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RELATIONS & FUNCTIONS

TYPE OF RELATIONS

* **Reflexive Relation** : Every element is related to itself, i.e R is reflexive in A <=> $(a,a) \in R$ for every $a \in A$

* **Symmetric Relation** · R is Symmetric in A if e $(a,b) \in R \Rightarrow (b,a) \in R$ for every $a,b \in A$

* **Transitive Relation** · R is transitive in A if $(a,b) \in R$, $(b,c) \in R \Rightarrow (a,c) \in R$ for every $a,b,c \in A$

* **Equivalence relation**: If R is reflexive, symmetric and transitive then R is equivalence

Equivalence Class

Let $R = \{(1,1)(2,2)(3,3)(1,3)(3,1)\}$ is an equivalence relation defined on $A = \{1,2,3\}$

Equivalence class of 1 is $[1] = \{1,3\}$
Equivalence class of 2 is $[2] = \{2\}$
Equivalence class of 3 is $[3] = \{3,1\}$
$[1] \cup [2] \cup [3] = A$

Functions

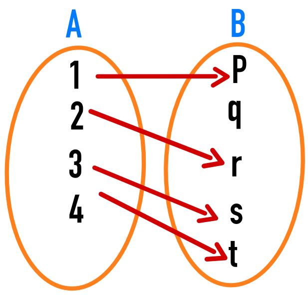
A relation $(f : A \to B)$ where every element of A has only one image in B

Dofn $(f) = \{a : (a,b) \in F\}$
Range $(f) = \{b ; (a,b) \in F\}$

Type of functions

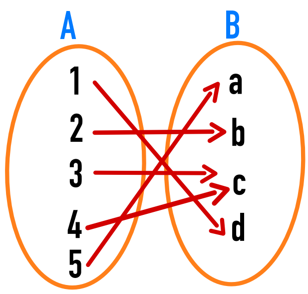
* **One – one (injective) function**

No two elements of A has same image in B
A function is not one - one then it is Many one



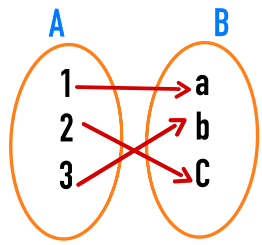
* **Onto (Surjective) function**

- All elements of B have at least one pre - Image in A
- Range = co domain



Bijective Function

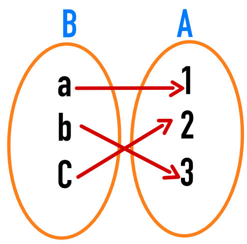
If a function is both one-one and onto then it is bijective



Inverse function

inverse of a function exist if it is bijective

If $f : A \to B$ is a function
its inverse is $F^{-1} : B \to A$



Composition of function

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[rr, bend right] & B \arrow[r, "G"] & C \end{tikzcd}$$

$g \circ f : A \to C$

eg: $f(x) = \sin x$, $g(x) = x^2$

$f \circ g = f(g(x)) = f(x^2) = \sin(x^2)$

$g \circ f = (g(f x) = g(\sin x) = \sin^2 x$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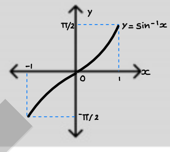
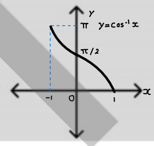
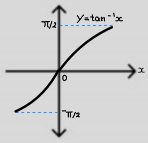
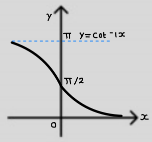
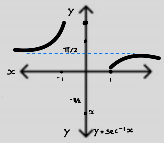
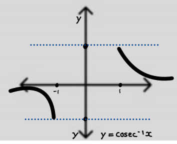
INVERSE TRIGONOMETRIC FUNCTIONS

Function	Domain	Range	Graph
$y=\sin^{-1}x$	[-1,1]	[-π/2,π/2]	
$y=\cos^{-1}x$	[-1,1]	[0,π]	
$y=\tan^{-1}x$	R	(-π/2,π/2)	
$y=\cot^{-1}x$	R	(0,π)	
$y=\operatorname{cosec}^{-1}x$	R-(-1,1)	[-π/2,π/2]-{0}	
$y=\sec^{-1}x$	R-(-1,1)	[0,π]-{π/2}	

