$$\Rightarrow (e_{00}, e_{11}, \Delta_e) \Rightarrow \boxed{\Gamma = \frac{\Gamma_M - e_{00}}{\Gamma_M e_{11} - \Delta_e}}$$

error de-embedding

- ideally, in the OSM calibration,

$$\Gamma_1 = \Gamma_o = 1$$
$$\Gamma_2 = \Gamma_s = -1$$
$$\Gamma_3 = \Gamma_m = 0$$

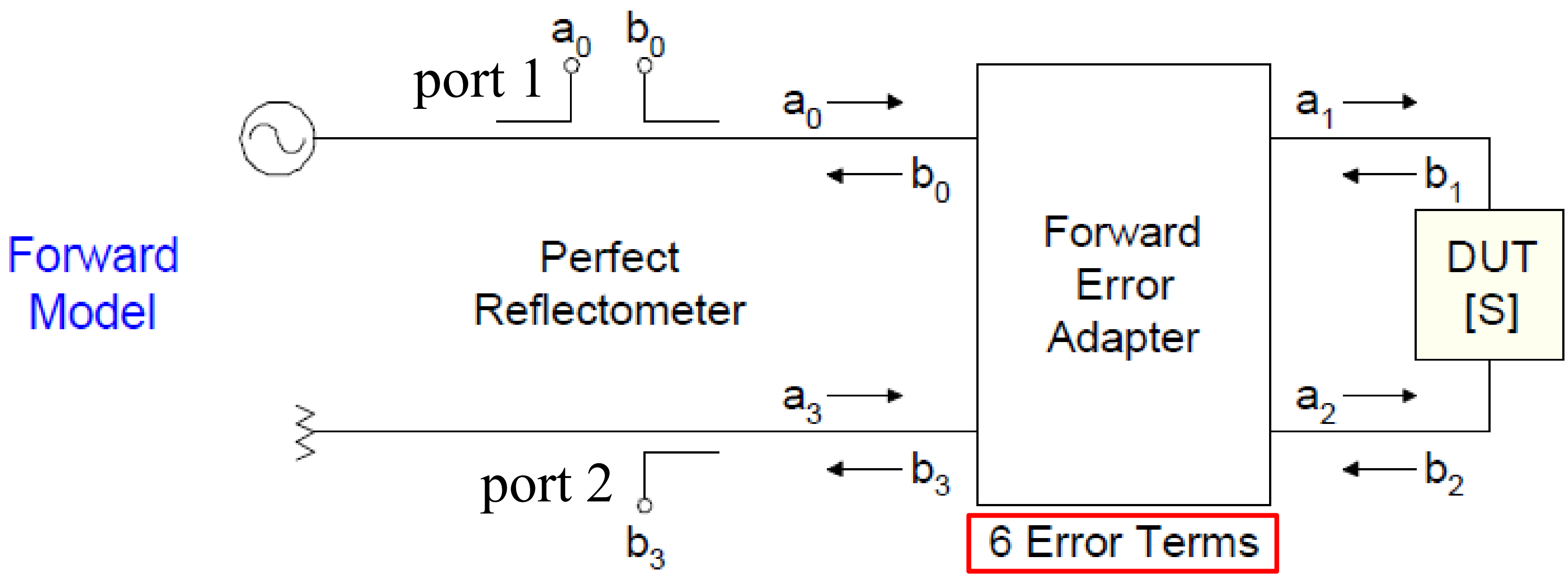
- for accurate results, one has to know the exact values of Γ_o, Γ_s and Γ_m – use manufacturer's cal kits!

2-port Calibration: Classical 12-term Error Model

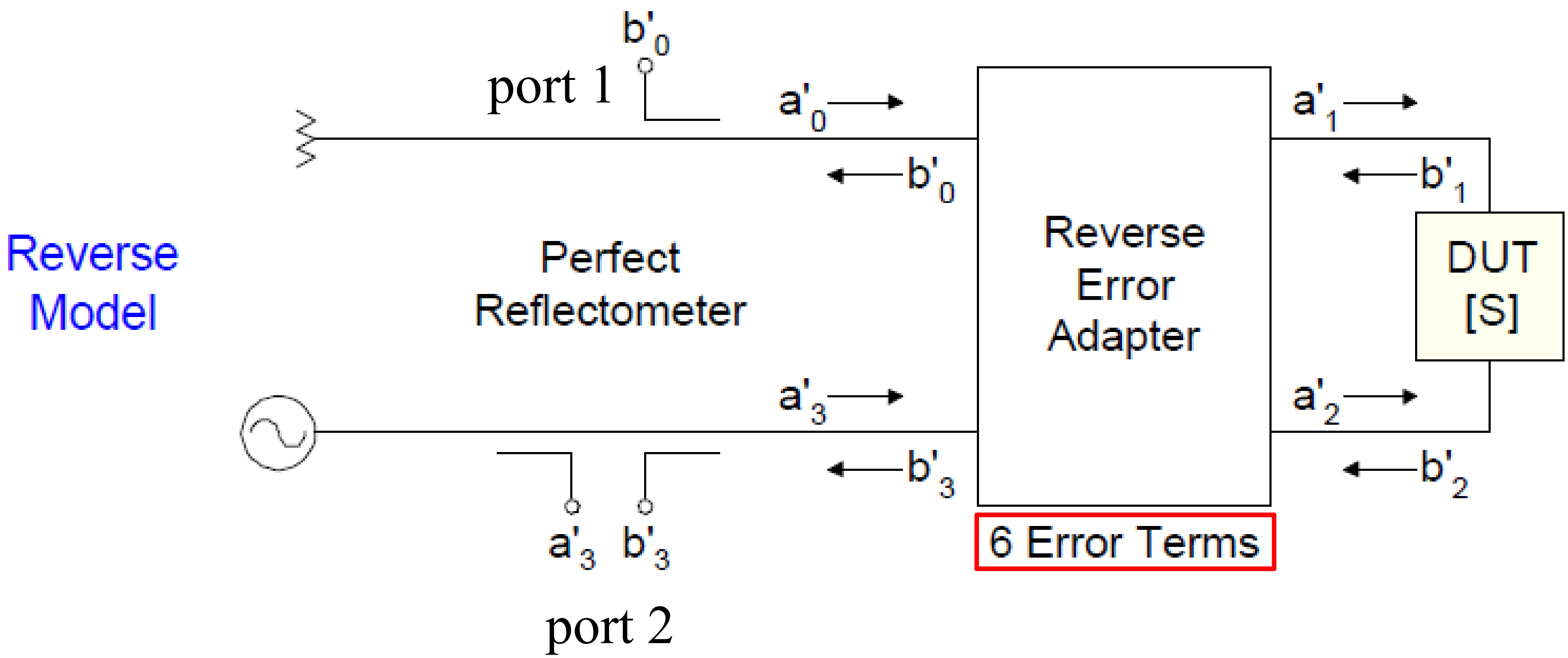
[Rytting, *Network Analyzer Error Models and Calibration Methods*]

consists of two models:

