Single qubit and its logic gates

$$|0\rangle - \boxed{H} - |+\rangle \qquad |1\rangle - \boxed{H} - |-\rangle$$

Hadamard gate

$$\boxed{X}$$

$$\boxed{Y}$$

$$\boxed{Z}$$

$$\boxed{R}$$

Blouch sphere
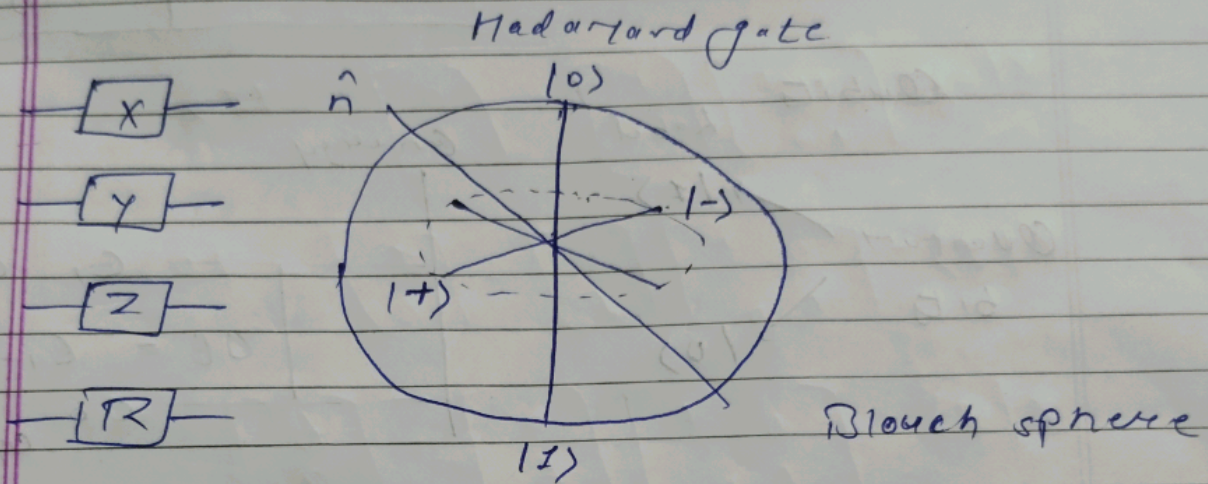
Quantum gates

Classical bits   Physical representation

       State   ⊕ ⊕ ⊕   ON

Classical bits — 1

         ⊖ ⊖ ⊖   off

       0
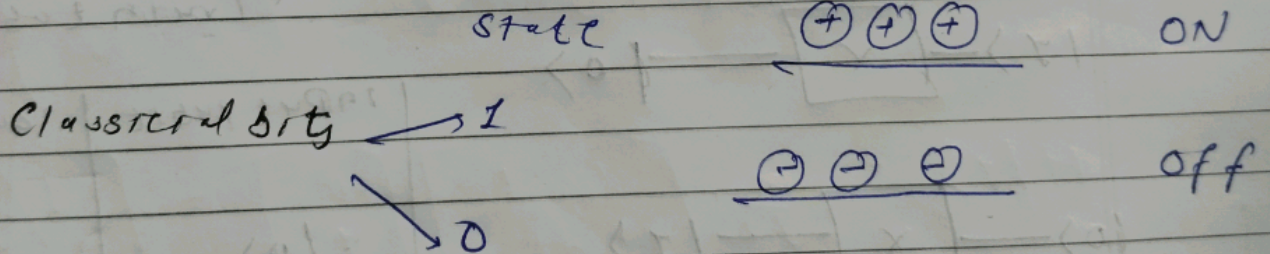
NOT GATE

Circuit representation

$$A - \triangleright\!\circ - \text{NOT } A$$

$0 \longrightarrow$ ▷o $\longrightarrow 1$

$1 \longrightarrow$ ▷o $\longrightarrow 0$
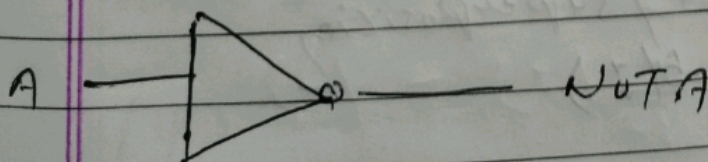
Truth table

