UTM

UNIVERSITI TEKNOLOGI MALAYSIA

DISCRETE STRUCTURE

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No. Date

ASSIGNMENT 2 (CHAPTER 2)

Q1 : RELATION

1. $A = \{2, 3, 4, 5, 6, 7, 8\}$ xRy if $x - y = 3n$ $n \in Z$

$R = \{(2,5), (2,8), (3,6), (4,7), (5,2), (5,8), (6,3), (7,4), (8,5), (8,2)\}$

2. $A = \{1, 2, 3\}$ and $B = \{9, 8, 7\}$

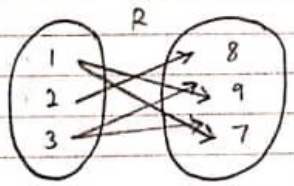
R : A to B $(a,b) \in A \times B$, $a \, R \, b \leftrightarrow$ even number

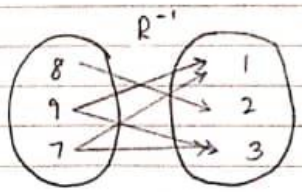
a. Determine R and R^{-1}

$R = \{(1,9), (1,7), (2,8), (3,9), (3,7)\}$

$R^{-1} = \{(9,1), (7,1), (8,2), (9,3), (7,3)\}$

b. Draw arrow diagram for both.





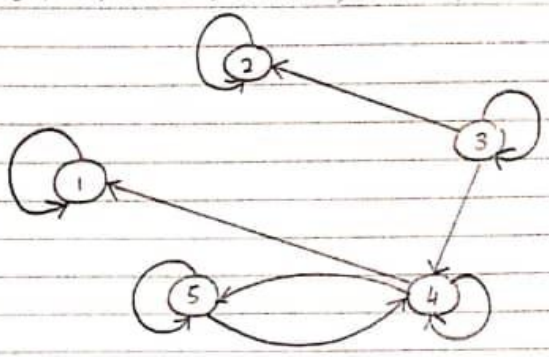
c. Describe R^{-1} in words.

The R^{-1} can be described as pairs elements of R in reverse order. such that if R pairs elements (a, b), the inverse relation pairs them as (b, a).

3. $A = \{1, 2, 3, 4, 5\}$

	1	2	3	4	5
1	1	0	0	0	0
2	0	1	0	0	0
3	0	1	1	1	0
4	1	0	0	1	1
5	0	0	0	1	1

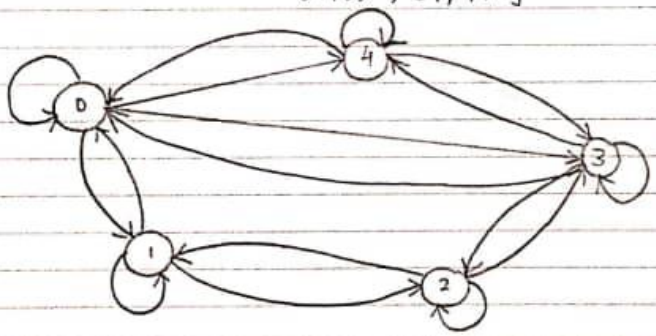
(1,1), (2,2), (3,2), (3,3), (3,4)
(4,1), (4,4), (4,5), (5,4), (5,5)



	1	2	3	4	5
In degree	2	2	1	3	2
Out degree	1	1	3	3	2

4. $A = \{0, 1, 2, 3, 4\}$ $R = \{(0,0), (0,1), (0,3), (0,4), (1,0), (1,1), (1,2), (2,1), (2,2), (2,3), (3,0), (3,2), (3,3), (3,4), (4,0), (4,3), (4,4)\}$

Digraph :



Find is R reflexive, symmetric or transitive.

R is reflexive because $(A, A) \in R$

M_R =

	0	1	2	3	4
0	1	1	0	1	1
1	1	1	1	0	0
2	0	1	1	1	0
3	1	0	1	1	1
4	1	0	0	1	1

symmetric because $(0,1)$ and $(1,0) \in R$
$(0,3)$ and $(3,0) \in R$
$(0,4)$ and $(4,0) \in R$
$(1,2)$ and $(2,1) \in R$
$(2,3)$ and $(3,2) \in R$
$(3,4)$ and $(4,3) \in R$

M_R^T =

	1	1	0	1	1
	1	1	1	0	0
	0	1	1	1	0
	1	0	1	1	1
	1	0	0	1	1

$\therefore M_R$ is symmetric because $M_R = M_R^T$

transitive ?

