

# DAY-9 TASK

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Quantum Gates

$$U|\alpha\rangle = |0\rangle$$

↑      ↑  
qubit(s)      qubit(s)

transforming  
qubit(s)

unitary  
Matrix

$$\hookrightarrow U^\dagger U = U U^\dagger = I$$

$$U^\dagger = U^{-1}$$

$$A^T = A$$

$$\hookrightarrow H^\dagger = H$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$Y^\dagger = \begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix}$$

$$Y^\dagger = Y$$

$$Y^\dagger Y = \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix} \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix} = \begin{vmatrix} -i \times i & 0 \\ 0 & -i \times i \end{vmatrix} \\ = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$



Gate

Matrix

Examples

[1] - Pauli-X  
U & H

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$X|0\rangle = |1\rangle$$

$$X|1\rangle = |0\rangle$$

$$X(\alpha|0\rangle + \beta|1\rangle)$$

$$= \alpha X|0\rangle + \beta X|1\rangle$$

$$= \alpha|1\rangle + \beta|0\rangle$$

[2] - U &amp; H

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$Z|0\rangle = |0\rangle$$

$$Z|1\rangle = -|1\rangle$$

$$Z(\alpha|0\rangle + \beta|1\rangle)$$

$$= \alpha|0\rangle - \beta|1\rangle$$

[3] - U &amp; H

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = |+\rangle$$

Madelung

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = |-\rangle$$

$$H|+\rangle = |0\rangle$$

$$H|-\rangle = |1\rangle$$

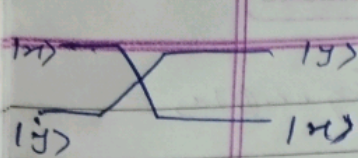
[4] - U & H  
H

$$R_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$R_\theta \neq R_\theta^\dagger$$

$$R_\theta^\dagger \cdot R_\theta = I$$



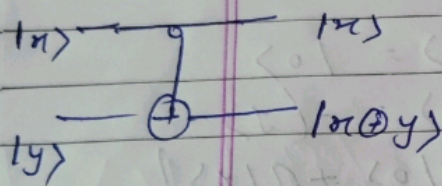


swap  
gate

$$\text{swap} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{swap} |01\rangle = |10\rangle$$

CNOT gate



CNOT =

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\text{CNOT} |10\rangle =$$

$$|11\rangle$$

$$\text{CNOT} |01\rangle = |01\rangle$$

I single qubit gates.

①  $\boxed{X}$   $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$   $X|0\rangle \rightarrow |1\rangle$   
 $X|1\rangle \rightarrow |0\rangle$

②  $\boxed{Y}$   $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$   $Y|0\rangle \rightarrow i|1\rangle$   
 $Y|1\rangle = -i|0\rangle$

③  $\boxed{Z}$   $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$   $Z|0\rangle = |0\rangle$   
 $Z|1\rangle = -|1\rangle$

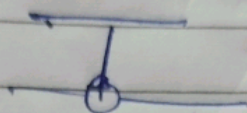
④  $\boxed{H}$   $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$   $H|0\rangle = \frac{1}{\sqrt{2}} [|0\rangle + |1\rangle]$   
 $H|1\rangle = \frac{1}{\sqrt{2}} [|0\rangle - |1\rangle]$

⑤  $\boxed{T}$   $\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$   $T|0\rangle = |0\rangle$   
 $T|1\rangle = e^{i\pi/4} |1\rangle$



## 2 Qubit Quantum Gates

① CNOT

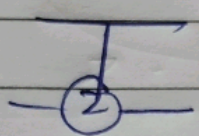


$$\begin{bmatrix} 1 & 0 \\ 0 & X \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$2 \leftrightarrow 3$   
 $|0\rangle \leftrightarrow |1\rangle$

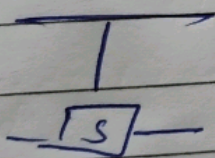
② CZ



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$3 \rightarrow -3$   
 $|11\rangle \rightarrow -|11\rangle$

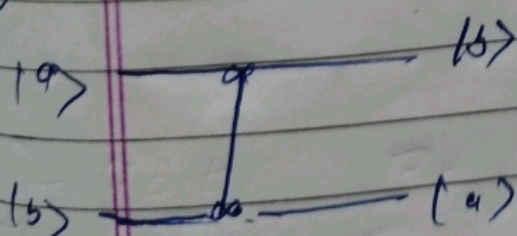
③ CPhase



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{bmatrix}$$

$3 \rightarrow i3$   
 $|11\rangle \rightarrow i|11\rangle$

④ swap



$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$|01\rangle \rightarrow |10\rangle$   
 $1 \rightarrow 2$