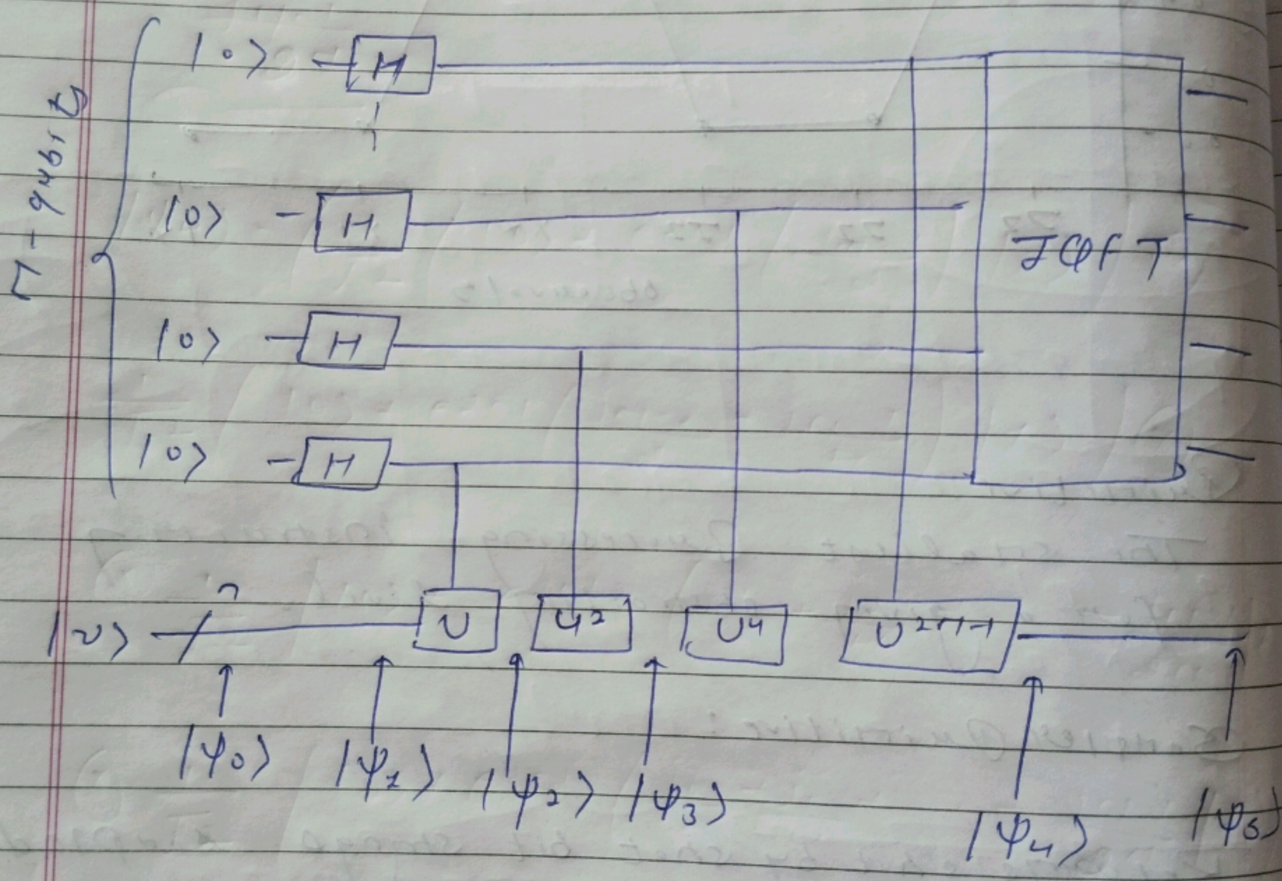
DAY-15 TASK.

Quantum Phase Estimation

$$U|v\rangle = e^{i\theta}|v\rangle$$

The Quantum Phase Estimating Algorithm (QPE) finds $e^{i\theta}$ given it's eigenvector $|v\rangle$ and matrix U.



$$|\psi_0\rangle = |0\rangle^{\otimes n}|v\rangle$$

$$|\psi_1\rangle = \left(\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \otimes \frac{1}{2}\left(|0\rangle + |1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\right)|v\rangle$$

$$|\psi_2\rangle = \left(\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \otimes \dots \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\theta}|1\rangle\right)\right)|u\rangle$$

$$|\psi_3\rangle = \left(\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \otimes \dots \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + e^{2i\theta}|1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\theta}|1\rangle\right)\right)|u\rangle$$

Let $\theta = 2\pi j$, where $j = 0.j_0 j_1 j_2 \dots j_{n-1}$ and each $j_i \in \{0,1\}$

$$|\psi_4\rangle = \left(\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2^{n-1}i\theta}|1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + e^{2^{n-2}i\theta}|1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + e^{2^{n-3}i\theta}|1\rangle\right) \otimes \dots \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + e^{2i\theta}|1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\theta}|1\rangle\right)\right)|u\rangle$$

$$|\psi_4\rangle = \left(\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i 2^{n-1}(0.j_0 j_1 \dots j_{n-1})}|1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i 2^{n-2}(0.j_0 j_1 \dots j_{n-1})}|1\rangle\right) \otimes \dots\right.$$

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$$\otimes \frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i 2(0.j_0 j_1 \ldots j_{7-1})}|1\rangle\right) \otimes$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i(0.j_0 j_1 \ldots j_{7-1})}|1\rangle\right)|v\rangle$$

$$\underbrace{j = 0.j_0 j_1 j_2 \ldots j_{7-1}}_{\text{Binary}} = \underbrace{\frac{j_0}{2} + \frac{j_1}{4} + \frac{j_2}{8} + \ldots + \frac{j_{7-1}}{2^7}}_{\text{Base}-10}$$

$$|\psi_4\rangle = \left(\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i\left(\frac{j_{7-1}}{2}\right)}|1\rangle\right) \otimes\right.$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i\left(\frac{j_{7-2}}{2} + \frac{j_{7-1}}{4}\right)}|1\rangle\right) \otimes \ldots$$

$$\otimes \frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i 2\left(\frac{j_1}{2} + \frac{j_2}{4} + \ldots + \frac{j_{7-1}}{2^{7-1}}\right)}|1\rangle\right)$$

$$\otimes \frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i\left(\frac{j_0}{2} + \frac{j_1}{4} + \ldots + \frac{j_{7-1}}{2^7}\right)}|1\rangle\right)|v\rangle$$

$$\boxed{|\psi_5\rangle = IQFT|\psi_4\rangle = |j\rangle}$$

$\theta = 2\pi j$ Eigenvalue $= e^{i\theta}$