$M_R \otimes M_R = M_R$

∴ M_R not transitive since (0,1) and (1,2) are present but not (0,2)

5. $A = \{ 1, 2, 3 \ldots 13, 14 \}$ $R = \{ (x, y) \cdot 3x - y = 0 \}$

Determine whether the relation is.

a. Reflexive.

$R = \{ (1,3), (2,6), \cancel{(3,9)}\ (3,9), (4,12), \}$

∴ ~~not~~ irreflexive because $(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8)$ $(9,9), (10,10), (11,11), (12,12) \notin R$. ②

b. Symmetric.

∴ ~~not~~ anti symmetric because $(1,3) \in R$ but $(3,1) \notin R$
$(2,6) \in R$ but $(6,2) \notin R$
$(3,9) \in R$ but $(9,3) \notin R$
$(4,12) \in R$ but $(12,4) \notin R$

∴ Transitive.

∴ not transitive because (x,y) and $(y,z) \in R$ but $(x,z) \notin R$.

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6. $R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

Using Boolean Arithmetic, Find :

a. RS

0	0	1	1	0	1	1	1
1	1	0	0	1	1	0	1
0	0	1	1	0	1	1	1
0	0	0	1	0	0	1	1

RS: $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

b. SR

1	0	0	1	0	0	1	1
0	1	0	1	1	1	0	1
0	1	1	0	1	1	1	1
0	0	1	1	0	0	1	1

SR: $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

Q2. FUNCTION

7. A relation can have many outputs for single input, but function has a single input for a single output.

8. (i) Set A is a function. It is one-to-one / onto.
(ii) Set A is a function. It is many-to-one.
(iii) Set A is not a function. It is one-to-many / input receives multiple output
(iv) Set A is not a function. It contains one-to-many / input receives many output

9. $R: \{(0,5), (1,6), (2,7), (3,8), (4,9), (5,10)\}$
domain, $x: \{0, 1, 2, 3, 4, 5\}$
range, $y: \{5, 6, 7, 8, 9, 10\}$

10. (v) $1-2x_1 = 1-2x_2$

$-2x_1 = -2x_2$

$x_1 = x_2$

$f(x)$ is one-one

$f(x) = 1-2x$

$y = 1-2x$

$x = \frac{(y-1)}{2}$

$x = \frac{1-y}{2}$

$f\left(\frac{1-y}{2}\right) = 1-2\left(\frac{1-y}{2}\right)$

$= 1 - \frac{2+2y}{2}$

$= 1 - 1 + y$

$= y$

$f(x)$ is onto //

(vi) $5x_1^2 - 1 = 5x_2^2 - 1$

$5x_1^2 = 5x_2^2$

$x_1^2 = x_2^2$

$x_1 = x_2$

$f(x)$ is one-one

$f(x) = 5x^2 - 1$

$y = 5x^2 - 1$

$x^2 = \frac{y+1}{5}$

$x = \sqrt{\left(\frac{y+1}{5}\right)}$

$f\left(\sqrt{\left(\frac{y+1}{5}\right)}\right) = 5\left(\left(\frac{y+1}{5}\right)^{\frac{1}{2}}\right)^2 - 1$

$= \frac{5y+5}{5} - \frac{1}{1}$

$= y + 1 - 1$

$f(x) = y$

f_x is onto //

(vii) $f(x) = x^4$

$x_1^4 = x_2^4$

$x_1 = x_2$

$f(x)$ is one-one

$f(x) = x^4$ $\quad f(\sqrt[4]{y}) = (\sqrt[4]{y})^4$

$y = x^4$ $\quad f(x) = y$

$x = \sqrt[4]{y}$ $\quad f(x)$ is onto

(viii) $f(x) = \left(\frac{x-2}{x-3}\right)$

$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

$\Rightarrow (x_2-3)(x_1-2) = (x_2-2)(x_1-3)$

$\Rightarrow x_2x_1 - 2x_2 - 3x_1 + 6 = x_2x_1 - 3x_2 - 2x_1 + 6$

$-2x_2 - 3x_1 = -3x_2 - 2x_1$

$2x_2 + 3x_1 = 3x_2 + 2x_1$

$f(x)$ is not one-one.