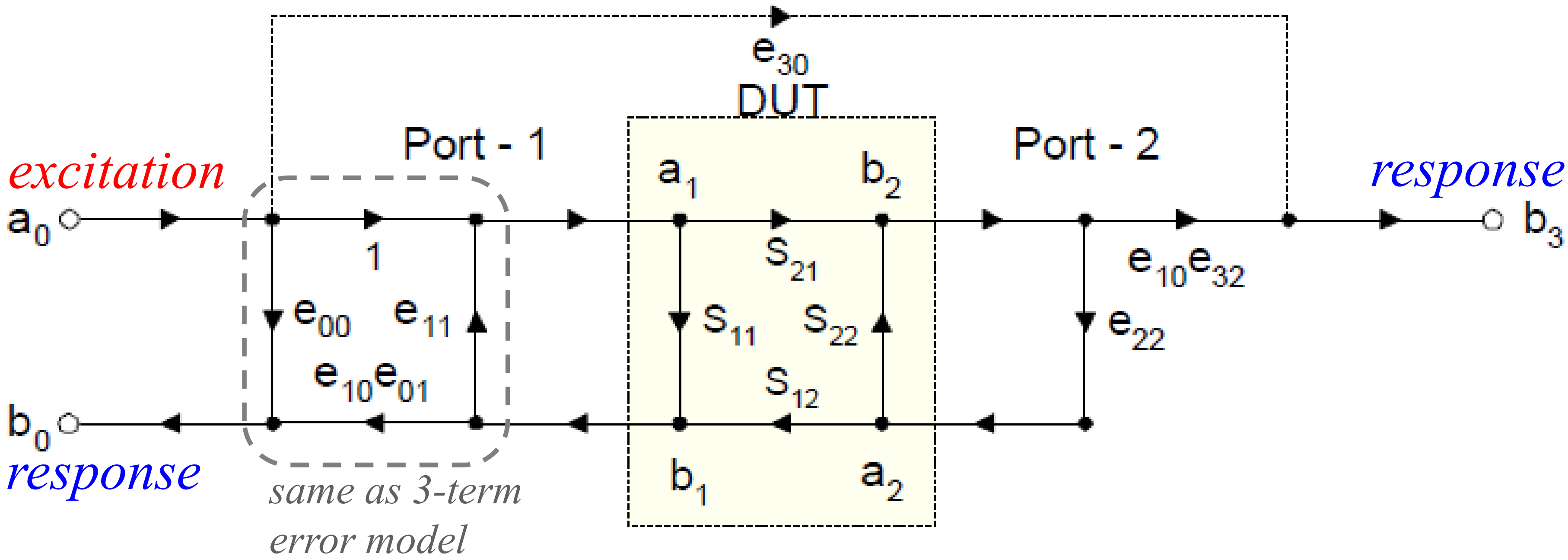
- ***forward*** (excitation at port 1): models errors in S_{11M} and S_{21M}
- ***reverse*** (excitation at port 2): models errors in S_{22M} and S_{12M}



12-term Error Model: Reverse Model



12-term Error Model: Forward-model SFG



e_{00} = Directivity

e_{11} = Port-1 Match

$(e_{10}e_{01})$ = Reflection Tracking

$(e_{10}e_{32})$ = Transmission Tracking

e_{22} = Port-2 Match

e_{30} = Leakage

$$S_{11M} = \frac{b_0}{a_0} = e_{00} + (e_{10}e_{01}) \frac{S_{11} - e_{22}\Delta_S}{1 - e_{11}S_{11} - e_{22}S_{22} + e_{11}e_{22}\Delta_S}$$

$$S_{21M} = \frac{b_3}{a_0} = e_{30} + (e_{10}e_{32}) \frac{S_{21}}{1 - e_{11}S_{11} - e_{22}S_{22} + e_{11}e_{22}\Delta_S}$$

(*) $\Delta_S = S_{11}S_{22} - S_{21}S_{12}$

12-term Error Model: Forward-model SFG

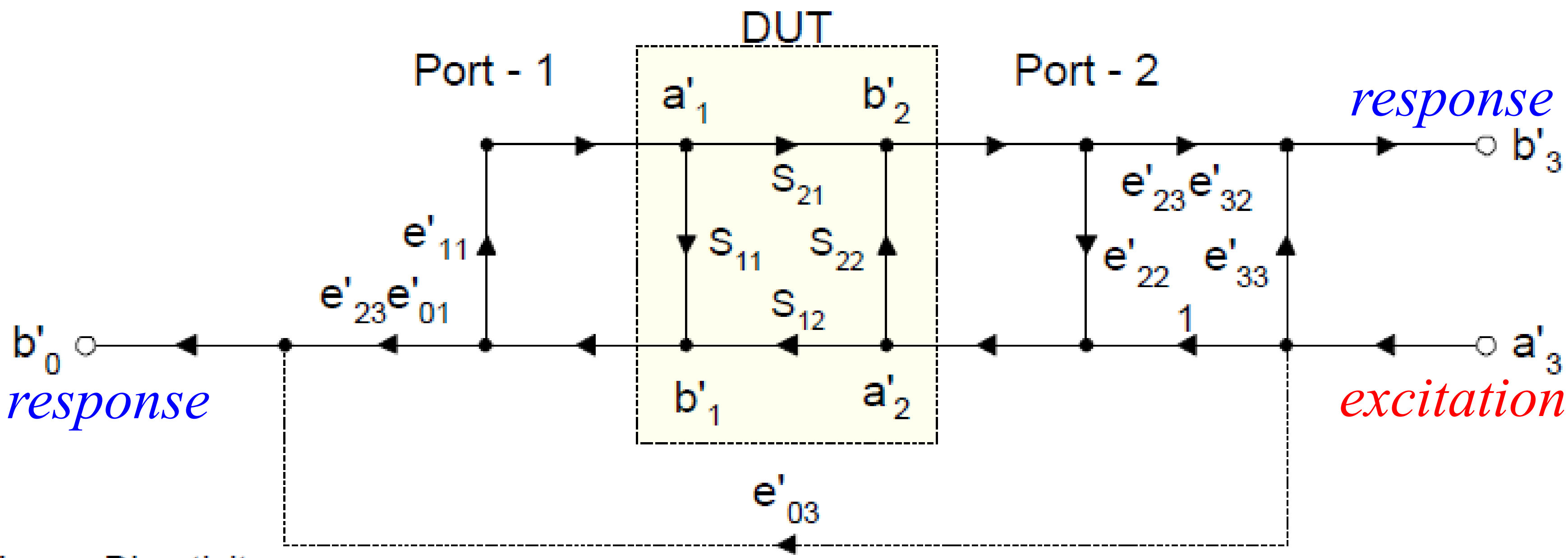
Using signal-flow graph transformations derive the formulas for S_{11M} and S_{21M} in the previous slide.