Input bit A	output bit NOT A
0	1
1	0

Qubits   Label   Energy

Quantum bits
$\longrightarrow |1\rangle$
$\longrightarrow |0\rangle$

— — — — E_1 Excited state

$\Delta E = E_1 - E_0$

— — — — E_0 Ground state
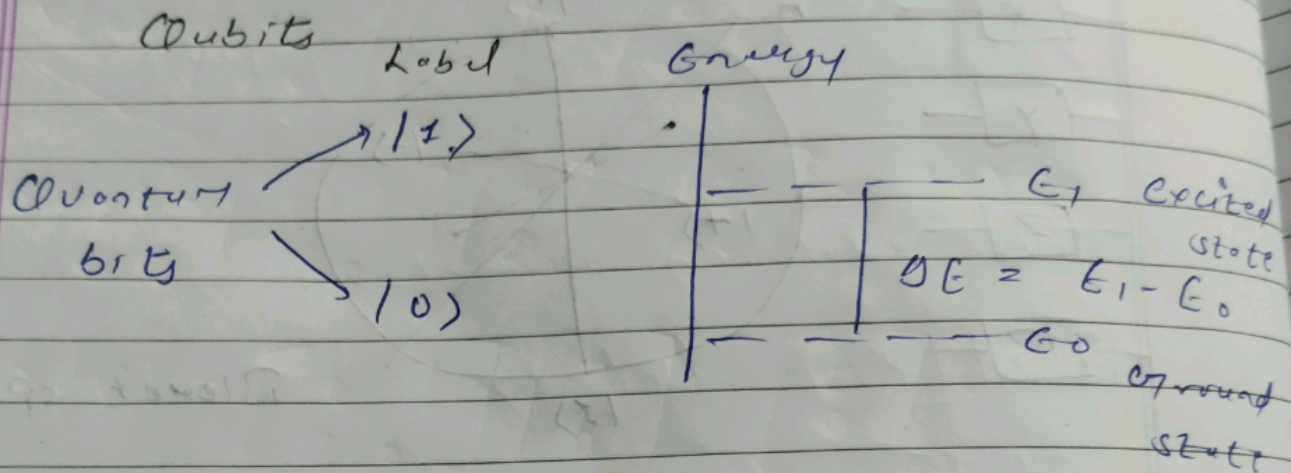
Pauli X gate

Circuit representation

$|1\rangle - \boxed{X} - |0\rangle$

$|0\rangle - \boxed{X} - |1\rangle$

Truth table
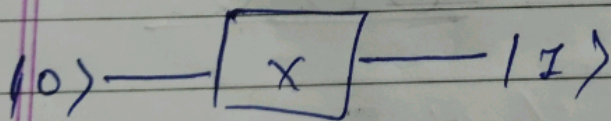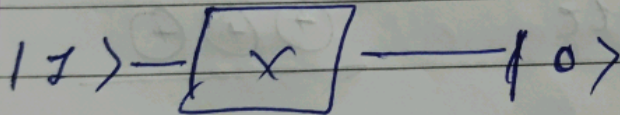
input qubit	output qubit
$|0\rangle$	$|1\rangle$
$|1\rangle$	$|0\rangle$

