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An adaptive observer-based SOC estimation method for lithium-ion batteries against sensor measurement uncertainties

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***Abstract*—Accurate estimation of battery state of charge (SOC) is critical for efficient and safe battery applications. The measurement uncertainties of sensors, including measurement noises and sensor bias will affect the estimation accuracy inevitably. Therefore, quantifying the relationship between the sensor measurement uncertainties and the SOC estimation errors and seeking better SOC estimation methods has been a hot topic of research. In this paper, model errors and SOC estimation errors under measurement noise and sensor bias are derived. The resulting analyses can be used to assess the robustness of the SOC estimates. In addition, an adaptive observer-based SOC estimation method for lithium-ion batteries is proposed to cope with the measurement uncertainty. Simulation experiments demonstrate the effectiveness of the proposed method.**

***Index Terms*—adaptive observer, lithium-ion battery, sensor bias, sensor noises, State-of-Charge (SOC) estimation**

I. Introduction

Lithium battery is an outstanding and popular energy storage device. Real-time monitoring of the battery state of charge (SOC) is one of the keys that ensure the safety and efficiency of battery application [1]–[3]. SOC refers to the percentage of available battery capacity to the maximum stored capacity in the battery [4]. Researchers have proposed various SOC estimation methods, which can be divided into three main classes: direct measurement methods, model-based method, and data-driven methods.

Due to the requirement for real-time SOC estimation in applications such as electric vehicles, commonly used SOC estimation algorithms use a suitable model to capture the battery dynamics such as the equivalent circuit model (ECM) [5]. It is often combined with methods such as the Luenberger observer, Kalman filter, particle filter and $H\infty$ filter for real-time SOC estimation [6], [7].

Considering the nonlinear time-varying characteristics in the battery model, real-time correction of the model is the key to ensure SOC estimation accuracy. In [8], Prof. Chow et al. proposed a joint estimation algorithm that uses a segmented linear approximation with variable coefficients to describe the nonlinear relationship between the open-circuit voltage (VOC) and SOC of the battery. A moving-window least squares method is used for parameter identification, and update the observer parameters online. In [9], forgetting factor least squares (LS) and the unscented Kalman filter algorithm are jointly applied for parameter and SOC estimation. [10] combines recursive least squares (RLS) with extended Kalman filter (EKF) for parameter and SOC estimation.

Sensor uncertainties, including sensor noises and bias, can inevitably affect the parameter and state estimation accuracy. Many researchers experimentally show SOC estimation errors from voltage and current sensor uncertainties with experiments [11], [12]. It is shown that considering sensor measurement uncertainties is necessary for the improvement of SOC estimation accuracy. Some theoretical analyses have been carried out to find a way to cope with it. In [13], The SOC estimation errors due to sensor uncertainties are derived analytically. The analysis is based on the Kalman filter method and the accurate battery equivalent circuit model parameters. [14] analyzed the parameter variation of the battery model under the disturbances of current and voltage measurement uncertainties. [15] derives SOC estimation errors theoretically in parameter mismatch for observer-based SOC estimation methods.

For measurement noises, the Kalman filter-based methods and the moving window adaptive observer [16] are viable options. For sensor bias, the augmented observer-based SOC estimation method is one of the widely adopted ways, which takes the voltage or current sensor bias as one of the estimated state variables, but still relies on an accurate model. The model error due to measurement uncertainties is a potential problem for its stability and convergence. In this paper, the impact of measurement noise and bias on least squares-based battery parameter estimation and observer-based SOC estimation is derived. In addition, a robust adaptive observer algorithm for battery parameter and SOC simultaneous estimation under measurement noise and bias is proposed. The stability and convergence of the method are guaranteed mathematically. The performance of the robust adaptive observer in SOC estimation against measurement uncertainty is verified with simulation experiments.

This paper is structured as follows. Section II briefly introduces the equivalent circuit model and sensor measurement uncertainties. Theoretical analysis of the effect of sensor measurement uncertainties on model errors and SOC estimation errors is given in Section III. The robust adaptive observer algorithm is presented in Section IV and Section V verifies its performance. Finally, Section VI summarises the paper.

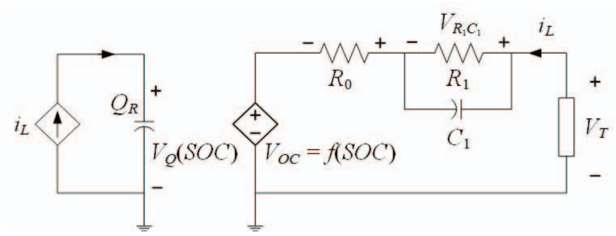

Fig. 1. Equivalent circuit model

II. Problem Formulation

A. Battery Equivalent Circuit Model

The first-order RC equivalent circuit battery model is shown in Fig. 1 [17]. It is composed of one RC pair C_1 and R_1, and an equivalent ohmic resistance R_0. The discrete-time battery dynamics are expressed below [18].

$$SOC_{k+1} = SOC_k + \frac{T}{Q_R} i_{L,k} \tag{1}$$

$$V_{RC,k+1} = e^{-\frac{T}{R_1C_1}} V_{RC,k} + R_1 \left(1 - e^{-\frac{T}{R_1C_1}}\right) i_{L,k} \tag{2}$$

$$V_{T,k} = V_{OC,k} + V_{RC,k} + R_0 i_{L,k} \tag{3}$$

$$V_{OC,k} = g\left(SOC_k\right) \tag{4}$$

where T denotes the sampling interval and k is the time step. The battery SOC is related to the integral of the load current i_L. Q_R is the total battery capacity. The RC pair is used to capture the instantaneous voltage dynamics. V_{RC} and V_{OC} are the voltage of RC pair and open-circuit voltage associated with the SOC, respectively. The terminal voltage V_T and current i_L are typically measured by sensors and used for battery model parameters and SOC estimation.