$$S_{11M} = \frac{b_0}{a_0} = e_{00} + (e_{10}e_{01}) \frac{S_{11} - e_{22}\Delta_S}{1 - e_{11}S_{11} - e_{22}S_{22} + e_{11}e_{22}\Delta_S}$$

$$S_{21M} = \frac{b_3}{a_0} = e_{30} + (e_{10}e_{32}) \frac{S_{21}}{1 - e_{11}S_{11} - e_{22}S_{22} + e_{11}e_{22}\Delta_S}$$

(*) $$\Delta_S = S_{11}S_{22} - S_{21}S_{12}$$ $\boxed{*}$

12-term Error Model: Reverse-model SFG



e'_{33} = Directivity

e'_{11} = Port-1 Match

$(e'_{23}e'_{32})$ = Reflection Tracking

$(e'_{23}e'_{01})$ = Transmission Tracking

e'_{22} = Port-2 Match

e'_{03} = Leakage

$$S_{22M} = \frac{b'_3}{a'_3} = e'_{33} + (e'_{23}e'_{32})\frac{S_{22} - e'_{11}\Delta_S}{1 - e'_{11}S_{11} - e'_{22}S_{22} + e'_{11}e'_{22}\Delta_S}$$

$$S_{12M} = \frac{b'_0}{a'_3} = e'_{03} + (e'_{23}e'_{01})\frac{S_{12}}{1 - e'_{11}S_{11} - e'_{22}S_{22} + e'_{11}e'_{22}\Delta_S}$$

(**)

$$\Delta_S = S_{11}S_{22} - S_{21}S_{12}$$

12-term Calibration Method

Step 1: (***Port 1 Calibration***) using the OSM 1-port procedure, obtain e_{11}, e_{00}, and Δ_e, from which ($e_{10}e_{01}$) is obtained. *(sl. 12)*

Step 2: (***Isolation***) Connect matched loads (Z_0) to both ports. ($S_{21} = 0$) The measured S_{21M} yields e_{30} directly. ($S_{12M} = e'_{03}$) *(sl. 17)*

Step 3: (***Thru***) Connect ports 1 and 2 directly. (S_{21}=S_{12}=1, S_{11}=S_{22}=0)

Obtain e_{22} and $e_{10}\,e_{32}$ from eqns. (*) using $S_{21} = S_{12} = 1$, $S_{11} = S_{22} = 0$. $\Rightarrow$

$$e_{22} = \frac{S_{11M} - e_{00}}{S_{11M}e_{11} - \Delta_e} \quad \leftarrow \textit{port 2 match}$$

$$\textit{transmission tracking} \rightarrow e_{10}e_{32} = (S_{21M} - e_{30})(1 - e_{11}e_{22})$$

- All 6 error terms of the forward model are now known.
- Same procedure is repeated for port 2.

12-term Calibration Method: Error De-embedding

$$S_{11} = \frac{\left(\frac{S_{11M}-e_{00}}{e_{10}e_{01}}\right)\left[1+\left(\frac{S_{22M}-e'_{33}}{e'_{23}e'_{32}}\right)e'_{22}\right] - e_{22}\left(\frac{S_{21M}-e_{30}}{e_{10}e_{32}}\right)\left(\frac{S_{12M}-e'_{03}}{e'_{23}e'_{01}}\right)}{D}$$

$$S_{21} = \frac{\left(\frac{S_{21M}-e_{30}}{e_{10}e_{32}}\right)\left[1+\left(\frac{S_{22M}-e'_{33}}{e'_{23}e'_{32}}\right)(e'_{22}-e_{22})\right]}{D}$$

$$S_{22} = \frac{\left(\frac{S_{22M}-e'_{33}}{e'_{23}e'_{32}}\right)\left[1+\left(\frac{S_{11M}-e_{00}}{e_{10}e_{01}}\right)e_{11}\right] - e'_{11}\left(\frac{S_{21M}-e_{30}}{e_{10}e_{32}}\right)\left(\frac{S_{12M}-e'_{03}}{e'_{23}e'_{01}}\right)}{D}$$

$$S_{12} = \frac{\left(\frac{S_{12M}-e'_{03}}{e'_{23}e'_{01}}\right)\left[1+\left(\frac{S_{11M}-e_{00}}{e_{10}e_{01}}\right)(e_{11}-e'_{11})\right]}{D}$$

$$D = \left[1+\left(\frac{S_{11M}-e_{00}}{e_{10}e_{01}}\right)e_{11}\right]\left[1+\left(\frac{S_{22M}-e'_{33}}{e'_{23}e'_{32}}\right)e'_{22}\right] - \left(\frac{S_{21M}-e_{30}}{e_{10}e_{32}}\right)\left(\frac{S_{12M}-e'_{03}}{e'_{23}e'_{01}}\right)e_{22}e'_{11}$$

2-port Thru-Reflect-Line Calibration

- TRL (Thru-Reflect-Line) calibration is used when classical standards such as open, short and matched load cannot be realized
- TRL is the calibration used when measuring devices with non-coaxial terminations (HMIC and MMIC)
- TRL calibration is based on an 8-term error model
- TRL calibration requires three (2-port) custom calibration structures

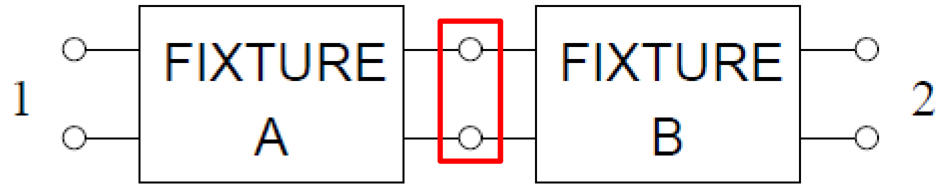
thru: the 2 ports must be connected directly, **sets the reference planes**

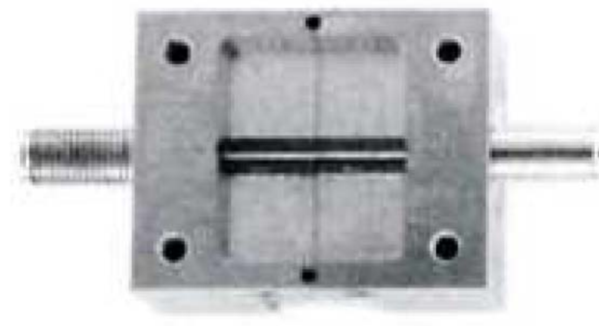
reflect: same load on each port (preferred); must have large reflection

line (or ***delay***): 2 ports connected with system interconnect (represents the IC interconnect for the measured DUT and **sets** $\boldsymbol{Z_0}$)