Qubits as quantum superposition

Quantum superposition

$$|\psi\rangle = a|0\rangle + b|1\rangle$$

states

a, b ∈ C

$|a|^2$ is the probability that the system is in state $|0\rangle$

$|b|^2$ is the probability that the system is in state $|1\rangle$

$|a|^2 + |b|^2 = 1$

Matrix representation of qubits

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
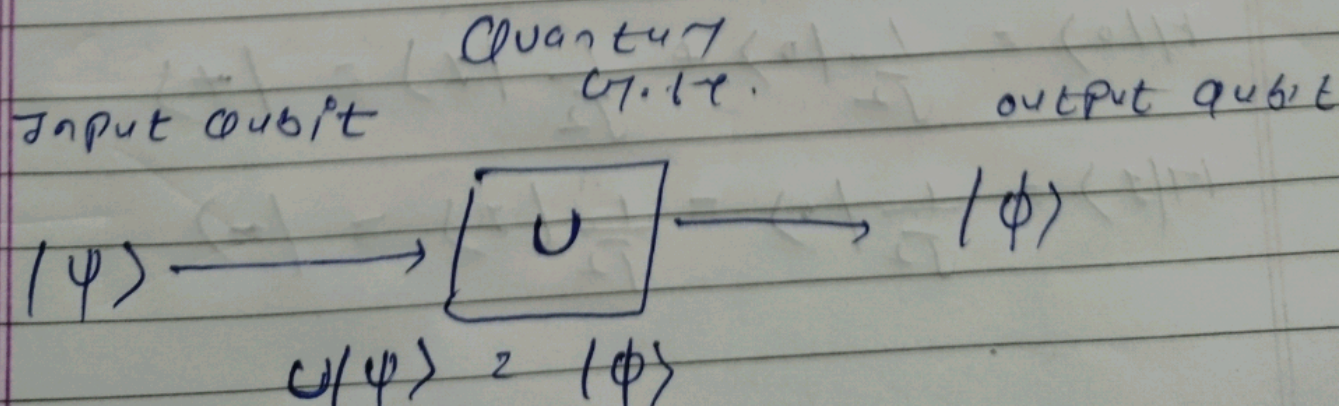
$$|\psi\rangle = a|0\rangle + b|1\rangle = a\begin{bmatrix} 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} a \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$|0\rangle$ and $|1\rangle$ span the 2D Hilbert Space.

Quantum Logic Gates

Input qubit   Quantum Gate   output qubit

$$|\psi\rangle \longrightarrow \boxed{U} \longrightarrow |\phi\rangle$$

$$U|\psi\rangle = |\phi\rangle$$

Pauli X gate.

Hadamard gate.

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

$$X|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$X|\psi\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ a \end{bmatrix}$$

$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$H|1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$
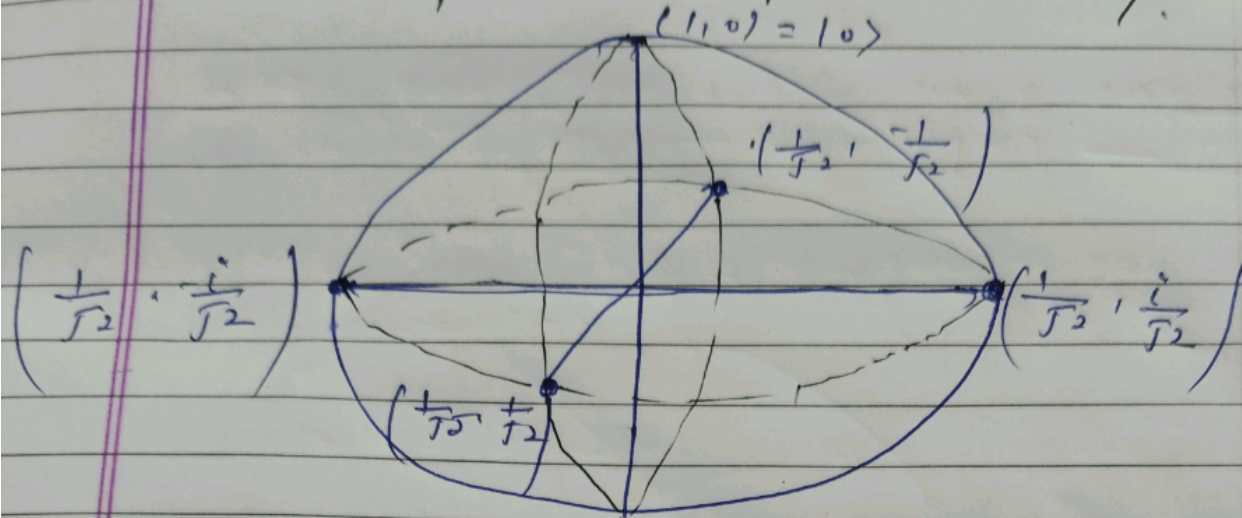
Hadamard gate.

$$H|0\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = |+\rangle$$

$$H|1\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle = |-\rangle$$

Bloch sphere Representation.

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

Quantum Logic Gates.

Hermitian

$$U = U^{\dagger}$$

Ensures real
eigenvalues

unitary

$$UU^{\dagger} = I$$

Ensures Conservation
of Probability