# An Accurate SOC Estimation System for Lithium-ion Batteries by EKF with Dynamic Noise Adjustment

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Abstract—Based on an SOC (State of Charge) estimation system for Lithium-ion battery by EKF (Extended Kalman filter), we have improved the accuracy by dynamic adjusting of noises in the battery model. Firstly, the battery model and the SOC (State of Charge) estimation algorithm by EKF are explained. Then, static optimization for the process noise and observation noise in the battery model is discussed. Afterward, an adaptive noise tuning method is proposed and its accuracy is evaluated by some examinations. The error by the static method is 0.97% and 1.21% in different test patterns. The average of the SOC estimation errors with the adaptive noise tuning is 0.87% and 1.16%. Significant improvement of accuracy has been achieved.

Keywords—battery management system, SOC estimation, Lithium-ion Battery

## I. INTRODUCTION

With the advent of the large-scale popularization of Lithiumion secondary batteries, the accuracy as well as low cost should be indispensable for measurement systems. Particularly, the SOC (state of charge) estimation is essential for capturing the state of Lithium-ion secondary batteries. Generally, SOC estimation uses the external voltage and output current of battery. The self-discharge, the voltage drop by internal resistance, and polarization make direct SOC estimation from the external voltage and current of a battery very difficult.

In this paper, a precise SOC estimation system is devised by means of the Extended Kalman Filter (EKF). Compared with the conventional techniques, this proposed method can get high precision, such as OCV method [1], internal resistance method [2], current accumulation method [3], etc.

The EKF algorithm for li-ion batteries is constructed on a discrete-time state-space model of battery model using a numerical analysis method, and utilizes a SOC-OCV curve. It can estimate the state of unknown quantities by the battery model [4,5]. To get high precision, we must set the noise of discrete-time state-space model or get the parameters of battery model accurately. To solve the noise setting, we provide adaptive noise tuning method using observation SOC.

#### II. PRELIMINARIES

## A. Battery Model

To describe the characteristics of the battery, we use the equivalent circuit model as shown in Fig.1. The model consists of resistors, capacitors and a voltage source. The  $R_0$  is solution resistance, two RC networks  $(R_1, C_1, R_2, C_2)$ 

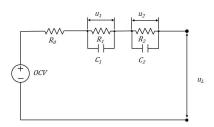


Fig. 1. Equivalent circuit model of a Li-ion battery

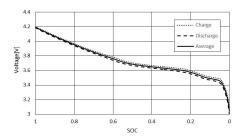


Fig. 2. OCV-SOC curve

represent the battery polarization. i is the terminal current, and  $u_L$  is the terminal voltage of the battery,  $u_1$  and  $u_2$  are the voltages of the two RC networks. Equations (1) and (2) show the relationship of i,  $u_1$  and  $u_2$ . Equation (3) shows relationship between  $u_L$  and i.

$$i = C_1 \frac{du_1}{dt} + \frac{u_1}{R_1}$$

$$i = C_2 \frac{du_2}{dt} + \frac{u_2}{R_2}$$

$$u_L = u_1 + u_2 + iR_0 + u_{OCV}$$
(1)
(2)

$$i = C_2 \frac{du_2}{dt} + \frac{u_2}{R} \tag{2}$$

$$u_L = u_1 + u_2 + iR_0 + u_{OCV} \tag{3}$$

The voltage source represents the open circuit voltage (OCV). The OCV is a function of SOC. We obtained the relationship between the OCV and SOC by experiment.

The average of 0.02C discharge and charge approximates the OCV. The SOC-OCV curve is obtained by averaging the voltage curves for 0.02C discharge and 0.02C charge. The experimental result is shown in Fig.2. In order to use in the EKF algorithm, we approximate the SOC-OCV curve by a 12th order polynomial as in Equation (4).

$$u_{OCV}(SOC) = \sum_{i=0}^{n} a_i SOC^i$$
 (4)

Though a logarithmic function is used in the references [4,5], we employ the polynomial function because it can be calculated more efficiently than the logarithmic function on a microcomputer.