PROPERTY-1

$\sin^{-1}(-x) = -\sin^{-1}(x)$

$\tan^{-1}(-x) = -\tan^{-1}(x)$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}(-x)$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$

$\sec^{-1}(-x) = \pi - \sec^{-1}x$

$\cot^{-1}(-x) = \pi - \cot^{-1}x$

PROPERTY-2

$\operatorname{cosec}^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$

$\sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)$

$\cot^{-1}x = \cot^{-1}\left(\frac{1}{x}\right)$

PROPERTY-3

$\sin^{-1}(\sin x) = x, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos^{-1}(\cos x) = x, x \in [0, \pi]$

$\tan^{-1}(\tan x) = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\cot^{-1}(\cot x) = x, x \in (0, \pi)$

$\operatorname{cosec}^{-1}(\text{cosex}) = x, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], x \neq 0$

$\sec^{-1}(\sec x) = x, x \in [0, \pi], x \neq \frac{\pi}{2}$

Identities to Remeber

$\sin^2 x + \cos^2 x = 1$

$\sec^2 x - \tan^2 x = 1$

$\operatorname{cosec}^2 x - \cot^2 x = 1$

$\sin 2x = 2\sin x \cos x$

$= \dfrac{2\tan x}{1+\tan^2 x}$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

$= 2\cos^2 x - 1$

$= \dfrac{1-\tan^2 x}{1+\tan^2 x}$

$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$\sin 3x = 3\sin x - 4\sin^3 x$

$\cos 3x = 4\cos^3 x - 3\cos x$

$\tan 3x = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

$\tan\left(\dfrac{\pi}{4} + x\right) = \dfrac{1+\tan x}{1-\tan x}$

$\tan\left(\dfrac{\pi}{4} - x\right) = \dfrac{1-\tan x}{1+\tan x}$

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MATRICES

Definition and its types

A Matrix of order $m \times n$ is an ordered rectangular array of numbers or functions having 'm' rows and 'n' columns. The matrix

$$A = [a_{ij}]_{m \times n} \cdot 1 \le i \le m, 1 \le j \le n; i, j \in N \text{ is given by}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ a_{m1} & a_{m2} \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

* **Column matrix:** It is of the form $[a_{ij}]_{m \times 1}$
* **Raw matrix:** It is of the form $[a_{ij}]_{1 \times n}$
* **Square matrix:** Here, $m = n$ (no. of rows = no of columns)
* **Diagonal matrix:** All non-diagonal entries are zero
* **Scalar matrix:** $a_{ij} = 0, i \ne j$ and $a_{ij} = k$ (Scalar), $i = j$
* **Identity matrix:** $a_{ij} = 0, i \ne j$ and $a_{ij} = 1, i = j$
* **Zero matrix:** All elements are zero

Equality of two matrices

$A = [a_{ij}] = [b_{ij}] = B$ if. A and B are of Same order and $a_{ij} = b_{ij}$ $\forall$ i and j

eg: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$a = 1,\ b = 2,\ c = 3,\ d = 4$

Operations on matrices

Multiplication

If $A = [a_{ij}]_{m \times n}$ an $B = [b_{jk}]_{n \times p}$, then $AB = C$, $[c_{jk}] = \sum_{j=1}^{n} a_{ij} b_{jk}$

Also, $A(BC) = (AB)C$, $A(B+C) = AB + AC$ and $(A+B)C = AC + BC$,

but $AB \neq BA$ (always)

Addition

If A, B are two matrices of same order, then

$A+B = [a_{ij} + b_{ij}]$. The addition of A and B follows

$A+B = B+A$, $(A+B)+C = A+(B+C)$, $-A = (-1)A$,

$k(A+B) = kA + kB$, k is Scalar

eg: If $A = \begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix}$ ten $A+B = \begin{bmatrix} -1 & 5 \\ -2 & 9 \end{bmatrix}$

If $A = \begin{bmatrix} 2 & 3 \end{bmatrix}_{1 \times 2}$, $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}_{2 \times 1}$ then $AB = [2 \times 4 + 3 \times 5] = [23]_{1 \times 1}$

Transpose of a matrix

If $A = [a_{ij}]_{m \times n}$, then its transpose $A' = (A^T) = [a_{ji}]_{n \times m}$ i.e. if $A = \begin{bmatrix} 2 & 1 \end{bmatrix}$ then $A^T = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Also, $(A')' = A$, $(kA)' = kA'$, $(A+B)' = A' + B'$, $(AB)' = B'A'$

A is Symmetric matrix if $A = A'$ i.e. $A' = A$.

