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Li-ion Battery SOC Estimation Using Particle Filter Based on an Equivalent Circuit Model

Du Jiani, Wang Youyi, *Senior Member*, *IEEE* and Wen Changyun, *Fellow*, *IEEE*

Abstract— **In this paper, a method for Li-ion battery state of charge (SOC) estimation using particle filter (PF) is proposed. The equivalent circuit model for Li-ion battery is established based on the available battery block in MATLAB/Simulink. To improve the model's accuracy, the circuit parameters are represented by functions of SOC. Then, the PF algorithm is utilized to do SOC estimation for the battery model. From simulation it reveals that PF provides accurate SOC estimation. It is demonstrated that the proposed method is effective on Li-ion battery SOC estimation.**

I. INTRODUCTION

Compared with other kinds of batteries, Li-ion battery has the advantages of high energy ratio, no memory effect and long service life [1]. Therefore, Li-ion battery is used in electric vehicles (EVs) and hybrid electric vehicles (HEVs) as the power source recently. The battery management system (BMS) is established to control Li-ion battery's charging and discharging while guaranteeing reliable and safe operation [2]. Since over-charging and over-discharging brings inevitable damage to Li-ion battery, BMS must supervise the battery's status and avoid such phenomenon. An important variable describing Li-ion battery's status is state of charge (SOC) [3]. Therefore, to estimate battery SOC accurately is important in BMS.

The elementary step for SOC estimation is battery modeling. The most accurate model for Li-ion battery is the electrochemical model which describes the electrochemical reaction inside the battery. However, this kind of model is difficult to control and estimate since it is described by partial differential equations. Another common-used Li-ion battery model is the equivalent circuit model which describes the battery's characteristic by a circuit [4]. This kind of model is established based on the external reflection of the battery. The equivalent circuit model can achieve high accuracy. In [5], an accurate equivalent circuit model for Li-ion battery is proposed with the circuit parameters represented by SOC functions. The model output error is quite small.

In [6], various SOC estimation techniques are reviewed. The most widely-used and simplest technique is the Coulomb counting method. This technique does current integration directly based on SOC's definition. However, the open-loop form of this method may bring big error with integration. In [7], an enhanced Coulomb counting method is proposed, considering more effecting factors. For Li-ion battery, the open circuit voltage (OCV) has a direct relationship to SOC. However, the steady state of Li-ion battery is hard to obtain when the battery is at work. Artificial intelligence methods including fuzzy logic and neural networks are also utilized to estimate SOC more accurately. In [8], the electrochemical impedance spectroscopy combined with fuzzy logic provides accurate SOC while this method is also not suitable for online work. Recently, Kalman filter (KF)-based methods are widely used to do Li-ion battery SOC estimation. In [9, 10], the extended Kalman filter (EKF) is utilized to do SOC estimation and model parameter identification online. However, such methods are dependent on the model's simplicity and accuracy. For example, EKF can't achieve good estimation performance on highly nonlinear system [11]. On the other hand, KF requires the system noise to be Gaussian noise.

Particle filter (PF) is a probability-based estimator with high accuracy [12]. PF uses a set of weighted particles sampled by Monte Carlo method to approximate the posterior distribution of the system without any explicit assumption about the form of the distribution [13, 14]. PF is suitable to do state estimation for complex model with strong nonlinearity [12]. Therefore, PF can be utilized to do SOC estimation for nonlinear Li-ion battery model with complex form in condition of non-Gaussian distributed system noise.

In this paper, the algorithm of PF is used to do SOC estimation for Li-ion battery. In Section II, the equivalent circuit model of Li-ion battery is established in simulation. The parameters in the model are represented by functions of SOC. Then, the battery SOC is estimated by PF in Section III. The conclusion is given in Section IV.

II. LI-ION BATTERY MODEL

Firstly, the equivalent circuit model for Li-ion battery is established in MATLAB/Simulink. Apparently, the circuit parameters in this model are changing to SOC. Therefore, the parameters are approximated by functions of SOC in this section.

Du Jiani is with the school of Electrical and Electronic Engineering, Nanyang Technological University, 50 Nanyang Avenue, 639798 Singapore (email: jdu2@e.ntu.edu.sg).
Wang Youyi is with the school of Electrical and Electronic Engineering, Nanyang Technological University, 50 Nanyang Avenue, 639798 Singapore (corresponding author, email: EYYWANG@e.ntu.edu.sg).
Wen Changyun is with the school of Electrical and Electronic Engineering, Nanyang Technological University, 50 Nanyang Avenue, 639798 Singapore (email: ECYWEN@e.ntu.edu.sg).