## B. SOC Estimation by EKF

Our SOC estimation uses the EKF since the state-space of the battery is nonlinear as described below. The EKF is an approximately optimal state estimator for the nonlinear stochastic process subject to Gaussian white noises [6]. The discrete-time nonlinear state-space model is given by Equations (5) and (6), where Equation (5) is the state Equation, and Equation (6) is the observation equation. f(x(k), u(k)) is the functional relationship of the state vector x(k) and the next sample time x(k+1). h(x(k), u(k)) is the functional relationship of the state vector x(k) and observation vector y(k). u(k) is the control vector, w(k) is the process noise, and v(k) is the observation noise.

$$x(k+1) = f(x(k), u(k)) + bw(k)$$
(5)

$$y(k) = h(x(k), u(k)) + v(k)$$
(6)

In the battery model presented in [7,8], we define the state vector as  $x(k) = [SOC(k) \ u_1(k) \ u_2(k)]^T$ , and the observation vector as  $y(k) = U_L(k)$ . By applying the forward Euler approximation with the sampling interval  $\Delta t$  to Equations (1) and (2), we obtain the following difference equations.

$$u_1(k+1) = \left(1 - \frac{\Delta t}{R_1 c_1}\right) u_1(k) + \frac{i}{c_1} \Delta t \tag{7}$$

$$u_2(k+1) = \left(1 - \frac{\Delta t}{R_2 C_2}\right) u_2(k) + \frac{i}{C_2} \Delta t$$
 (8)

Putting Equations (7) and (8) together yields the state equation of (9). The observation equation (10) is directly obtained from Equation (3). We assume that the process noise w(k) comes from the variations of the internal impedance parameters of the equivalent circuit model.

$$x(k+1) = Ax(k) + b_u i(k) + bw(k)$$
(9)

$$y(k) = u_{OCV}(SOC) + i(k)R_0(k) + u_1(k) + u_2(k) + v(k)$$
 (10)  
A,  $b_u$  and b in Equations. (9) and (10) are defined by

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\Delta t}{c} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \left(1 - \frac{\Delta t}{R_1 c_1}\right) & 0 \\ 0 & 0 & \left(1 - \frac{\Delta t}{R_2 c_2}\right) \end{bmatrix}, \quad b_u = \begin{bmatrix} \frac{\Delta t}{c} \\ \frac{\Delta t}{c_1} \\ \frac{\Delta t}{c_2} \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (11)$$

Equation (9) is a linear equation. On the other hand, the observation equation (10) is nonlinear because the SOC-OCV curve is a nonlinear function as shown in Fig. 2. Therefore, Equation (10) defines a nonlinear state-space model of the battery. To apply the EKF to this state-space model, we introduce

$$C(k) = \frac{\partial h(x(k), u(k))}{\partial x(k)} \Big|_{x(k) = \hat{x}(k)} = \left[ \frac{dOCV}{dSOC} \Big|_{SOC = \widehat{SOC}^-}, \quad 1, \quad 1 \right]$$
(12)

The EKF algorithm for the state-space model (9),(10) is described below. This algorithm consists of 3 steps: Initialization, Prediction, and Filtering. In addition,  $\hat{x}^-$  is the one-step prediction vector,  $\hat{x}$  is the filtered estimate vector,  $P^$ is the prediction error covariance matrix, and P is the filtered error covariance matrix.  $\sigma_w^2(k)$  is the covariance of the process noise w(k), the  $\sigma_v^2(k)$  is the covariance of the observation noise v(k).

Initialization

$$\hat{x}(0) = \mathbf{E}[x(0)], P(0) = \mathbf{E}[x(0) - \hat{x}(0)][x(0) - \hat{x}(0)]^{\mathsf{T}}$$

**Prediction Step** 

$$\hat{x}^{-}(k+1) = A\hat{x}(k) + b_u u(k)$$

$$P^{-}(k+1) = AP(k)A^T + bb^{\mathsf{T}}\sigma_w^2(k)$$

Filtering Step

$$G(k+1) = \frac{P^{-}(k+1)C^{T}(k+1)}{C(k+1)P^{-}(k+1)C^{T}(k+1) + \sigma_{v}^{2}(k)}$$

$$\hat{x}(k+1) = \hat{x}^{-}(k+1) + G(k+1)\{y(k+1) - h(\hat{x}^{-}(k+1), u(k+1))\}$$

$$P(k+1) = \{I - G(k+1)C(k+1)\}P^{-}(k+1)$$

In Initialization, we estimate the initial SOC as  $f_{SOC-OCV}(u_L(0))$  using the initial external voltage. However,  $u_1(k)$  and  $u_2(k)$  are internal state that cannot be observed, so we set the initial  $u_1(k)$  and  $u_2(k)$  to 0. P(0) is the initial error covariance matrix.

