Bubble Sort (Comparison Based Sorting)

⟹ [20, 20, 50, 30, 90, 8, 15] $n = 7$

	0	1	2	3	4	5	6	
Approach	20	20	50	70	5	90	90	70 > 50
	i	50	30			15		70 < 90

Pass 1 (20, 50, 30, 70, 8, 15) 90 — $(n-1)$

	0	1	2	3	4	5	
	20	50	30	70	8	15	90
		30	50	5	70	70	
					15		

Pass 2 (20, 30, 50, 8, 15) 70, 90 — $(n-2)$

	0	1	2	3	4	5	6
	20	30	50	8	15	70	90
			5	50	50		
				15			

Pass 3 (20, 30, 8, 15) 50, 70, 90 — $(n-3)$

	0	1	2	3	4	5	6
	20	30	8	15	50	70	90
		5	30	30			
			15				

Pass 4 (20, 8, 15) 30, 50, 70, 90

	0	1	2	3	4	5	6
	20	8	15	30	50	70	90
	5	20	20				
		15					

Pass 5 (5, 15) 20, 30, 50, 70, 90

	0	1	2	3	4	5	6
	5	15	20	30	50	70	90

Pass 6 5, 15, 20, 30, 50, 70, 90 — (1)

In each pass we will be having the array which is sorted at right end. Ex; for Pass1 one element at right end will be sorted, For Pass 2 two elements at right end will be sorted. Therefore, in every pass we will apply bubble sort on the elements that are not sorted after removing/ignoring the SORTED elements at the right end.

Points to Note :-

1) $n = 7 \longrightarrow$ Pass 6

$n \longrightarrow (n-1)$ Passes

For an array with Size n we will require n-1 number of passes in order to sort that array using Bubble sort approach

2) # comparisons $\longrightarrow \frac{(n-1)(n-1+1)}{2} = \frac{n(n-1)}{2}$

$\hookrightarrow (n-1) + (n-2) + (n-3) + \cdots + 3 + 2 + 1$

↓ No. of comparisons needed in Pass1 ↓ No. of comparisons needed in Last pass

Sum of n natural numbers

$\hookrightarrow \frac{n(n+1)}{2}$

We can clearly observe that the above summation of comparison sequence represents the summation of first (n-1) natural numbers. ie; (n-1)(n-1 + 1)/2 which is of order square

$\Rightarrow O(n^2)$

Time complexity in the worst case scenario

3) Swaps
- Best case $\rightarrow 0$
- Worst case $\rightarrow \frac{n(n-1)}{2}$

$\hookrightarrow O(n^2)$

Bubble sort is doing 2 things that comparison followed by swapping

Bubble Sort $\rightarrow$ comparisons + swaps

$\downarrow\downarrow \rightarrow O(n^2) + O(n^2)$

Implementation $\rightarrow O(n^2)$

$\downarrow\downarrow$

Loops

$\hookrightarrow O(n^2)$

Space complexity $\rightarrow O(1) \Rightarrow$ constant