A is knew - Symmetric if $A = -A'$ i.e. $A' = -A$

A is any square matrix, then -

$$A = \frac{1}{2}\Big[\underbrace{(A + A')}_{S.M.} + \underbrace{(A - A')}_{\text{Skew } S.m}\Big]$$ sum of a Symmetric and a skew - Symmetric matrix

For example if $A = \begin{pmatrix} 2 & 8 \\ 6 & 4 \end{pmatrix}$, then $A = \frac{1}{2}\left[\begin{pmatrix} 4 & 14 \\ 14 & 8 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}\right]$


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DETERMINANTS

If $A = [a_{11}]_{1\times1}$, then $|A| = a_{11}$

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2}$ then $|A| = a_{11}\,a_{22} - a_{12}\,a_{21}$

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$, then

$$|A| = a_{11}(a_{22}\cdot a_{33} - a_{23}\cdot a_{32}) - a_{12}(a_{21}\cdot a_{33} - a_{23}\cdot a_{31}) + a_{13}(a_{21}\,a_{32} - a_{22}\,a_{31})$$

e.g., If $A = \begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$, then $|A| = 2\times4 - 3\times2 = 2$

Area of a triangle

If (x_1,y_1), (x_2,y_2) and (x_3,y_3) are the vertices of triangle, Area of $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

e.g., if $(1,2)$, $(3,4)$ and $(-2,5)$ are the vertices, then triangle is

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 4 & 1 \\ -2 & 5 & 1 \end{vmatrix} = \frac{1}{2}\left|1(4-5) - 2(3+2) + 1(15+8)\right| = 6 \text{ sq. units}$$

we take positive value of the determinant because area is considered positive.

Equation of line joining the point (x,y) and (x_2,y_2)

is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

Minors & confactors of a matrix

Minor of an element a_{ij} in a determinant of matrix A is the determinant obtained by deleting i^{th} row and j^{th} Column and is denoted by M_{ij}. If M_{ij} is the Minor Of a_{ij} and Cofactor of a_{ij} is A_{ij} given by $A_{ij} = (-1)^{i+j} M_{ij}$

* If A is 3x3 a matrix, then $|A| = a_{11} \cdot A_{11} + a_{12} \cdot A_{12} + a_{13} \cdot A_{13}$
* If elements of one row (or column) are multiplied with Cofactors of elements of any other row (or column), then their sum is zero. for e.g., $a_{11} A_{21} + a_{12} A_{22} + a_{13} A_{33} = 0$.

e.g., if $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$, then $M_{11} = 4$ and $A_{11} = (-1)^{1+1} 4 = 4$

$M_{12} = -3$ and $A_{12} = (-1)^{1+2} (-3) = 4$

Adjoint of a matrix(Transpose of cofactor matrix)

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then Adj $A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

where A_{ij} is the Cofactor of a_{ij}.

* $A(\text{adj} \cdot A) = (\text{adj} \cdot A) \cdot A = |A| I$, A - Square matrix of order 'n'
* If $|A| = 0$, then A is Singular. otherwise, A is non-Singuler

Inverse of a square matrix

* If $AB = BA = I$, where B is a square matrix then B is called the inverse of A, $A^{-1} = B$ or $B^{-1} = A$, $(A^{-1})^{-1} = A$