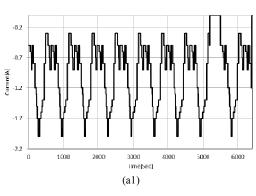
#### III. TUNING OF NOISE COVARIANCE

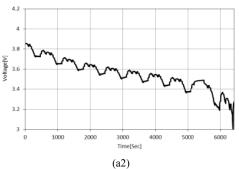
In this section, we show the optimization noise setting method. We make the static noise set method and adaptive noise tuning method and compare them.

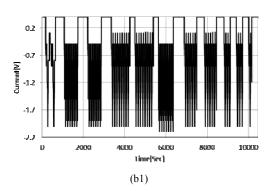
In order to evaluate the accuracy of the SOC estimation method, we show the experimental results that compare the results by the proposed method to those by the commercial battery test system Kikusui PFX2021S. Fig. 3 shows experimentation of SOC estimation for the two patterns of discharge current. The pattern 1 is the discharge with a periodic current waveform discharge, while the pattern 2 is the discharge with a pseudorandom current waveform. The battery which is used in this experiment is the type 18650, which nominal capacity is 2250mAh.

#### A. Static Noise Set Method

To compare to the adaptive noise covariance tuning method, we make the static noise covariance tuning method for comparison. The static noise covariance tuning method is the exhaustive search for the optimal noise covariance using the results of the test experiments. We use the rooted mean square (RMS) errors to evaluate the performance of a selected noise covariance. Since the terminal voltage is about 0.2[V] for this experiment, we set the search region as  $10^{-7} \le \sigma_w \le$  $10^{-2}$ ,  $10^{-3} \le \sigma_v \le 0.2$ , with logarithmic grid size. The search result is as Fig. 6. For the figure, the optimal values are  $\sigma_w = 10^{-5}, \, \sigma_v = 0.05.$ 







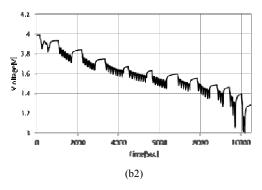


Fig. 3. External voltage and discharge current of the battery of experiment, (a1) discharge current of pattern 1, (a2) external voltage of pattern 1, (b1) discharge current of pattern 2, (b2) external voltage of pattern 2

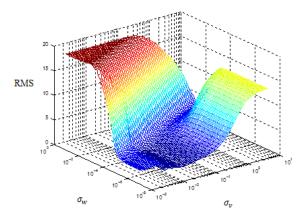
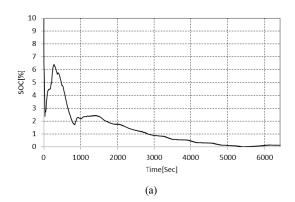


Fig. 4. SOC error result by the noise region search



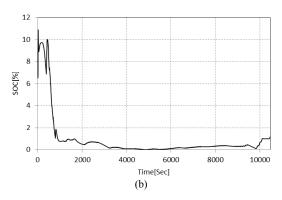


Fig. 5. The SOC estimation error by static noise set method, (a) for pattern 1, (b) for pattern 2

## B. Adaptive Noise Tuning Method

This season addresses the optimal noise covariance tuning method. Therefore, we improve the SOC estimation accuracy by incorporating the noise covariance tuning mechanism into the aforementioned EKF algorithm [9]. To be more specific, we adaptively estimate the observation noise covariance  $\sigma_v^2(k)$  from the observed data  $y(0), \ldots, y(k)$ .

For this purpose, we introduce the innovation (observation residuals)  $\boldsymbol{\tilde{y}}(k)$  as

$$\tilde{y}(k) = y(k) - h(\hat{x}^{-}(k), u(k)).$$
 (13)

We also compute the Taylor expansion of h(x(k), u(k)) $h(x(k), u(k)) = h(\hat{x}^{-}(k), u(k)) +$ around  $\hat{x}^-(k)$  $C(k)(x(k) - \hat{x}^{-}(k)) + (high order terms)$  where C(k) is the Jacobian matrix

$$C(k) = \frac{\partial h}{\partial x} \Big|_{(x,u) = (\hat{x}^{-}(k), u(k))}$$
(14)