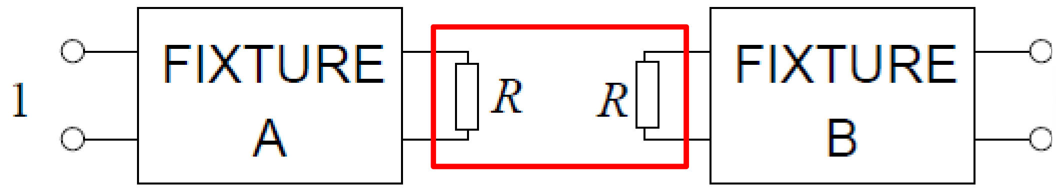
Thru, Reflect, and Line Calibration Connections

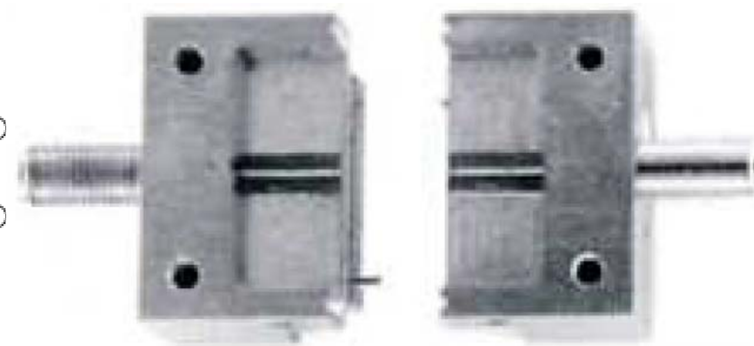
(a) ***thru***



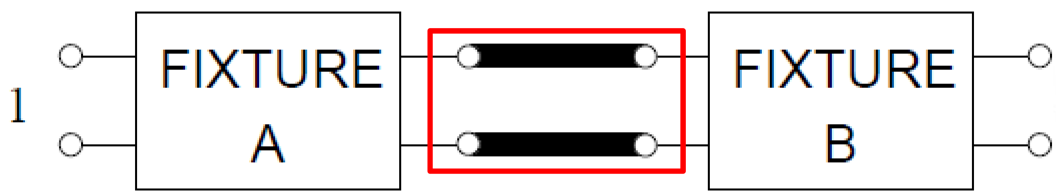


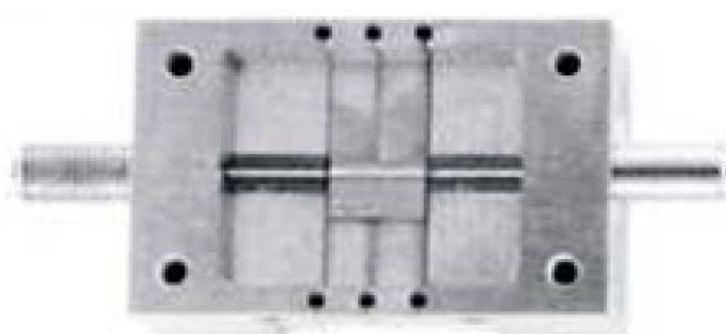
(b) ***reflect***



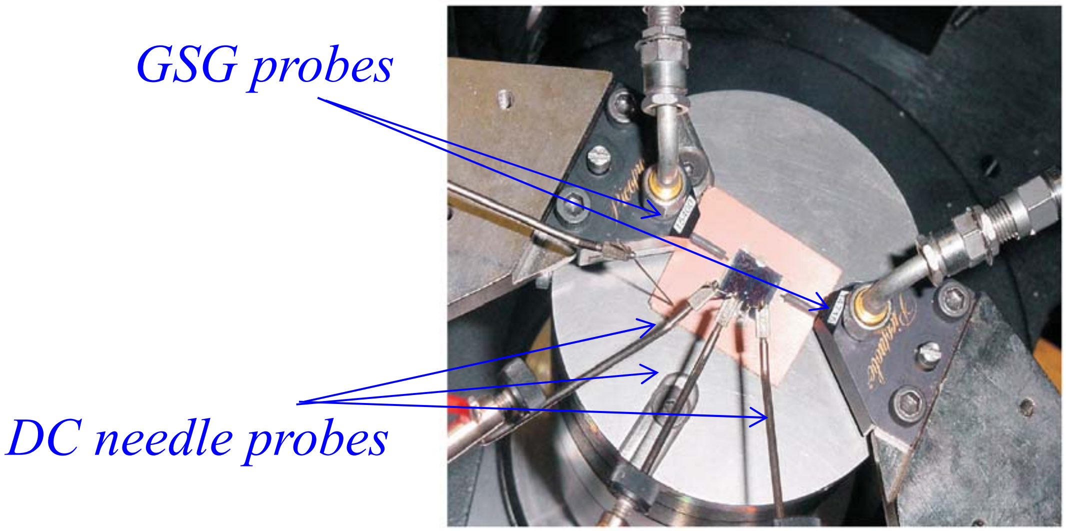


(c) ***line***

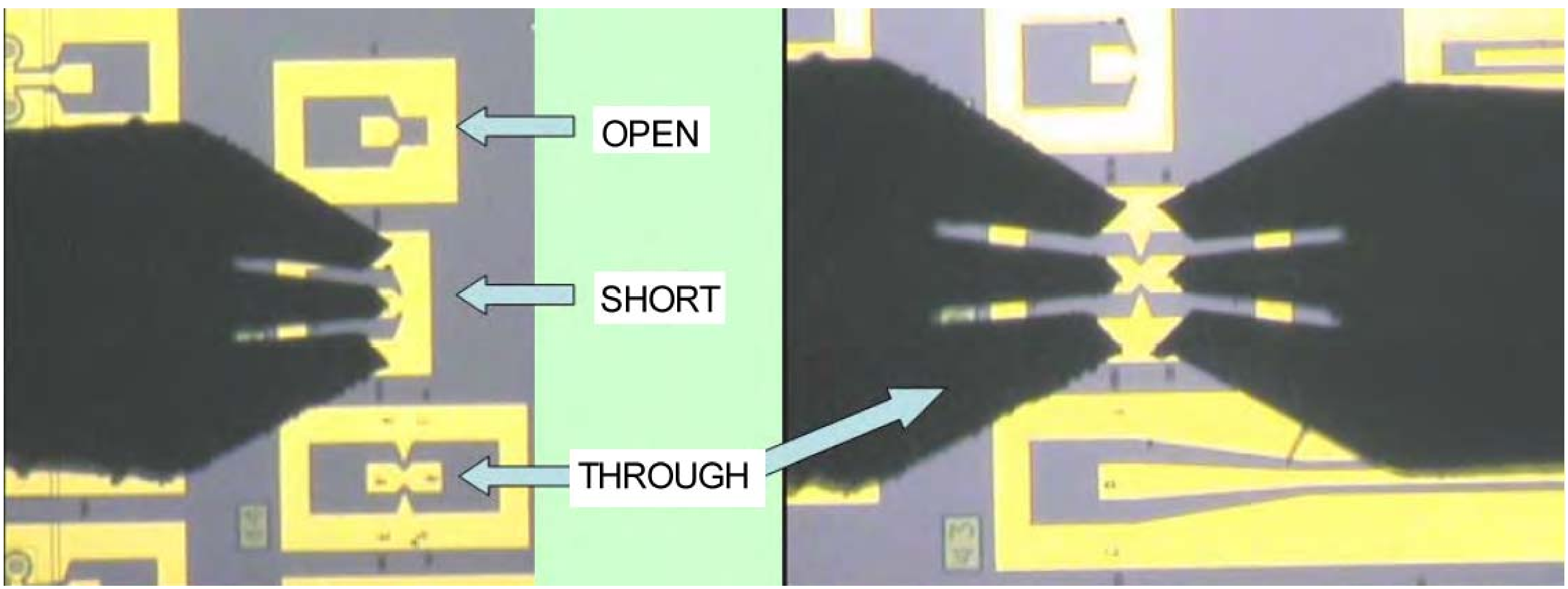




Thru-Reflect-Line Calibration Fixtures

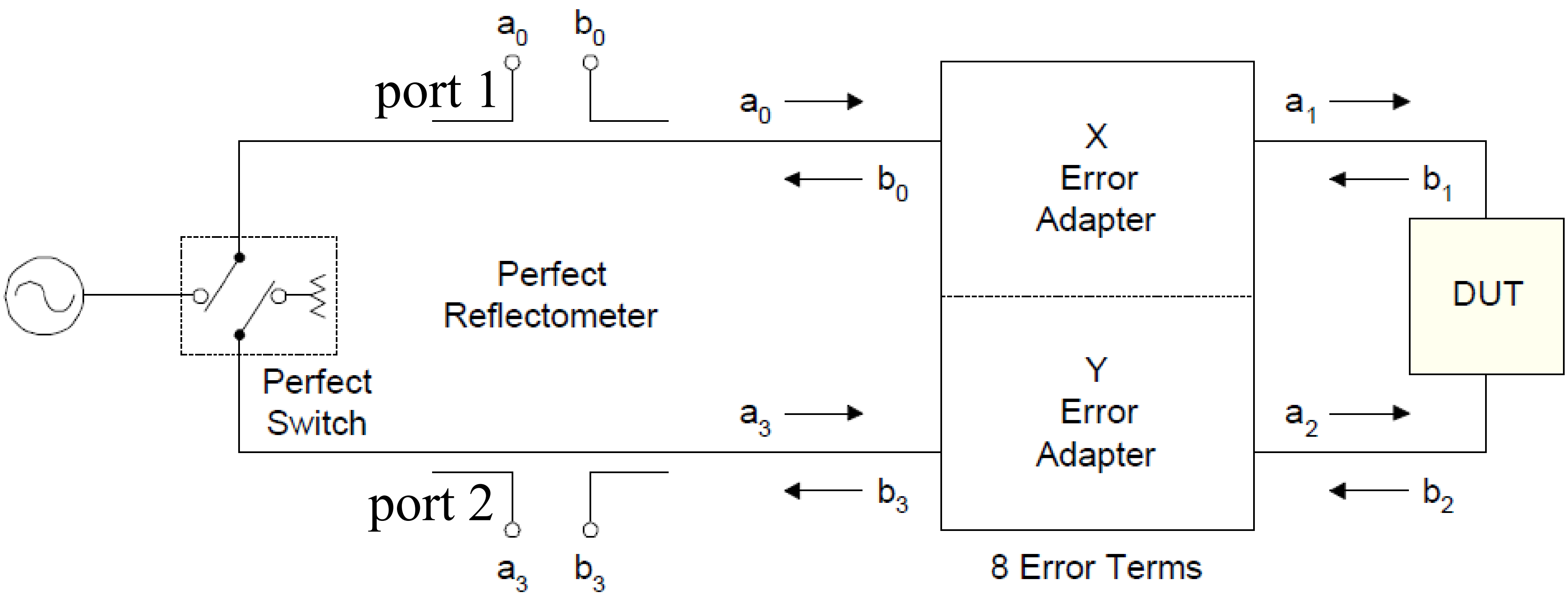


[Steer, *Microwave and RF Design*]