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A. State of Charge

As the variable to be estimated, SOC has its own process equation. According to SOC's definition, its updating process can be described with the following equation:

$$SOC(t) = SOC(t_0) - \int_{t_0}^{t} \frac{I(\tau)}{C_n} d\tau \qquad (1)$$

where C_n is the battery's capacity, $SOC(t_0)$ is the SOC at time t_0, $I(t)$ is the battery current with positive value for discharging.

B. The Battery Model

The equivalent circuit model for Li-ion battery is a combination of voltage source, resistors and capacitors. The battery dynamics is described by RC ladders. With more RC ladders the model is more accurate but more complicated. In this paper, considering the model's accuracy and simplicity, the circuit with 1 RC ladder is selected to describe Li-ion battery. The model circuit is shown in Fig.1.

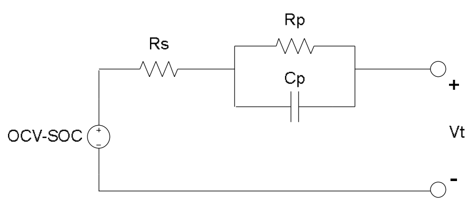

Fig.1 The proposed battery circuit model.

The state-space model is obtained based on the circuit in Fig. 1. Take the voltage across the RC ladder as U_p. Define the state variables as U_p and SOC. According to the circuit structure, the state-space model is described by:

$$\begin{bmatrix} S\dot{O}C \\ \dot{U}_p \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{R_p C_p} \end{bmatrix} \begin{bmatrix} SOC \\ U_p \end{bmatrix} + \begin{bmatrix} -\frac{1}{C_n} \\ -\frac{1}{C_p} \end{bmatrix} I \qquad (2)$$

$$V_t = OCV(SOC) - U_p - R_s I \qquad (3)$$

where R_s represents the series resistance, R_p and C_p are the resistance and capacitance of the RC ladder.

C. Extraction of the Model Parameters

The battery block in MATLAB/Simulink is selected to simulate Li-ion battery. The battery's nominal capacity is set to 0.85Ah. The fully charged voltage is 3.5V. A current test is applied in simulation. The circuit parameters of the proposed model are extracted by external reflection of the battery. The testing circuit is shown in Fig. 2. The battery is set fully charged before the test. The testing process stops when the battery voltage reaches the cut-off voltage.

The constant pulse discharging current test is utilized in the test. In simulation, a controlled current source is used in the circuit. A pulse generator is used to provide current signal to the current source. The testing current is depicted in Fig. 3. In each discharging period, the battery is discharged down to a level of 0.1 SOC with 1C and then allowed to rest for a period. The resting period is to make the battery obtain the steady state. The battery terminal voltage V_t with the pulse discharging current is shown in Fig. 4.

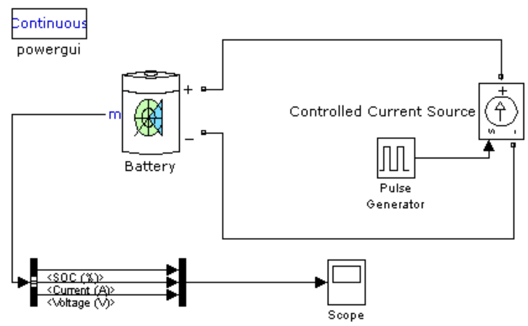

Fig. 2 The battery testing circuit in MATLAB/Simulink.

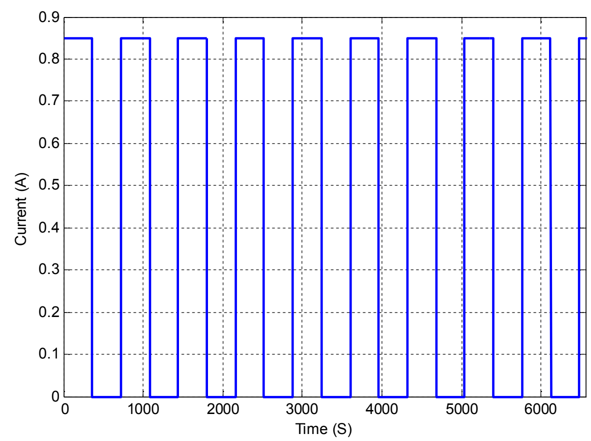

Fig. 3 Constant pulse discharging current.