$f(x) = \left(\frac{x-2}{x-3}\right)$

$y = \frac{x-2}{(x-3)}$

$xy - 3y = x - 2$

$xy - x = 3y - 2$

$x(y-1) = 3y - 2$

$x = \frac{3y-2}{(y-1)}$

$f\left(\frac{3y-2}{y-1}\right) = \frac{2y-2}{y-1} - \frac{2}{1}$

$= \frac{3y-2-2y+2}{y-1}$

$\frac{3y-2-3y+3}{y-1}$

$= \frac{y}{y-1} \times \frac{y-1}{1}$

$= \frac{y}{1} = y$

$f(x)$ is onto

11 (ix) $f(x) = 3x - 1$; $g(x) = x^2 - 1$

$f(g(x)) = 3(x^2 - 1) - 1$

$= 3x^2 - 3 - 1$

$= 3x^2 - 4$

for $x = 0$, $f(g(x)) = 3(0)^2 - 4$

$= -4$

for $x = 1$, $f(g(x)) = 3(1)^2 - 4$

$= -1$

for $x = 2$, $f(g(x)) = 3(2)^2 - 4$

$= 8$

for $x = 3$, $f(g(x)) = 3(3)^2 - 4$

$= 23$

(x) $f(x) = x^2$; $g(x) = 5x - 6$

$f(g(x)) = (5x - 6)^2$

$= 25x^2 - 60x + 36$

for $x = 0$: $25(0)^2 - 60(0) + 36$

$= 36$

for $x = 1$, $f(g(x)) = 25(1)^2 - 60(1) + 36$

$= 1$

for $x = 2$, $f(g(x)) = 25(2)^2 - 60(2) + 36$

$= 16$

for $x = 3$, $f(g(x)) = 25(3)^2 - 60(3) + 36$

$= 81$

(xi) $f(x) = x-1$; $g(x) = x^3+1$

$f(g(x)) = (x^3+1) - 1$

$= x^3$

for $x = 0$, $f(g(x)) = (0)^3$

$= 0$

for $x = 1$, $f(g(x)) = 1^3$

$= 1$

for $x = 2$, $f(g(x)) = 2^3$

$= 8$

for $x = 3$, $f(g(x)) = 3^3$

$= 27$

Q3 : RECURRENCE RELATION

12 (xii) $a_n = 6a_{n-1} - 9a_{n-2}$, $n \geq 2$, $n_0 = 1$ and $n_1 = 6$

$a_2 = 6(6) - 9(1) = 27$

$a_3 = 6(27) - 9(6) = 108$

$a_4 = 6(108) - 9(27) = 405$

$a_5 = 6(405) - 9(108) = 1458$

First six sequence : 1, 6, 27, 108, 405, 1458 ...

(xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$, $n \geq 3$, $a_0 = 2$, $a_1 = 5$, $a_2 = 15$

$a_3 = 6(15) - 11(5) + 6(2) = 47$

$a_4 = 6(47) - 11(15) + 6(5) = 147$

$a_5 = 6(147) - 11(47) + 6(15) = 455$

$a_6 = 6(455) - 11(147) + 6(47) = 1395$

first seven sequence : 2, 5, 15, 47, 147, 455, 1395..

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(iv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$, $n \geq 3$, $a_0 = 1$, $a_1 = -2$, $a_2 = -1$

$a_3 = -3(-1) - 3(-2) + 1 = 10$

$a_4 = -3(10) - 3(-1) + (-2) = -29$

$a_5 = -3(-29) - 3(10) + (-1) = 56$

$a_6 = -3(56) - 3(-29) + 10 = -71$

first seven sequence : $1, -2, -1, 10, -29, 56, -71$

13 (i) $a_4 = a_{3+1} = 5a_3 - 3$, $a_1 = k$

$a_2 = a_{1+1} = 5a_1 - 3$

$= 5k - 3$

$a_3 = a_{2+1} = 5a_2 - 3$

$= 5(5k-3) - 3$

$= 25k - 15 - 3$

$= 25k - 18$

$a_4 = 5(25k - 18) - 3$

$= 125k - 90 - 3$

$= 125k - 93$ #

(ii) $a_4 = 7$, $7 = 125k - 93$

$125k = 100$

$k = \frac{100}{125}$

$k = \frac{4}{5}$ #