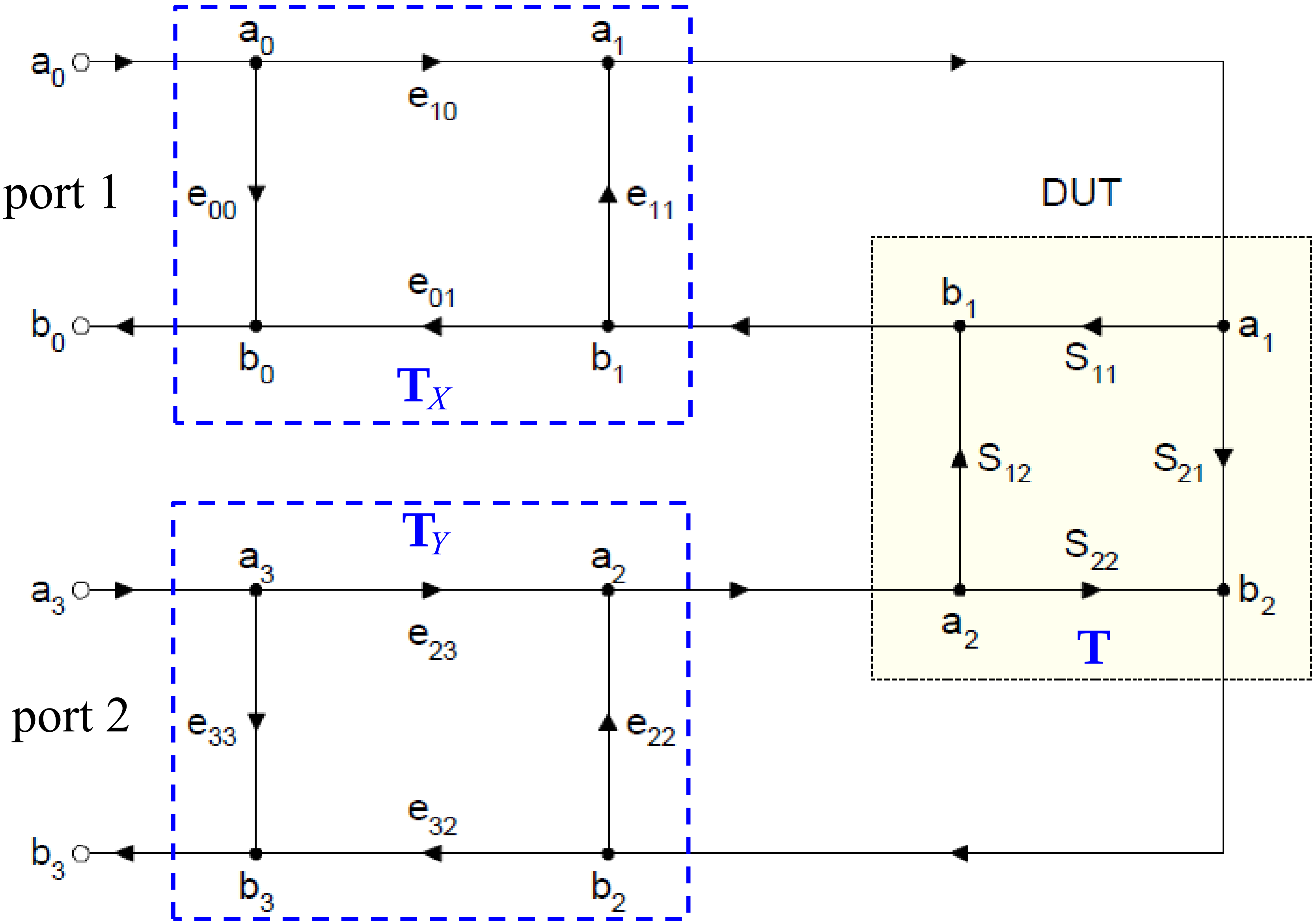


2-port Calibration: 8-term Error Model

[Rytting, *Network Analyzer Error Models and Calibration Methods*]