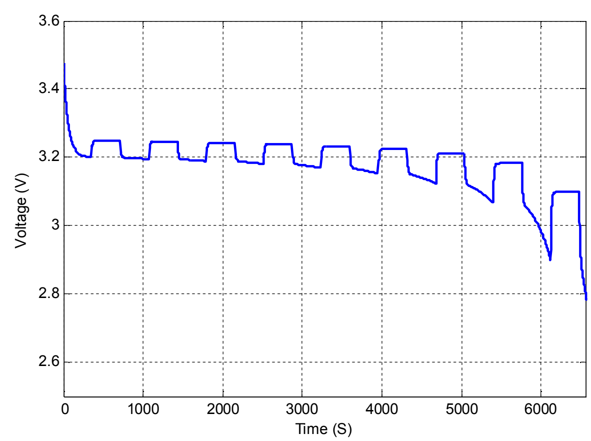

Fig. 4 The battery terminal voltage.

The battery voltage at the end of each resting period is taken as the OCV for every 0.1 SOC. The MATLAB curve fitting tool box is utilized to approximate the nonlinear relationship between OCV and SOC. The OCV-SOC relationship is represented by the following fitting equation:

$$OCV(SOC) = 3.919 \cdot SOC^5 - 12.24 \cdot SOC^4 + 14.8 \cdot SOC^3 - 8.709 \cdot SOC^2 + 2.576 \cdot SOC + 2.914 \quad (4)$$

The fitting curve is depicted in Fig. 5.

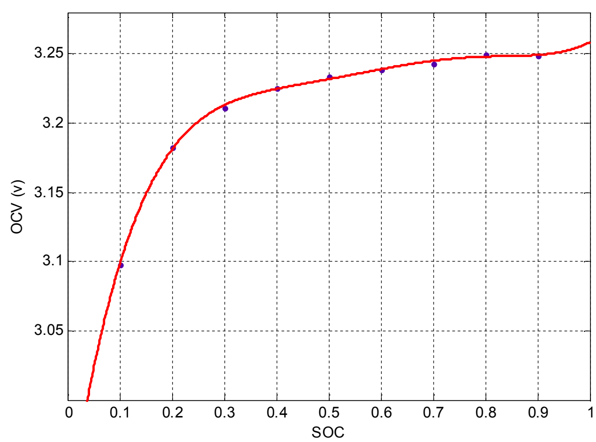

Fig. 5 The OCV-SOC relationship.

The series resistance R_s is calculated by the ratio between the instantaneous voltage drop and the discharging current at the instant when the discharging current starts. The voltage drop caused by R_s, $V(R_s)$, is shown in Fig. 6. R_s is calculated at every 0.1 SOC by:

$$R_s = \frac{V(R_s)}{I} \quad (5)$$

It is verified that R_s has similar value in the whole SOC range. Therefore, in this case, a constant R_s is utilized in the model.

The RC ladder's parameters R_p and C_p can be determined from the battery terminal voltage in the resting period [15]. The voltage curve at the transient period in Fig. 7 is utilized. At every 0.1 SOC, the voltage curve is approximated by the curve fitting tool box in MATLAB. This technique does not need prior knowledge of the RC circuit's time constant. Based on U_p's form in (2), the voltage curve of each transient period is approximated by the following equation:

$$V_t(t) = OCV + a \cdot e^{bt}, \quad (6)$$

where a and b are the coefficients determined by curve fitting. Then, the RC circuit parameters are calculated at every 0.1 SOC by $R_p = -\frac{a}{I}$ and $C_p = -\frac{1}{R_p b}$. The value of R_p in the whole SOC range is described by the curve in Fig. 8. It is obvious that R_p is changing to SOC. Then, the R_p curve is approximated by the function of SOC:

$$R_p(SOC) = 0.4006 \cdot e^{(-7.573 \cdot SOC)} + 0.02995 \quad (7)$$

The SOC function for C_p is similar to (7).

After determining the circuit parameters' value and function, the battery model is established. To verify the model's accuracy, the model output voltage and the actual voltage from the battery block is depicted in Fig. 9. It is shown that the model's error is very small. The largest error is smaller than 3%.