We obtain the linearized observation

$$y(k) = h(\hat{x}^{-}(k), u(k)) + C(k)(x - \hat{x}^{-}(k)) + v(k)$$
(15)

by neglecting the high-order terms. Then, the innovation  $\tilde{y}(k)$ in (16) is rewritten as

$$\tilde{y}(k) = C(k)(x(k) - \hat{x}^{-}(k)) + v(k)$$
 (16)

Since the innovation  $\tilde{y}(k)$  and the noise v(k) are uncorrelated by the Kalman filtering theory, the covariance of  $\tilde{y}(k)$  is given by

$$E\{\tilde{\mathbf{y}}(\mathbf{k})\tilde{\mathbf{y}}(\mathbf{k})^{\mathsf{T}}\}$$

$$= C(\mathbf{k})E\left\{ \left( \mathbf{x} - \hat{\mathbf{x}}^{-}(\mathbf{k}) \right) \left( \mathbf{x} - \hat{\mathbf{x}}^{-}(\mathbf{k}) \right)^{\mathsf{T}} \right\} C(\mathbf{k})^{\mathsf{T}} + \sigma_{\mathbf{v}}^{2}(\mathbf{k})$$

$$= C(\mathbf{k})P^{-}(\mathbf{k})C(\mathbf{k})^{\mathsf{T}} + \sigma_{\mathbf{v}}^{2}(\mathbf{k})$$
(17)

By replacing the sampling average with the time average, the above equation reduces to

$$Y(k) = C(k)P^{-}(k)C(k)^{\mathsf{T}} + \sigma_{\mathsf{v}}^{2}(k) \tag{18}$$

$$Y(k) \stackrel{\text{def}}{=} \frac{1}{k+1} \sum_{i=0}^{k} \tilde{y}(k) \tilde{y}(k)^{\mathsf{T}}$$
(19)

 $Y(k) = C(k)P^{-}(k)C(k)^{T} + \sigma_{v}^{2}(k) \tag{18}$   $Y(k) \stackrel{\text{def}}{=} \frac{1}{k+1} \sum_{i=0}^{k} \tilde{y}(k) \tilde{y}(k)^{T} \tag{19}$ Using Equations (18) and (19), the adaptive noise covariance tuning mechanism is given by

$$\sigma_{\mathbf{v}}^{2}(\mathbf{k}) = \mathbf{Y}(\mathbf{k}) - \mathbf{C}(\mathbf{k})\mathbf{P}^{-}(\mathbf{k})\mathbf{C}(\mathbf{k})^{\mathsf{T}}$$
(20)

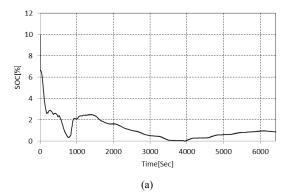
$$\sigma_{v}^{2}(k) = Y(k) - C(k)P^{-}(k)C(k)^{T}$$

$$Y(k) = \frac{1}{k+1}\tilde{y}(k)\tilde{y}(k)^{T} + \frac{k}{k+1}Y(k-1)$$
Experiment for the adaptive noise tuning method is

executed for the same input to the static noise set method shown in Fig. 3. The initial noise covariance of adaptive noise covariance tuning method are  $\sigma_w = 10^{-5}$ ,  $\sigma_v = 0.05$ . Fig. 6 shows the SOC estimation errors by the adaptive noise tuning method, noise covariance tuning, for the two input patterns. The averages of the SOC estimation errors by the adaptive noise tuning method are 1.16% for pattern 1, and 0.87% for pattern 2. Significant accuracy improvement from the static noise set method has been achieved. Moreover, the adaptive noise tuning method is much more efficient since it does not need pre-evaluation of SOC noises as in Fig. 4.

# IV. CONCLUSION

We have proposed the SOC estimation system for Lithiumion batteries using EKF with an adaptive noise tuning. We verified the accuracy of the SOC estimation system using an evaluation experiment. The error by the static method is 0.97% and 1.21% in different test patterns. The average of the SOC estimation errors with the adaptive noise tuning is 0.87% and 1.16%. Significant improvement of accuracy has been achieved from the static noise set method. Moreover, the adaptive noise tuning method is very efficient and reliable since it does not need pre-evaluation of SOC noises.



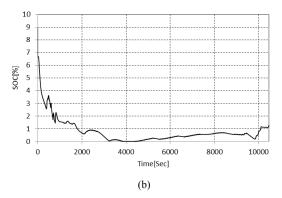


Fig. 6. The SOC estimation error by adaptive noise tuning method, (a) for pattern 1, (b) for pattern 2

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