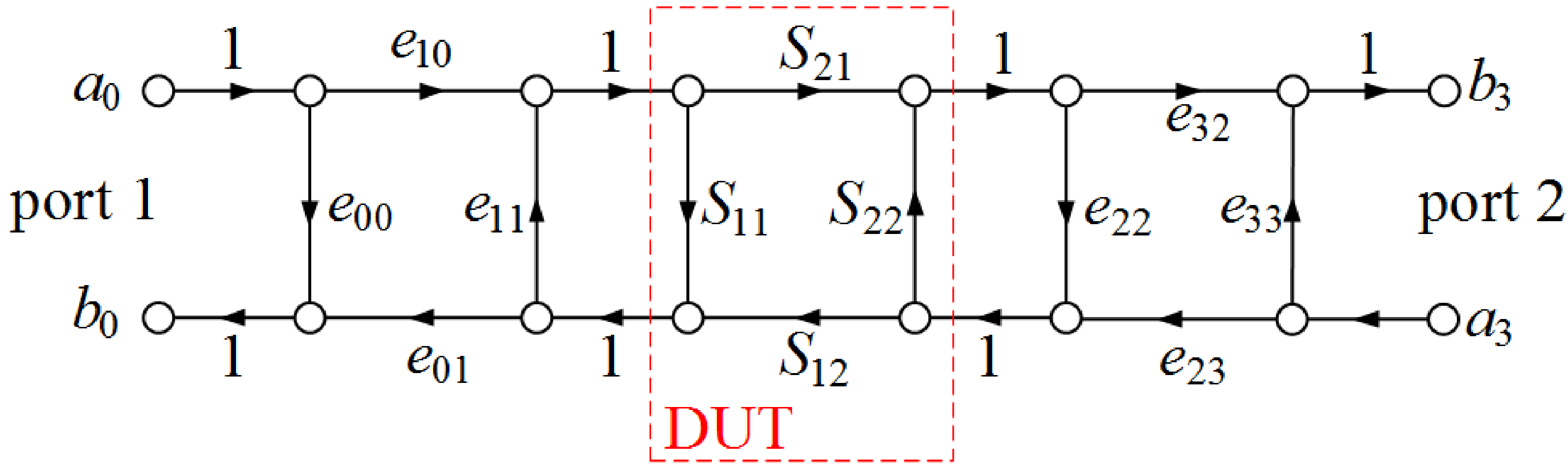


Signal-flow Graph of 8-term Error Model



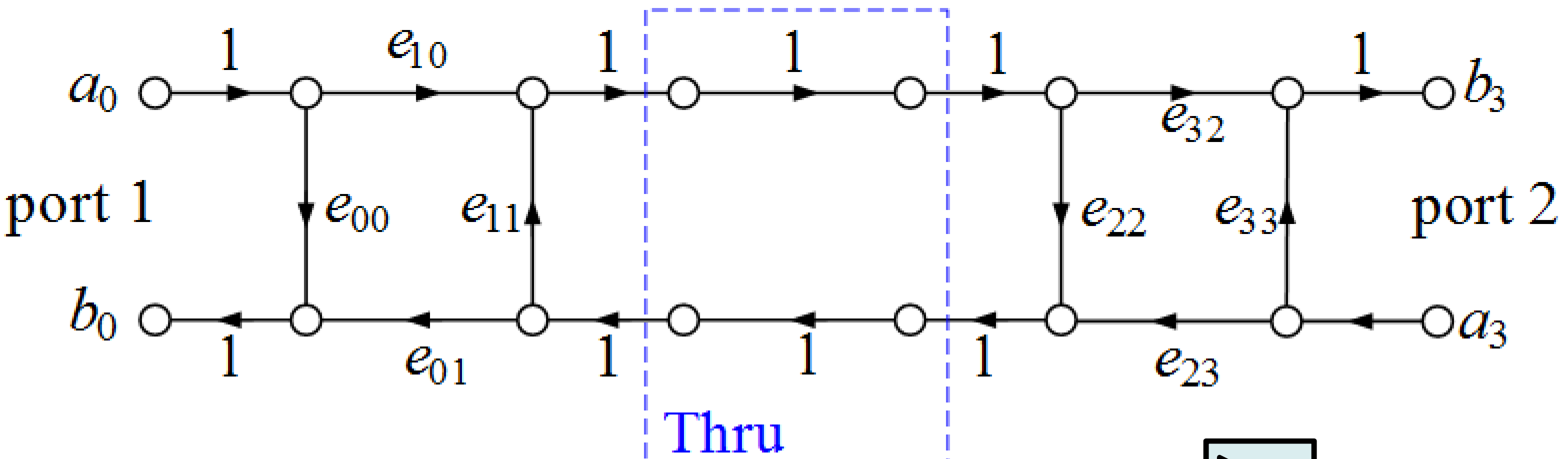
TRL Calibration: SFG with DUT

- unfolded SFG of the DUT measurement



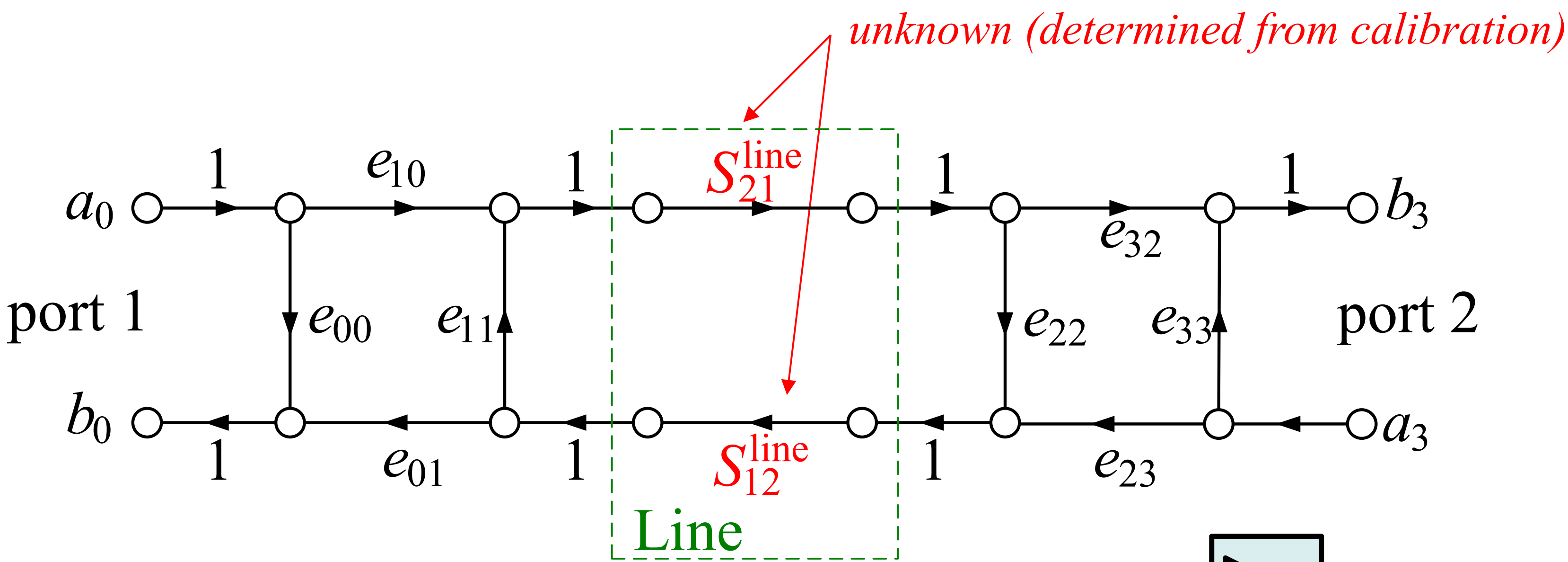
TRL Calibration: SFG of *Thru* Measurement

- we must know all 4 ***Thru*** S-parameters
- if ***Thru*** is assumed of zero length, then reference plane for all ports is set in its middle: $S_{21}^{\text{thru}} = S_{12}^{\text{thru}} = 1$
- if ***Thru*** assumed perfectly matched, then $S_{11}^{\text{thru}} = S_{22}^{\text{thru}} = 0$ (then it must be made with the same line as that in the ***Line*** standard, which determines Z_0)