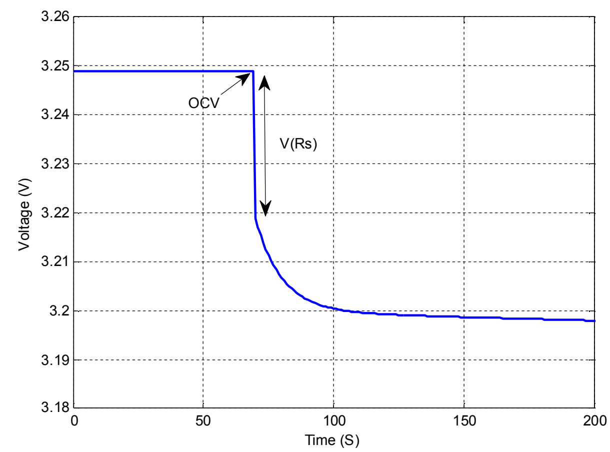

Fig. 6 The voltage drop caused by R_s.

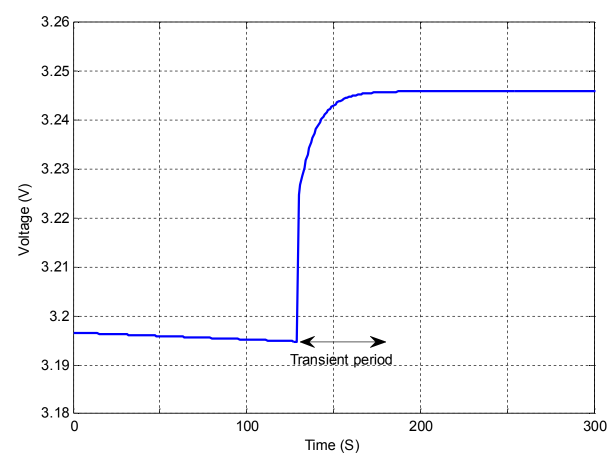

Fig. 7 The battery voltage during the resting period.

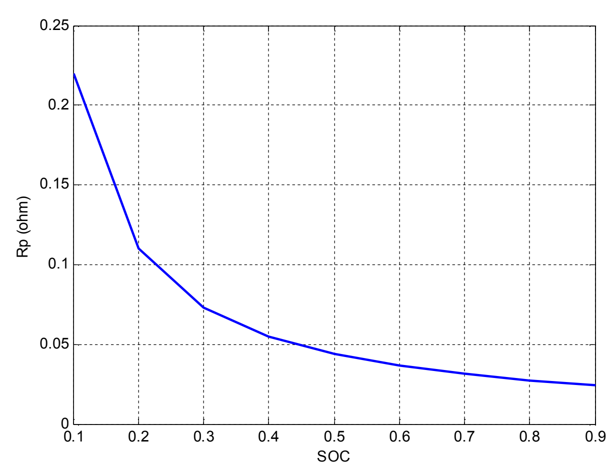

Fig. 8 R_p value changing to SOC.

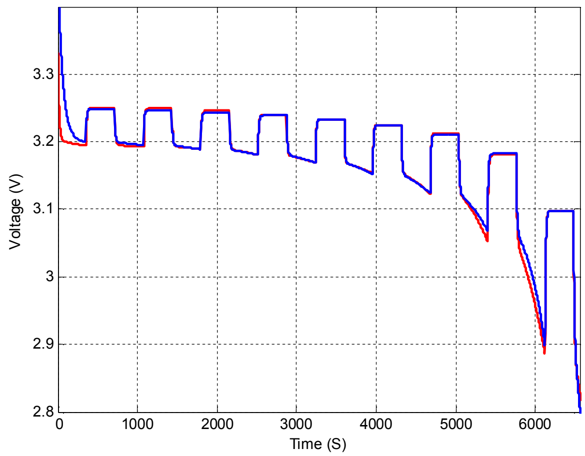

Fig. 9 The model voltage compared with the actual voltage.

III. Particle Filter

In the proposed model, the parameters R_p and C_p are nonlinear functions of SOC, making the model complex. PF can do state estimation for highly nonlinear system with high accuracy. In this section, the algorithm of PF is utilized to do SOC estimation based on the proposed nonlinear model. To execute PF, the discrete state-space model should be obtained first.

