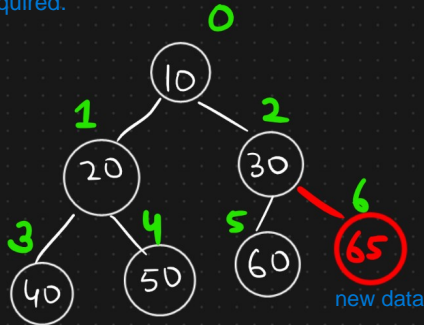


## Deletion in Minheap

Insertion

- worst & average  
 $= O(\log n)$   $\longleftrightarrow$   $n$  elements
- Best case  
 $= O(1) = \text{constant}$

Best case occurs when only 1 comparison and 0 swap is required.  
 Eg;



$30 < 65 \Rightarrow$  valid Minheap  
1 comparison &  
0 swap

Question: What is time complexity for inserting an element in Min heap in case we are provided with  $n \times 2^n$  elements?

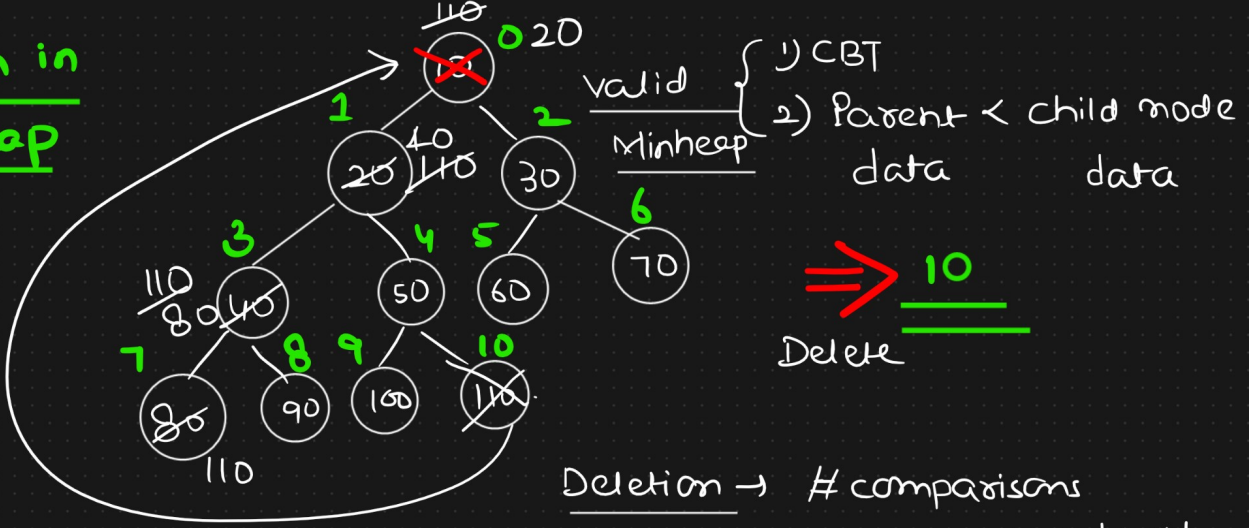
Ans: If not provided we will consider that the time complexity is asked for the worst or average case scenario which is equivalent to the number of levels. that is;  
 for  $n$  elements number of levels is given as;  
 $k = \log(n)_{\text{base } 2}$  where we will get the TC of order  $O(n)$

for  $n \times 2^n$  elements number levels will be  
 $k = \log(n \times 2^n)_{\text{base } 2}$  which is order of  $O(n)$ . See elaboration on the right hand side

$$\begin{aligned}
 \# \text{ elements} &= n \times 2^n \quad (\text{Insertion}) \\
 &= \log_2(n \times 2^n) \\
 &= \log_2 n + \log_2 2^n \\
 &= \log_2 n + n \\
 &= O(n) \quad \leftarrow \text{(Since } n \text{ is greater than } \log n)
 \end{aligned}$$

# Deletion in minheap

Deletion in MinHeap happens in the following 2 steps:  
 1. Deletion happens at the root level. Therefore, element at the 0th index will be Popped out.  
 2. Element at the last index of the MinHeap tree will be moved to the 0th index followed by performing continuous comparison and swapping until MinHeap property is again restored in case it gets destroyed



Deletion  $\rightarrow$  # comparisons

$$\begin{aligned} & \hookrightarrow 2 \times \log N \\ & \downarrow \text{no. of levels} \\ & \text{Left child} \\ & \quad \uparrow \\ & \text{Right child} \end{aligned}$$

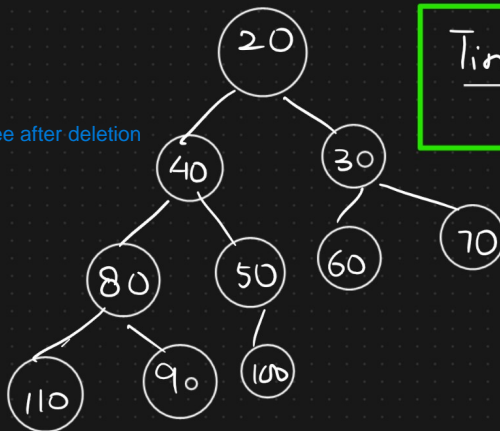
Root Node to  
Leaf Node

# swaps  $\rightarrow \log N$

Swaps will happen in each level and either left child or right child will be involved

Time complexity  $\rightarrow O(\log N)$

MinHeap tree after deletion



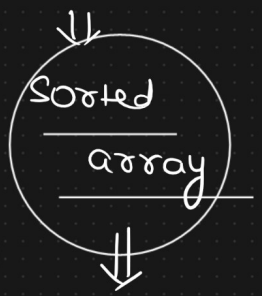
comparison based  
 $\uparrow$  sorting

Heapsort

If we perform continuous deletion and store the respective deleted element in a new Array then we will get the sorted array that will be containing all the elements of the MinHeap tree in the sorted fashion. This is called Heap Sort which is comparison based sorting algorithm

Deletion - 1st time  $\rightarrow$  1st smallest  $\log N$   
 2nd time  $\rightarrow$  2nd smallest  $\log N$   
 3rd time  $\rightarrow$  3rd smallest  $\log N$   
 ...  
 n time  $\rightarrow$  nth smallest element  $\log N$

Array



increasing  
order

Overall time complexity =  $O(n \log n)$  (Since n times deletion is performed)

Maxheap  $\rightarrow$  sorted array

Deletion

$n$  number  
of times

$\rightarrow$  Decreasing  
order

Heapsort  $\rightarrow$   $O(n \log n)$