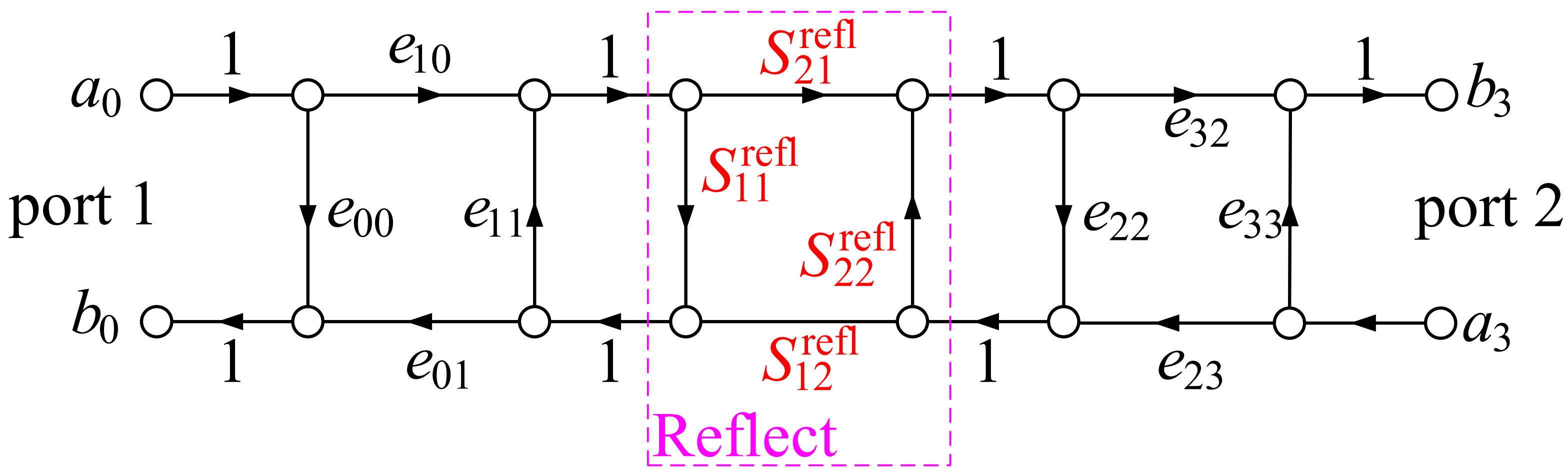
TRL Calibration: SFG of *Line* Measurements

- we need to know only 2 ***Line*** S-parameters
- ***Line*** determines Z_0 and is, therefore, assumed perfectly matched to Z_0: $S_{11}^{\text{line}} = S_{22}^{\text{line}} = 0$ (2 known parameters)
- must have different physical length compared to ***Thru***



TRL Calibration: SFG of *Reflect* Measurements

- must have high reflection ($S_{11}^{\text{refl}}, S_{22}^{\text{refl}}$) on both ports!
- only one piece of information is needed, for example
  - $S_{11}^{\text{refl}} = S_{22}^{\text{refl}}$ (most common)
  - $S_{21}^{\text{refl}} = S_{12}^{\text{refl}}$