A. Discrete State-Space Model

To obtain the discrete model, the model is assumed time-invariant in one sample interval. The discrete form for (1) is:

$$SOC(k+1) = SOC(k) - \frac{\Delta t}{C_n} I(k) \tag{8}$$

where Δt is the sampling interval, k is the sampling step. The other state variable U_p is represented by the sum of RC circuit's discrete-time zero-state response $R_p(1-e^{-\frac{\Delta t}{R_p C_p}})I(k)$ and discrete-time zero-input response $e^{-\frac{\Delta t}{R_p C_p}} U_p(k)$ approximately [16]. In this way, the discrete form for the equivalent circuit model is described by:

$$\begin{bmatrix} SOC(k+1) \\ U_p(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & e^{-\frac{\Delta t}{R_p(SOC)C_p(SOC)}} \end{bmatrix} \begin{bmatrix} SOC(k) \\ U_p(k) \end{bmatrix} + \begin{bmatrix} -\frac{\Delta t}{C_n} \\ R_p(SOC)(1-e^{-\frac{\Delta t}{R_p(SOC)C_p(SOC)}}) \end{bmatrix} I(k) \tag{9}$$

$$V_t(k) = OCV(SOC(k)) - U_p(k) - R_s I(k) \tag{10}$$

where R_p and C_p are functions of SOC. In (9) and (10), the state vector is defined by:

$$x_k = \begin{bmatrix} SOC(k) & U_p(k) \end{bmatrix}^T \tag{11}$$

The battery current $I(k)$ is defined as the system input u_k. The terminal voltage V_t is defined as the output.

B. Particle Filter

For a nonlinear system:

$$x_k = f(x_{k-1}, u_{k-1}, w_{k-1}) \tag{12}$$

$$y_k = h(x_k, u_k, v_k) \tag{13}$$

where ω_k and v_k are independent white noise with known probability distribution function (PDF) [12]. The system states are Markovian, i.e.,

$$P(x_k \mid x_{k-1}, x_{k-2}, \dots, x_0) = P(x_k \mid x_{k-1}) \,. \tag{14}$$

Then PF can be utilized to do state estimation for the system. PF is a Monte Carlo method for implementing a recursive Bayesian filter. The posterior state is approximated by a set of weighted particles. The key part for PF is to obtain $P(x_t \mid y_{0:t})$ by generating a set of samples with associated weights. The PF algorithm is described as follows:

1) Initialize:

Determine the number of particles M according to the system's form and computation cost. Draw the state particles $x_0^i (i = 1, 2, \dots, M)$ from Guassian distribution $N(\hat{x}_0, \sigma_0^2)$, where $\hat{x}_0$ and σ_0^2 are the initial state mean and error covariance, i represents the *ith* particle. So, the initial particles are generated by:

$$x_0^i = \hat{x}_0 + N(0, \sigma_0^2) \,. \tag{15}$$

2) State predict:

For i=1, 2…, M particles, propagate the state particles $x_{k-1}^i (i = 1, 2, \dots, M)$ to the next step by the system process equation:

$$x_k^i = f(x_{k-1}^i, u_{k-1}) + N(0, \sigma_{k-1}^2) \,, \tag{16}$$

where x_{k-1}^i represents the prior state, σ_{k-1}^2 represents the error covariance.

3) Determine the particle weights:

Update and normalize the important weights for each particle. Calculate the *ith* particle's likelihood as:

$$q_i = \exp\left(-\frac{1}{2\sqrt{R}}(y_k - h(x_k^i, u_k))^2\right) / \sqrt{2\pi R} \tag{17}$$

where y_k represents the measurement at step k. $h(x_k^i, u_k)$ is the *ith* particle's output calculated with the system measurement equation. Then normalize the likelihood q_i of each particle by:

$$\omega_k^i = \frac{q_i}{\sum_{j=1}^{M} q_j} \tag{18}$$

where ω_k^i is the normalized weight for the *ith* particle. ω_k^i represents the probability of observing the measurement from the corresponding particle's state.

4) Re-sample:

A new particle set $\{\bar{x}_k^i\}_{i=1}^M$ is obtained by re-sampling. The basic idea of re-sampling is to eliminate particles with small weights and to concentrate on particles with large weights. For i=1, 2…, M, execute the re-sampling process with the following two steps. Firstly, a random number λ uniformly distributed on [0,1] is chosen. Then, accumulate the particle weight into a sum successively until the sum is greater than λ [12]. When $\sum_{n=1}^{j-1}\omega_k^n < \lambda$ and $\sum_{n=1}^{j}\omega_k^n > \lambda$, set the particle $\bar{x}_k^i$ to x_k^j with the weight ω_k^j.

