DAY- 6 TASK

Linear function definition :-

A linear functional is any linear map L that goes from the vector space to a scalar number.

$L\vec{U} \longrightarrow C$

$$L_n = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C_n = \begin{bmatrix} 1 & 0 \end{bmatrix}$$ row matrix

all linear functional

$\mathbb{R}^2 \longrightarrow \mathbb{R}^1$

$$\begin{bmatrix} a & b \end{bmatrix}$$

form its own vector space this basis basis is called dual space

vector space dual space

$$\begin{bmatrix} a \\ b \end{bmatrix} \qquad \begin{bmatrix} a & b \end{bmatrix}$$

Dual ~~Defi.~~ Space Definition

given a vector space V the dual space V^* is the vector space of all linear functionals in V.

$$L \in V^*$$

$$\begin{bmatrix} a & b \end{bmatrix} \vec{v} = \begin{bmatrix} a \\ b \end{bmatrix} \cdot \vec{v}$$

Riesz Representation Theorem

for any linear functional $L\phi$, the action of $L\phi$ is equivalent to taking the inner product with unique vector $\vec{\phi}$

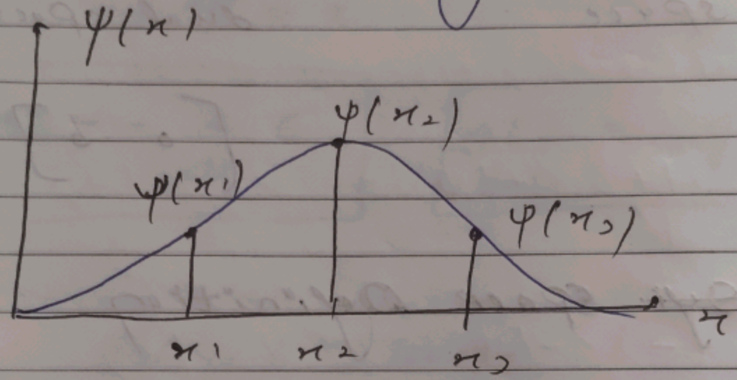
$$L\phi \vec{v} = \text{InProd}(\vec{\phi}, \vec{v})$$

$\langle x\, \vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \vec{v}$

Bra-ket notation

$\langle \phi |$ $\quad | \psi \rangle$

$\langle \phi | \psi \rangle$ completely different mathematically analysis



$$\psi = \begin{bmatrix} \psi(x_1) \\ \psi(x_2) \\ \psi(x_3) \\ \vdots \end{bmatrix}$$

$$\langle \psi | = [\psi(x_1)^*, \psi(x_2)^*, \psi(x_3)^*, \ldots]$$

$$\langle \psi | \psi \rangle = [\phi_1^*, \phi_2^*, \phi_3^* \ldots \phi_n^*] \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \vdots \\ \psi_n \end{bmatrix}$$

$\psi_1 = \psi(x_1)$

$\phi_1^* \psi_1 + \phi_2^* \psi_2 + \phi_3^* \psi_3 + \dots + \phi_n^* \psi_n$

$\langle \phi | \psi \rangle = \sum_{i=1}^{n} \phi_i^* \psi_i$

$\langle \phi | \psi \rangle = \int dx \, \phi^*(x) \psi(x)$

ket: $|\psi\rangle$

Bra:- $\langle \psi|$

$\psi(x) = \psi^*(x)$

$\int \psi^*(x) \psi(x) dx = \langle \psi | \psi \rangle$

Inner Product / Scalar Product

Operators:-

$\hat{A} |\psi\rangle = \lambda |\psi\rangle$

Energy: $\hat{H} |\psi_1\rangle = E_1 |\psi_1\rangle$

$\hat{H} |\psi_2\rangle = E_2 |\psi_2\rangle$ physical information

Properties:-

$|\psi\rangle + |\phi\rangle = |r\rangle$