eduport

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Integrals

It is the inverse of differentiation. Let, $\frac{d}{dx}F(x)=f(x)$
Then, $\int f(x)dx = F(x)+c$, 'c'.

Constant of integral. These integrals are called indefinite or general Integrals Properties of indefinite integrals are

(i) $\int [f(x) \pm g(x)]\, dx = \int f(x)dx \pm \int g(x)dx$, (ii) $\int kf(x)dx = k\int f(x)dx$,
eg: $\int (3x^2+2x)\, dx = x^3+x^2+c$, where c is real

(i) $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$ like, $\int dx = x+c$

(ii) $\int \cos x\, dx = \sin x + c$

(iii) $\int \sin x\, dx = -\cos x + c$

(iv) $\int \sec^2 x\, dx = \tan x + c$

(v) $\int \operatorname{cosec}^2 x + c$

(vi) $\int \sec x \tan x\, dx = \sec x + c$

(vii) $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + c$

(viii) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$

(ix) $\int \frac{dx}{\sqrt{1-x^2}} = \cos^{-1} x + c$

(x) $\int \frac{dx}{1-x^2} = \tan^{-1} x + c$

(xi) $\int \frac{dx}{1-x^2} = -\cot^{-1} x + c$

(xii) $\int e^x dx = e^x + c$

(xiii) $\int a^x dx = \frac{a^x}{\log a} + c$

(xiv) $\int \frac{dx}{x\sqrt{x-1}} = \sec^{-1} x + c$

(xv) $\int \frac{dx}{x\sqrt{x^2-1}} = -\operatorname{cosec}^{-1} x \quad c$

(xvi) $\int \frac{1}{x}\, dx = \log|x| + c$

The method in which we change the variable to some other variables is called the method of substitution. Below problems can be solved by substitution

$\int \tan x\, dx = \log|\sec x| + c$

$\int \cot x\, dx = \log|\sin x| + c$

$\int \sec x\, dx \log|\sec x + \tan x| + c$

$\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x| + c$

Integration using trigonometric identities: $\cos^2 x = \frac{1+\cos 2x}{2}$

Example:

$$\int_{-\pi/4}^{\pi/4} \sin^2 x dx = 2\int_0^{\pi/4} \sin^2 x dx = 2\int_0^{1/4} \left(\frac{1-\cos 2x}{2}\right) dx = \int_0^{\pi/4} (1-\cos 2x) dx = \left[x - \frac{\sin 2x}{2}\right]_0^{\pi/4} = \frac{\pi}{4} - \frac{1}{2}$$

$$\sin^3 x = \frac{3\sin x - \sin 3x}{4}, \cos^3 x = \frac{\cos 3x + 3\cos x}{4}$$

(i) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + c$

(ii) $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c$

(iii) $\int \frac{dx}{x^2 - a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

(iv) $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + c$

(v) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$

(vi) $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + c$

(vii) $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \log\left|x + \sqrt{x^2 - a^2}\right| + c$

(viii) $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \log\left|x + \sqrt{x^2 + a^2}\right| c$

(ix) $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$

Integration by partical fractions

consider $\frac{f(x)}{g(x)}$ defines a rational polynominal function

If the degree of numerator i.e., $f(x)$ is greater than equal to the degree of denominator i,e., $g(x)$ then, this type of rational function is called an improper rational function. and if degree of $f(x)$ is smaller than the degree of denominator i,e; $g(x)$, then this type of rational function is called a proper rational function.

In rational polynomial function if the degree i.e., highest power of the variable) of numerator (Nr.) is greater than or equal to the degree of denominator (Dr.), then (without any doubt) always perform the division i.e, divide the Nr. by Dr. before doing anything and thereafter use the following.

$$\frac{\text{Numerator}}{\text{Denominator}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Denominator}}$$

From of the Rational Function	From of the Partial Fraction
$\frac{Px+q}{(x-a)(x-b)}$, $a \neq b$	$\frac{A}{x-a} + \frac{B}{x-a}$
$\frac{Px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
$\frac{Px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
$\frac{Px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
$\frac{Px^2+qx+r}{(x-a)(x^2+bx+c)}$ Where x^2+bx+c can't be factorized further	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$

$$\int f_1(x) f_2(x)\,dx = f_1(x)\int f_2(x)\,dx - \int\left[\frac{d}{dx} f_1(x)\int f_2(x)\,dx\right]dx$$

$$\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x)+c$$

Let the area function be defined by $A(x)=\int_a^x f(x)\,dx \;\forall x \geq a$, where f is continuous on $[a,b]$ then $A'(x)=f(x)\;\forall x \in [a,b]$.

Let f be a continuous function of x defined on $[a,b]$ and let F be another function such that $\frac{d}{dx}F(x)=f(x)\;\forall x \in$ domain of f, then $\int_a^b f(x)\,dx = \left[F(x)+c\right]_a^b = F(b)-F(a)$ This is called definite integral of f over the range $[a,b]$ where a and b are called the limit of integration, a being the lower limit and b be the upper limit.

Let the area function be defined by $A(x)=\int_a^x f(x)\,dx \;\forall x \geq a$, where f is continuous on $[a,b]$ then $A'(x) \longrightarrow f(x)\;\forall x \subset [a,b]$.

Let f be a continuous function of x defined on $[a,b]$ and let F be another function such that $\frac{d}{dx}F(x)=f(x)\;\forall x \in$ domain of f, then $\int_a^b f(x)\,dx = \left[F(x)+c\right]_a^b = F(b)-F(a)$ This is called definite integral of f over the range $[a,b]$ where a and b are called the limit of integration, a being the lower limit and b be the upper limit.