5) Determine the estimation:

Calculate the mean of the re-sampled particles to obtain particle filter's estimation result $\hat{x}_k$:

$$\hat{x}_k = \frac{1}{M}\sum_{i=1}^{M}\bar{x}_k^i \, . \tag{19}$$

The steps (2)-(5) are executed recursively for k=1, 2…

C. Results and Discussion

To verify PF's estimation performance, a complex current pattern is applied in simulation. The current has a random discharging pattern depicted in Fig. 10. The current changes drastically in short time, which is possible in practice. The SOC value provided by the battery block is taken as the reference. For PF, the number of particles M is set to 1000. The estimated SOC from PF compared with the actual SOC is shown in Fig. 11. A clear description for SOC estimation error is shown in Fig. 12. The maximum error is smaller than 3%. The simulation result verifies that PF has good performance on Li-ion battery SOC estimation.

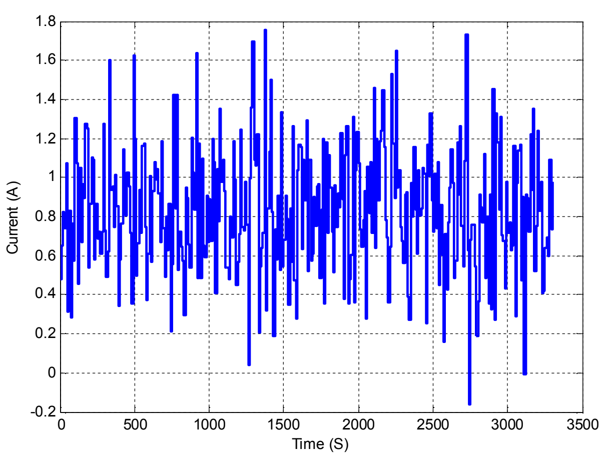

Fig. 10 Battery current.

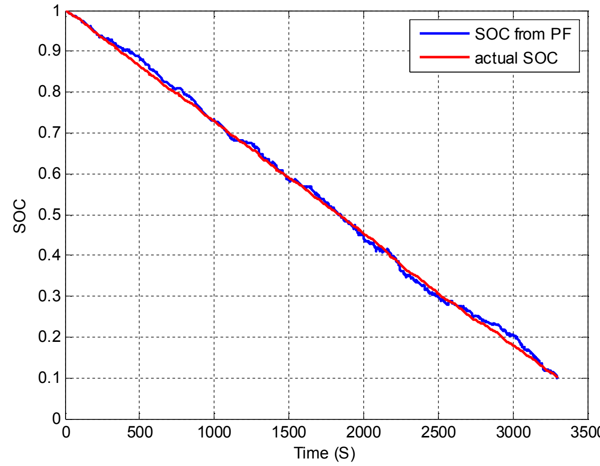

Fig. 11 The estimated SOC from PF compared with actual SOC.

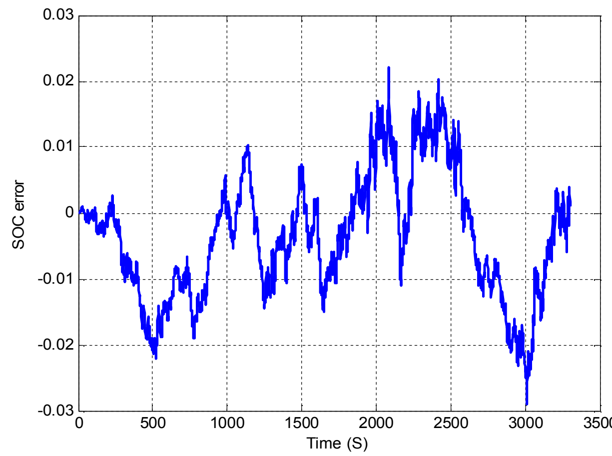

Fig. 12 SOC error.

IV. Conclusion

In this paper, a method for Li-ion battery SOC estimation using the algorithm of PF is proposed. The accurate equivalent circuit model with parameters represented by functions of SOC for Li-ion battery is established in MATLAB/Simulink. Then the algorithm of PF is utilized to estimate SOC based on the established model. From simulation, it is demonstrated that PF provides accurate SOC estimation. In conclusion, the proposed method has good performance on SOC estimation for Li-ion battery.

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