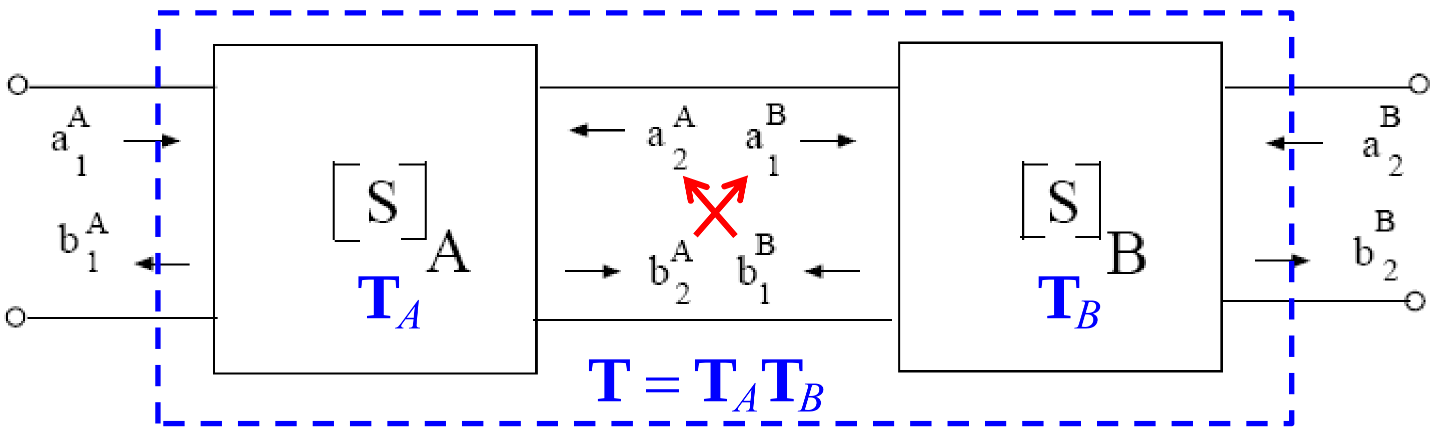
Scattering Transfer (or Cascade) Parameters

- when a network is a cascade of 2-port networks, often the scattering transfer (T-parameters) are used

$$\begin{bmatrix} V_1^- \\ V_1^+ \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} V_2^+ \\ V_2^- \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$$

- relation to S-parameters

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = S_{21}^{-1} \begin{bmatrix} -\Delta_S & S_{11} \\ -S_{22} & 1 \end{bmatrix}, \quad \Delta_S = S_{11}S_{22} - S_{12}S_{21}$$



8-term Error Model in Terms of *T*-parameters for TRL Calibration

MEASURED

$$\mathbf{T}_M = \mathbf{T}_X \mathbf{T}\,\mathbf{T}_Y$$

ACTUAL

$$\boxed{\mathbf{T} = \mathbf{T}_X^{-1}\,\mathbf{T}_M\,\mathbf{T}_Y^{-1}}$$

error de-embedding

$$\mathbf{T} = \frac{1}{S_{21}}\begin{bmatrix} -\Delta_S & S_{11} \\ -S_{22} & 1 \end{bmatrix}$$

$$\Delta_S = S_{11}S_{22} - S_{12}S_{21}$$

T *in terms of* **S**

$$\mathbf{T}_M = \frac{1}{S_{21M}}\begin{bmatrix} -\Delta_M & S_{11M} \\ -S_{22M} & 1 \end{bmatrix}$$

$$\Delta_M = S_{11M}S_{22M} - S_{12M}S_{21M}$$

$$\mathbf{T}_X = \frac{1}{e_{10}}\begin{bmatrix} -\Delta_X & e_{00} \\ -e_{11} & 1 \end{bmatrix}$$

$$\Delta_X = e_{00}e_{11} - e_{10}e_{01}$$

T *matrices of error boxes*

$$\mathbf{T}_Y = \frac{1}{e_{32}}\begin{bmatrix} -\Delta_Y & e_{22} \\ -e_{33} & 1 \end{bmatrix}$$

$$\Delta_Y = e_{22}e_{33} - e_{32}e_{23}$$

8-term Error Model for TRL Calibration

- the number of unknown error terms is actually 7 in the simple cascaded TRL network (see sl. 26)

$$\mathbf{T}_M = \frac{1}{(e_{10}e_{32})}\underbrace{\begin{bmatrix} -\Delta_X & e_{00} \\ -e_{11} & 1 \end{bmatrix}}_{\mathbf{A}}\mathbf{T}\underbrace{\begin{bmatrix} -\Delta_Y & e_{22} \\ -e_{33} & 1 \end{bmatrix}}_{\mathbf{B}} = \frac{1}{(e_{10}e_{32})}\mathbf{ATB}$$

$$\Rightarrow \mathbf{T} = (e_{10}e_{32})\mathbf{A}^{-1}\mathbf{T}_M\mathbf{B}^{-1}$$

- TRL measurement procedure

(1) $\mathbf{T}_{M1} = \mathbf{T}_X\mathbf{T}_{C1}\mathbf{T}_Y \;\rightarrow$ measured with 2-port cal standard #1
(2) $\mathbf{T}_{M2} = \mathbf{T}_X\mathbf{T}_{C2}\mathbf{T}_Y \;\rightarrow$ measured with 2-port cal standard #2
(3) $\mathbf{T}_{M3} = \mathbf{T}_X\mathbf{T}_{C3}\mathbf{T}_Y \;\rightarrow$ measured with 2-port cal standard #3
(4) $\mathbf{T}_M = \mathbf{T}_X\mathbf{T}\mathbf{T}_Y \;\rightarrow$ measured with DUT

- we need to find the 7 error terms from (1), (2) and (3)

8-term Error Model for TRL Calibration

- measuring the 3 two-port cal standards yields 12 independent equations while we have only 7 error terms

THRU (4) + LINE(4) + REFLECT(4)

- thus 5 parameters of the 3 cal standards need not be known and can be determined from the calibration measurements

- which 5 parameters are chosen for which cal standards is important in order to reduce errors and avoid singular matrices
  - cal standard #1 **(thru)** $\mathbf{T}_{C1}$ must be completely known *(sl. 28)*
    (common choice: ref. plane in the middle, perfect match)
  - cal standard #2 **(line)** $\mathbf{T}_{C2}$ can have 2 unknowns *(sl. 29)*
  - cal standard #3 **(reflect)** $\mathbf{T}_{C3}$ can have 3 unknowns *(sl. 30)*

VNA Calibration – Summary

- errors are introduced when measuring a device due to parasitic coupling, leakage, reflections and imperfect directivity
- these errors must be de-embedded from the overall measured *S*-parameters
- the de-embedding relies on the measurement of known or partially known cal standards – calibration measurements, which precede the measurement of the DUT
- 1-port calibration uses the 3-term error model and the OSM method
- 2-port calibration may use 12-term or 8-term error models
- the 12-term error model requires ***OSM*** at each port, ***isolation***, and ***thru*** measurements
- the 8-term error model with the TRL technique is widely used for non-coaxial devices – requires custom fixtures for ***thru***, ***reflect*** & ***line***
- there exists also a 16-term error model, many other cal techniques