Inverse of a square matrix exists if A is non singular i.e. $|A| \neq 0$, and is given by

$$A^{-1} = \frac{1}{|A|}(\text{adj} \cdot A)$$

Applications of determinants & matrix

If $a_1x + b_1y + c_1z = d_1$,
$a_2x + b_2y + c_2z = d_2$,
$a_3x + b_3y + c_3z = d_3$ then we can write $AX = B$,

where $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

* Unique solution of $AX = B$ is $X = A^{-1}B$, $|A| \neq 0$.
* $AX = B$ is consistent or inconsistent according as the solution exists or not
* For a square matrix A in $AX = B$, if

(i) $|A| \neq 0$ then there exists unique solution
(ii) $|A| = 0$ and $(\text{adj} \cdot A)B \neq 0$, then no solution
(iii) if $|A| = 0$ and $(\text{adj} \cdot A) \cdot B = 0$ then system may or may not be consistent

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CONTINUITY & DIFFERENTIABILITY

Continuous function

Suppose f ia real function on a subset of the real numbers and let 'c' be a point in the domain of f. Then f is continuous at c if $\lim_{x \to c} f(x) = f(c)$

A real functions f is said to be continuous if it is continuous at every point in the domain of f.

eg: The function $f(x) = \frac{1}{x}, x \neq 0$ is continuous

Let 'c' be any non-zero real number, then $\lim_{x \to c} f(x) = \lim_{x \to c} \frac{1}{x} = \frac{1}{c}$. for $c = 0$, $f(c) = \frac{1}{c}$ so $\lim f(x) = f(c)$

and hence f is continuous at every point in the domain of f

Algebra of continuous functions

Suppose f and g are two real functions continuous at a real number c, then, $f+g, f-g, f \cdot g$, and $\frac{f}{g}$ are continuous at x

Differentiability

Suppose f is a real function and c is a point in its domain. The derivative of f at c is $f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$

Every differentiable function is continuous, but the converse is not true.

Some standard derivatives

$\frac{dx}{dx} = 1$

$\frac{d}{dx} x^n = nx^{n-1}$

$\frac{d}{dx} \log x = \frac{1}{x}$

$\frac{d}{dx} e^x = e^x$

$\frac{dk}{dx} = 0$ k; Constant

$\frac{d}{dx} \sin x = \cos x$

$\frac{d}{dx} \cos x = -\sin x$

$\frac{d}{dx} \tan x = \sec^2 x$

$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$

$\frac{d}{dx} \sec x = \sec x \cdot \tan x$

$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cdot \cot x$

Chain rule

If $y = f(u)$, where $u = g(x)$ and if both $\frac{dy}{du}, \frac{du}{dx}$ exists, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Derivatives of implicit functions

If two variables are expressed by some relation then one will be the implicit function of other, is called Implicit function.

eg: Let $y = \cos x - \sin y$, then $\frac{dy}{dx} = \frac{d}{dx} \cos x - \frac{d}{dx} \sin y$

or, $\frac{dy}{dx} = -\sin x - \cos y \cdot \frac{dy}{dx}$ or $\frac{dy}{dx} = \frac{-\sin x}{1+\cos y}$, where $y \neq (2n+1)\pi$

Derivatives of inverse trigonometric functions

(i) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

(ii) $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

(iii) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

(iv) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

(v) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$

(vi) $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = \frac{-1}{x\sqrt{x^2-1}}$

Logarithmic differentiation

Let $y = f(x) = [u(x)^{v(x)}]$

$\log y = v(x) \log [u(x)]$

$\frac{1}{y}\frac{d}{dx} = v(x)\frac{1}{u(x)}u'(x) + v'(x)\log[u(x)]$

$\frac{d}{dx} = y\left[\frac{v(x)}{u(x}u'(x) + v'(x)\log[u(x)]\right]$

e.g.: Let $y = a^x$ Then $\log y = x \log a$

$\frac{1}{y}\cdot\frac{dy}{dx} = \log a$

$\frac{dy}{dx} = y \log a = a^x \log a.$

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6

APPLICATIONS OF DERIVATIVES

Rate of change of quantities

If a quantity 'y' varies with another quantity x so that $y=f(x)$, then

$\frac{dy}{dx}$ $[f'(x)]$ represents the rate of change of y w.r.t x and $\left.\frac{dy}{dx}\right|_{x=x_0}$ $(f'(x_0))$

represents the rate of change of y w.r.t. x at $x=x_0$

If 'x' and 'y' varies with another variable 't' i.e. if $x=f(t)$

and $y=g(t)$, then by chain rule $\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$, if $\frac{dx}{dt} \neq 0$.

