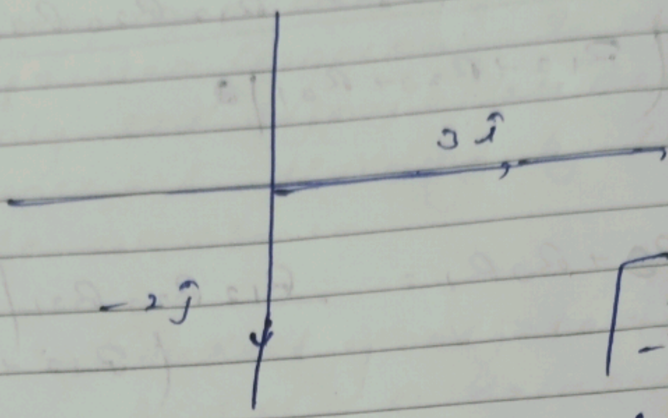
Day 1 - task 1 -

↳ Vectors

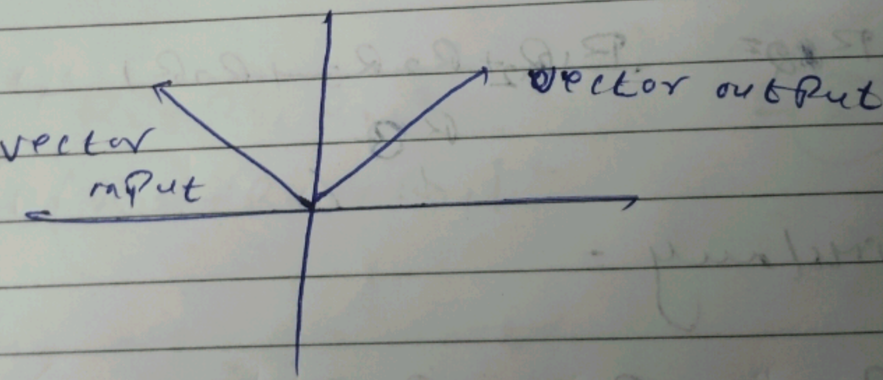


$$\begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$3(\hat{i}) + (-2\hat{j})$$

↳ Linear transformation.

Vector input → Vector output



$\vec{v} = -1\hat{i} + 2\hat{j}$

Transformed $\vec{v}$ = -1 (Transformed $\hat{i}$) + 2 (Transformed $\hat{j}$)

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$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = x\begin{bmatrix} a \\ c \end{bmatrix} + y\begin{bmatrix} b \\ d \end{bmatrix} =$$

$$\begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

↳ Determinant:

$$\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right) = a\det\left(\begin{bmatrix} e & f \\ h & i \end{bmatrix}\right) -$$

$$-b\det\left(\begin{bmatrix} d & f \\ g & i \end{bmatrix}\right)$$

$$+c\det\left(\begin{bmatrix} d & e \\ g & h \end{bmatrix}\right)$$

Transform

$$\begin{bmatrix} 1 & -2 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix} \rightarrow 4\cdot 1 + 3\cdot -2$$

vector

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \det\left(\begin{bmatrix} \hat{i} & v_1 & w_1 \\ \hat{j} & v_2 & w_2 \\ \hat{k} & v_3 & w_3 \end{bmatrix}\right)$$

$$\hat{i}(v_2w_3 - v_3w_2) + \hat{j}(v_3w_1 - v_1w_3) +$$

$$\hat{k}(v_1w_2 - v_2w_1)$$

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$$\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \Rightarrow \text{Area} = \det(A)x$$

$$x = \frac{\text{Area}}{\det(A)} = \frac{\det\left(\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}\right)}{\det\left(\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}\right)}$$

Eigenvalue and Eigenvectors:

Transformation Matrix

$$A\vec{v} = \vec{dv}$$

Eigenvalue

Eigenvector

Speed

$$\det(A - \lambda I) = 0$$

$$\frac{1}{2} tr\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \frac{a+d}{2} = \frac{\lambda_1 + \lambda_2}{2} = 7 \quad \text{(mean)}$$

$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc = \lambda_1 \lambda_2 = P \quad \text{(product)}$$

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formal definition of linearity

Additivity: $L(\vec{v}+\vec{w}) = L(\vec{v}) + L(\vec{w})$

Scaling: $L(c\vec{v}) = cL(\vec{v})$

Rules for vectors adding and scaling

1. $\vec{u} + (\vec{v}+\vec{w}) = (\vec{u}+\vec{v}) + \vec{w}$
2. $\vec{v} + \vec{w} = \vec{w} + \vec{v}$
3. There is a vector 0 such that $0 + \vec{v} = \vec{v}$ for all $\vec{v}$
4. for every vector $\vec{v}$ there is a vector $-\vec{v}$ so that $\vec{v} + (-\vec{v}) = 0$
5. $a(b\vec{v}) = (ab)\vec{v}$
6. $1\vec{v} = \vec{v}$
7. $a(\vec{v}+\vec{w}) = a\vec{v} + a\vec{w}$
8. $(a+b)\vec{v} = a\vec{v} + b\vec{v}$