DAY 05 TASK.

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Tensor Products:

Three dimensions more than one Particle.

$$\mathcal{V} = \mathcal{V}_1 \otimes \mathcal{V}_2$$

① Linear with respect to scaler Multiplication

$$a(|\psi\rangle_1 \otimes |\phi\rangle_2) = (a|\psi\rangle_1) \otimes |\phi\rangle)_2$$

$$= |\psi\rangle_1 \otimes (a|\phi\rangle_2), \quad a \in \mathbb{C}$$

② Distributive with respect to vector addition

$$(|\psi_1\rangle_1 \oplus |\psi_2\rangle_1) \otimes |\phi\rangle_2 = |\psi_1\rangle_1 \otimes |\phi\rangle_2 + |\psi_2\rangle_1 \otimes |\phi\rangle_2$$

Basis states

$$\left.\begin{array}{l} \{|v_i\rangle\} \text{ in } V_1 \text{ of dimension } N_1 \\ \{|v_j\rangle\} \text{ in } V_2 \text{ of dimension } N_2 \end{array}\right\} \{|u_i\rangle_1 \otimes |v_j\rangle_2\} \text{ in } V \text{ of dimension } N_1 N_2$$

$$|\psi\rangle_1 \otimes |\phi\rangle_2 \in V$$

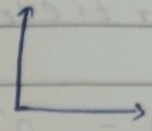
$$\left.\begin{array}{l} |\psi\rangle = \sum_i c_i |u_i\rangle \\ |\phi\rangle = \sum_j d_j |v_j\rangle \end{array}\right\} \begin{array}{l} |\psi\rangle_1 \otimes |\phi\rangle_2 = \\ \sum_{ij} c_i d_j |u_i\rangle_1 \otimes |v_j\rangle_2 \end{array}$$

$$a_{ij} = c_i d_j$$

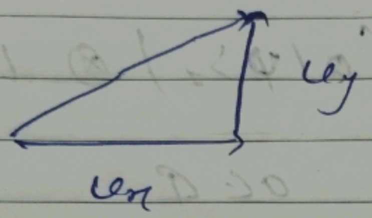
↳ Define a notion of angle.

$\vec{v}\cdot\vec{w} = v_1 w_1 + v_2 w_2 + v_3 w_3 \ldots$

$\vec{v}\cdot\vec{w} = |\vec{v}||\vec{w}| \cos\theta = 0$



↳ Define a notion of length



$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$

$:= \sqrt{\vec{v}\cdot\vec{v}}$

InProd($|\psi\rangle$, $|\phi\rangle$) = C

INNER PRODUCT DEFINATION

An inner product $\langle\psi|\phi\rangle$ is a map from vectors to scalars that satisfies the following.

$\langle\psi|\chi + \phi\rangle = \langle\psi|\zeta\rangle + |\langle\psi|\phi\rangle$

$\langle\psi|a\phi\rangle = a\langle\psi|\phi\rangle$

$\langle\psi|\phi\rangle = \langle\phi|\psi\rangle^*$

for $|\psi\rangle \neq 0$, $\langle\psi|\psi\rangle >$

$\langle a|b\rangle = (a_0^* \;\; a_1^*)\begin{pmatrix} b_0 \\ b_1 \end{pmatrix} = a_0^* b_0 + a_1^* b_1$

complex scalar

By changing the order, we get the outer product

$|b\rangle\langle a| = \begin{pmatrix} b_0 \\ b_1 \end{pmatrix}(a_0^* \;\; a_1^*) = \begin{pmatrix} a_0^* b_0 & a_1^* b_0 \\ a_0^* b_1 & a_1^* b_1 \end{pmatrix}$

How about measurements in other basis?

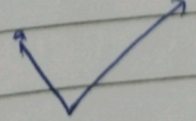
Pauli x basis $\quad |+\rangle\langle +|\psi\rangle = \frac{\alpha+\beta}{\sqrt{2}}|+\rangle$ outcome +1

$|-\rangle\langle -|\psi\rangle = \frac{\alpha-\beta}{\sqrt{2}}|-\rangle$ outcome -1

Pauli y basis $\quad |i\rangle\langle i|\psi\rangle = \frac{\alpha - i\beta}{\sqrt{2}}|i\rangle$ outcome +1

$|-i\rangle\langle -i|\psi\rangle = \frac{\alpha + i\beta}{\sqrt{2}}|-i\rangle$ outcome -1

Unitary operators



$|\phi\rangle$ $\quad |\psi|$

$\hat{U}^\dagger = \hat{U}^{-1}$

$|\psi||\phi|\cos(\theta) = |\hat{U}\psi||\hat{U}\phi|\cos(\theta_U)$

Generalized notation

$\langle \hat{U}\psi | \hat{U}\phi \rangle = \langle \psi | \phi \rangle$

The eigenvalues of a unitary operator must have magnitude one

$|\lambda|^2 = 1$

$$\boxed{\frac{\partial L}{\partial t} = -\frac{d}{dt} E}$$

$$\boxed{\frac{\partial L}{\partial x} = \frac{d}{dt} P}$$