eg: if the radius of a circle, $r=5$ cm, then the rate of change of the area of a circle per second w.r.t 'r' is -

$$\boxed{\left.\frac{dA}{dr}\right|_{r=5} = \frac{d}{dr}(\pi r^2)|_{r=5} = 2\pi r|_{r=5} = 10\pi}$$

Increasing and decreasing functions

A function f is said to be

(i) increasing on (a,b) if $x_1<x_2$ in $(a,b) \Rightarrow f(x_1) \leq f(x_2)$ $\forall x_1, x_2 \in (a,b)$, and

(ii) decreasing on (a,b) if $x_1<x_2$ in $(a,b) \Rightarrow f(x_1) \geq f(x_2)$ $\forall x_1, x_2 \in (a,b)$

If $f'(x) \geq 0$ $\forall x \in (a,b)$ then f is increasing in (a,b) and

If $f'(x) \leq 0$ $\forall x \in (a,b)$ then f is decreasing in (a,b)

eg: Let $f(x) = x^3 - 3x^2 + 4x$, $x \in R$, then

$f'(x) = 3x^2 - 6x + 4 = 3(x-1)^2 + 1 > 0$ $\forall x \in R$

So, the function f is strictly increasing on R

Maxima and Minima

A point C in the domain of 'f' at which either $f'(C)=0$ or is not differentiable is Called a critical point of f.

First derivative test

Let f be Continuous at a Critical point C in open interval. Then

(i) if $f'(x) > 0$ at every point left of c and $f'(x) < 0$ at every point right of c, then 'C' is a point of local maxima

(ii) if $f'(x) < 0$ at every point left of c and $f'(x) < 0$ at every point right of c, then 'C' is a point of local minima

(iii) if $f'(x)$ does not change sign as 'x' increases through C, then 'C' is called the point of inflection.

Second derivative test

Let f be a function defined on given interval f is twice differentiable at c. Then

(i) $x=c$ is a point of local maxima if $f'(c)=0$ and $f''(c)<0$, $f(c)$ is local maxima of f

(ii) $x=c$ is a point of local minima if $f'(c)=0$ and $f''(c)>0$, $f(c)$ is local maxima of f

(iii) The test fails if $f'(c)=0$ and $f''(c)=0$

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8

APPLICATIONS OF THE INTEGRATS

Area under simple curves

The area of the region bounded by the curve

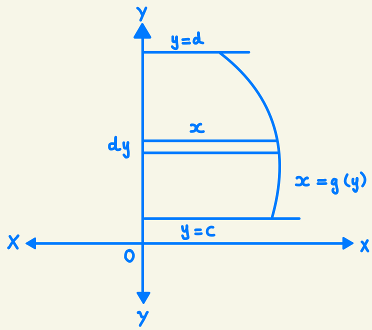
$x = f(y)$, y- axis and the lines $y = d\ (d > c)$

is given by $A = \int_c^d x\,dy$ or $\int_c^d f(x)dy$

eg: The area bounded by $x = y^3$, y - axis in the

I quadrant and the lines $y = 1$ and $y = 2$ is

$\int_a^b f(x)\,dx = \int_a^b f^3\,dy = \left[\frac{1}{4}y^4\right]_1^2 = \frac{1}{4}\left(2^4 - 1^4\right) = \frac{15}{4}$ sq. units



The area of the region bounded by the

curvey $y = f(x)$, x axis and the lines

$x = a$ and $x = b$ $(b > a)$ is given by

$A = \int_a^b y\,dx$ or $\int_a^b f(x)dx$

eg: The area bounded by $y = x^2$, x -axisin I quadrant

and the lines $x = 2$ and $x = 3$ is

$A = \int_2^3 y\,dx = \int_2^3 x^2 dx = \left[\frac{x^3}{3}\right]_2^3 = \frac{1}{3}(27 - 8) = \frac{19}{3}$ sq. units