DAY-17 TASK

Quadratic speedup for quantum walk search.

Random walks

A formula : $\phi(x) = C_1 (x_1, x_3, x_7) \, C_2 (x_2, x_5, x_6) \, C_3 (x_1, x_4, x_4)$

find x such that $\phi(x) = 1$

$x = 0010110 \qquad \phi(x) = 0$
$x = 0011110 \qquad \phi(x) = 0$
$x = 0011100 \qquad \phi(x) = 0$

$p[u, v] = $ Probability of going to v when

at $u = \begin{cases} \dfrac{1}{d_u} & \text{if } uv \in E \\ 0 & \text{else} \end{cases}$

Random walk search Algorithm:

Start at a random vertex u [S]
Repeat T times:

update [U]

Check [C]

If current vertex marked, output.

Complexity: $O(S + T(U+C))$

Hitting Time

Expected number of steps to get to a marked vertex starting from the stationary distribution $\bar{\pi}$.

$$P_{\bar{\pi}} = \bar{\pi}$$

Random Walk Search Algorithm:

Start at a vertex U sampled from $\bar{\pi}$ [S]
Repeat T times:

$$T \approx HT$$

update [U]

check [C]

If current vertex Marked, output.

Classical algorithm

Detecting $\quad O(S + HT(U+C))$

finding $\quad O(S + HT(U+C))$

If there is at least one Marked vertex, Hitting time $\leq HT$

Quantum walks

update : $|u, 0\rangle \longrightarrow \sum_v \sqrt{P(u,v)} |u,v\rangle$

check : $|v\rangle \longrightarrow \begin{cases} -|v\rangle & \text{if } v \text{ is marked} \\ |v\rangle & \text{else} \end{cases}$

setup : $|0\rangle \longrightarrow |\pi\rangle = \sum_v \sqrt{\pi(u)} |v\rangle$

	Classical Algorithm	Quantum algorithm
Detecting	$O(S + HT(U+C))$	$O(S + \sqrt{HT}(U+C))$
finding	$O(S + HT(U+C))$	

Detecting via Absorbing walk.

Original Markov walk : P

S - absorbing walk : P/S

Prob. S : stay (loop)
Prob. $1-S$: move.

If no vertex is marked : $P/S/ = P/S/^{HT}_{\pi = \pi}$