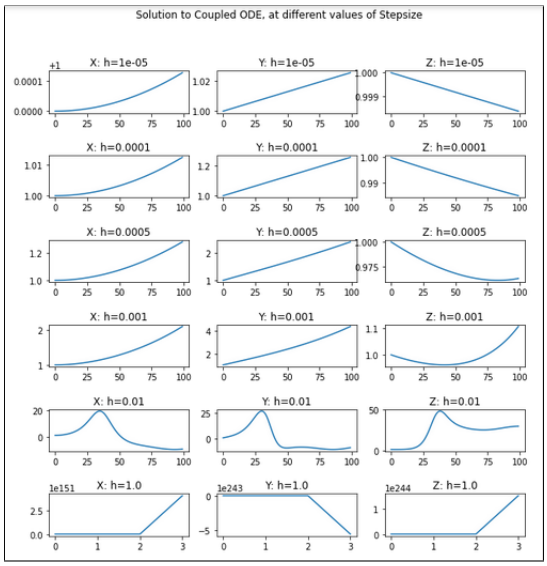
Monte Carlo Simulation Results for Coupled Differential Equation System

1 Results when Initial Conditions are known

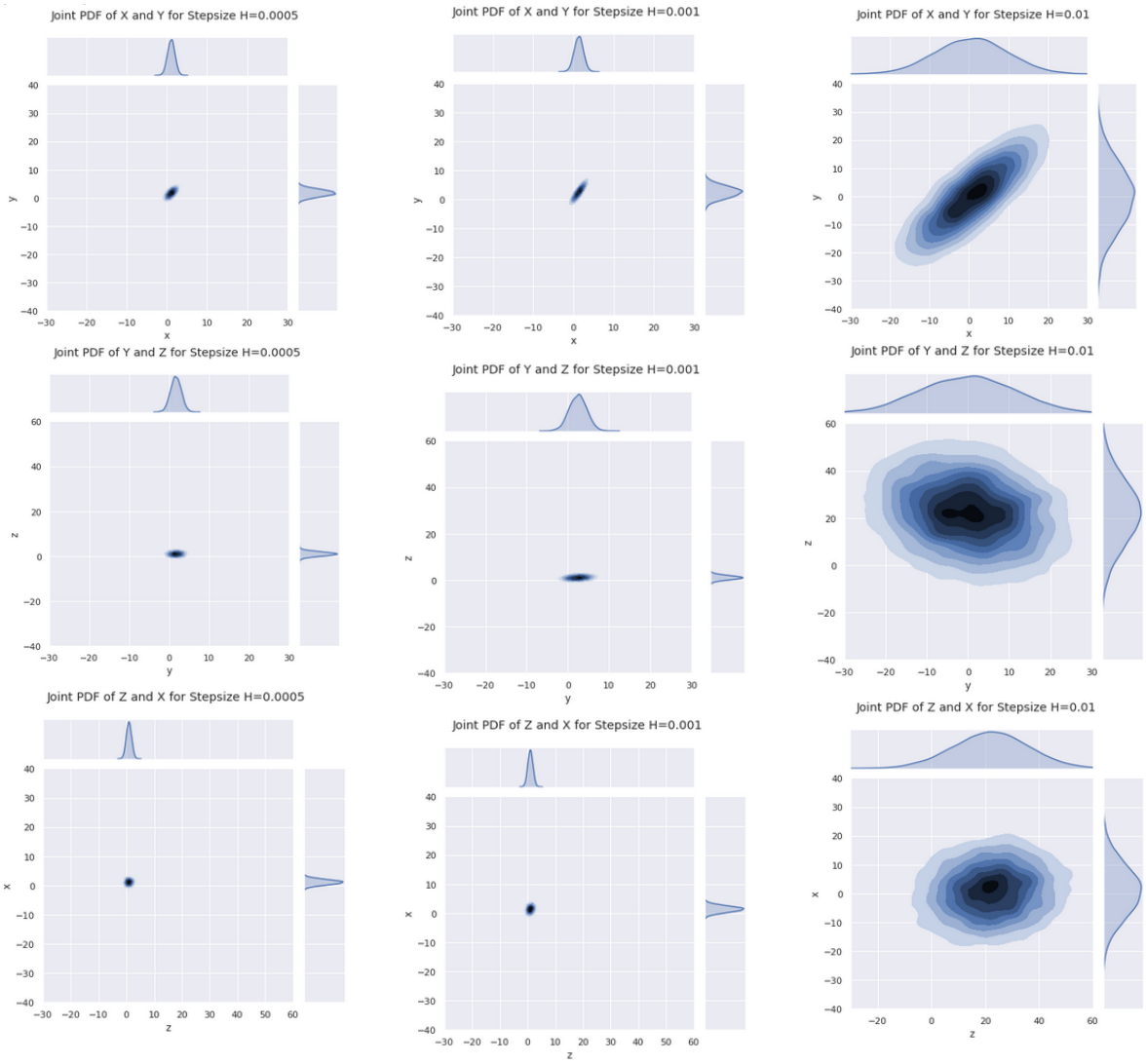


The above figure depicts that for step size 0.00001 and 0.0001 all the functions X,Y and Z behaves similarly and hence step size 0.0001 can be taken as equally accurate. With step size of 0.001, values of X and Y grow fastly, while Z follows a parabolic trajectory and hence the error increases. For the step size 0.0005 (mid point of step size 0.0001 and 0.001) the deviation is small and hence it is better step size. The error increases significantly for the step size of 0.01, as larger values are taken by X, Y and Z. With step size as 1 all the functions X,Y and Z shoots off fastly towards positive and negative infinite values while missing the important features of the function.

2 When Initial Conditions are unknown

With the given Joint PDF [mean (1,1,1) and identity matrix as co-variance] for initial values, 10000 initial value conditions were randomly sample. Afterwards Runge Kutta 4th Order was applied for each sampled initial condition, at different time step.

From the below figure it can be deduced that for smaller step size (h=0.0005) varying initial condition doesn't impact the values attained by functions X, Y and Z as the joint PDF remains tight around the mean values. With a somewhat larger step size h=0.001, the variance in all the functions X, Y and Z increases as the area of the ellipse increases.However, with the larger step size of h=0.01 the area of ellipse increases significantly, which shows that values taken by the functions disperse around the mean values. This shows that with varying initial conditions with bigger step size the functions take different values, which impacts the mean value also, for example for function Z the mean changes from 0 to approximately 20.


Joint PDF of X and Y for Stepsize H=0.0005
Joint PDF of X and Y for Stepsize H=0.001
Joint PDF of X and Y for Stepsize H=0.01
Joint PDF of Y and Z for Stepsize H=0.0005
Joint PDF of Y and Z for Stepsize H=0.001
Joint PDF of Y and Z for Stepsize H=0.01
Joint PDF of Z and X for Stepsize H=0.0005
Joint PDF of Z and X for Stepsize H=0.001
Joint PDF of Z and X for Stepsize H=0.01