Date

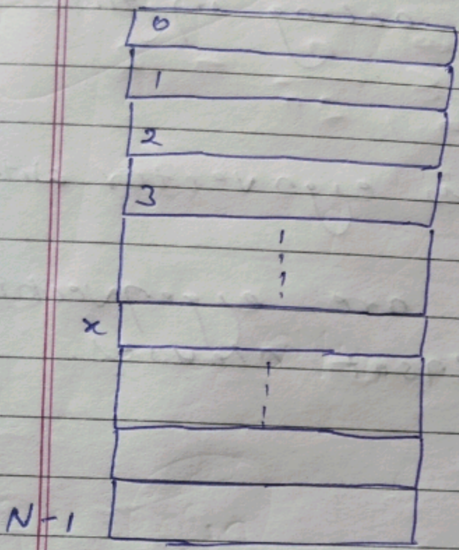
Page No.

DAY-13 TASK

ADIABATIC QUANTUM COMPUTATION

Unstructured search

"Digital haystack"



Theorem: Any Quantum algorithm must take at least $\sqrt{N}$ time

$H_0 \longrightarrow H_f$

$|\psi_0\rangle$ Gradually transform: $|\psi_f\rangle$

$$H(t) = (1-t)H_0 + tH_f$$

known Ground state

Output

↳ How fast $T = \dfrac{1}{\min_t g(t)^2}$ where $g(t)$ is the difference b/w 2 smallest eigenvalues of $H(t)$

3SAT as a local Hamiltonian Problem

$f(x_1, \ldots x_n) = c_1 \wedge \cdots \wedge c_m$

- n bits ——→ n qubits
- Clause $c_i = x_1 \vee x_2 \vee x_3$ Corresponds to 8×8 Hamiltonian matrix acting on first 3 qubits.
- Satisfying assignment is eigenvector with evalue 0.
- All truth assignments are eigenvectors with eigenvalue = # unsat clauses.

$$h_i \begin{bmatrix} 1 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & 0 & \\ & & & & 0 \end{bmatrix}$$

- Adiabatic optimization gives quadratic speedup for search, but exponential time in general.
- Exponential time for NP complete Problems but can tunnel through local optima in certain special circumstances.

↳ Anderson localization based arguments that is typically gets stuck in local optima

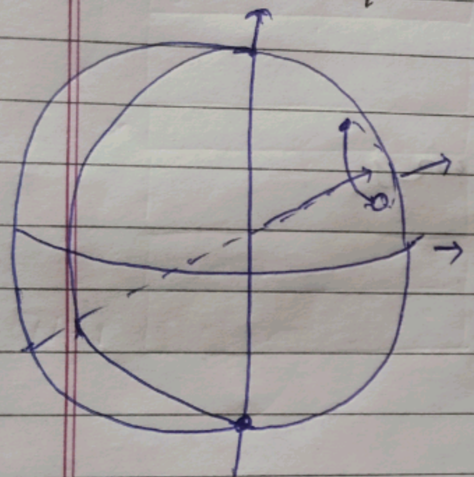
Computation without unitaries.

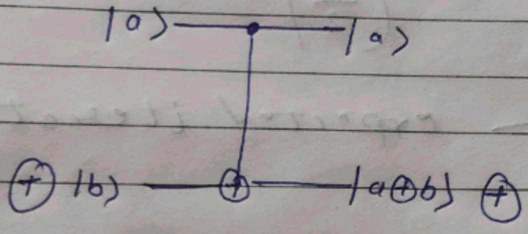
Arbitary quantum circuits

single- qubit operations

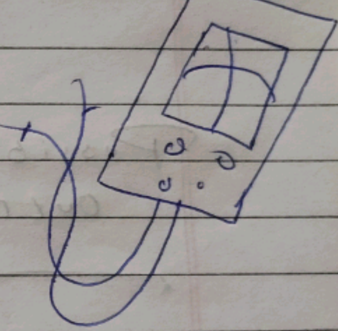
CNOT

measurement





$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



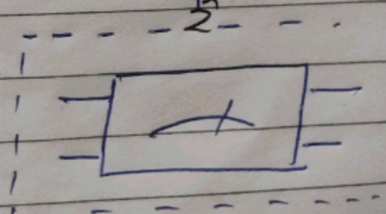
Emulating a unitary transformation with measurements only

measured observables

$\hat{B}$ $\hat{B}$ $\hat{Z}$

"ancilla" qubit → $|0\rangle$

qubit of interest → $|\psi\rangle$



$$\hat{B} := \frac{1}{2}(\hat{x} - i\hat{y}) \otimes \hat{U} + \frac{1}{2}(\hat{x} + i\hat{y}) \otimes \hat{U}^{+}$$

Date

Page No.

Properties:

1) $\hat{B}^{\dagger} = \hat{B}$ 2) $\hat{B}^2 = \hat{I}$

3) $\hat{B}(|0\rangle \otimes |\psi\rangle) = |1\rangle \otimes \hat{U}|\psi\rangle$

$$\underbrace{\langle n \rangle = \sum_{n=1}^{\infty} \underbrace{\text{Prob}(n)}_{\left(\frac{1}{2}\right)^n} \cdot n = \sum_{n=1}^{\infty} \frac{n}{2^n} = 2}_{\text{expected iterations}}$$

Possible outcomes

$$\underbrace{Z = +1 \qquad\qquad Z = -1}$$

$\leftarrow$ Prob $= 1/2$