B. Sensor Measurement Uncertainties and Estimation Error Analysis

Define the voltage and current sensor measurement bias as ΔV and ΔI. The measurement noise components are δV_k and δI_k, which are independently and identically distributed with mean zero and variance σ_V^2, σ_I^2. The true value is denoted as $(\cdot)^*$ and the measured value is denoted by $(\cdot)^m$. Then the measured voltage and current can be expressed as follows.

$$V_{T,k}^m = V_{T,k}^* + \Delta V + \delta V_k \tag{5}$$

$$i_{L,k}^m = i_{L,k}^* + \Delta I + \delta I_k \tag{6}$$

According to the derivation of the battery model input-output relationship in [19], the measured battery voltage can be represented as:

$$\begin{aligned} V_{T,k}^m = -a_1^* V_{T,k-1}^* - a_2^* V_{T,k-2}^* + c_0^* i_{L,k}^* + c_1^* i_{L,k-1}^* \\ + c_2^* i_{L,k-2}^* + \Delta V + \delta V_k \end{aligned} \tag{7}$$

where $a_1^*, a_2^*, c_0^*, c_1^*, c_2^*$ are related to the battery model parameters such as resistance, capacitance, and slope b_1 of the VOC-SOC nonlinear curve.

$$a_1 = \frac{-4Q_R R_1 C_1}{Q_R T + 2Q_R R_1 C_1} \tag{8}$$

$$a_2 = \frac{-Q_R T + 2Q_R R_1 C_1}{Q_R T + 2Q_R R_1 C} \tag{9}$$

$$c_0 = \frac{T^2 b_1 + 2Q_R R_0 T + 2Q_R R_1 T + 4Q_R R_0 R_1 C_1 + 2b_1 R_1 C_1 T}{2Q_R T + 4Q_R R_1 C_1} \tag{10}$$

$$c_1 = \frac{T^2 b_1 - 4Q_R R_0 R_1 C_1}{Q_R T + 2Q_R R_1 C_1} \tag{11}$$

$$c_2 = \frac{T^2 b_1 - 2Q_R R_0 T - 2Q_R R_1 T + 4Q_R R_0 R_1 C_1 - 2b_1 R_1 C_1 T}{2Q_R T + 4Q_R R_1 C_1} \tag{12}$$

Based on the measurements, traditional methods first identify the battery parameters and then construct an observer to estimate the SOC.

For the parameter identification part, there are many methods for real-time battery parameter identification, such as Least Squares (LS), Extended Kalman filter (EKF). These methods do not allow accurate estimation of the parameters in the presence of uncertainties in current and voltage measurements. Taking the moving window least square parameter identification algorithm in [19] as an example, the relationship between the measurements and the parameter estimates is as follows.

$$\begin{aligned} V_{T,k}^m = -\hat{a}_1 V_{T,k-1}^m - \hat{a}_2 V_{T,k-2}^m + \hat{c}_0 i_{L,k}^m + \hat{c}_1 i_{L,k-1}^m \\ + \hat{c}_2 i_{L,k-2}^m \end{aligned} \tag{13}$$

where $\hat{}$ denotes the estimated value.

For the SOC estimation part, if the model parameters are accurately identified and there is no measurement uncertainty, then after linearization, the SOC observer can be constructed in the following form [19]:

$$\begin{cases} \dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u^m + L(y^m - \hat{y}) \\ \hat{y} = \hat{C}\hat{x} + \hat{D}u^m + \hat{b}_0 \end{cases} \tag{14}$$

$$\hat{A} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{\hat{R}_1\hat{C}_1} \end{bmatrix}, \hat{B} = \begin{bmatrix} 1/\hat{Q}_R \\ 1/\hat{C}_1 \end{bmatrix}, \hat{x} = \begin{bmatrix} S\hat{O}C \\ \hat{V}_{RC} \end{bmatrix} \tag{15}$$

$$\hat{C} = \begin{bmatrix} \hat{b}_1 & 1 \end{bmatrix}, \hat{D} = \hat{R}_0 \tag{16}$$

where u^m and y^m denote input current and output voltage measurements, respectively. Under sensor measurement uncertainties, the solved battery parameters can not match the true values, which leads to model errors. In addition, measurement bias and noises will cause the SOC estimates to deviate from the accurate values.

III. Derivation for SOC Estimation Error

In this section, the theoretical analysis of the effect of sensor measurement uncertainties on model errors and SOC estimation errors is carried out for the least squares-based battery parameter identification method and the observer-based SOC estimation method.

A. Model error analysis

According to eqn. (7) and eqn. (13), the parameters a_1, a_2, c_0, c_1, c_2 can be calculated online as follows with the moving window least squares method.

$$\underbrace{\begin{bmatrix} V_{T,k}^m \\ V_{T,k+1}^m \\ \vdots \end{bmatrix}}_{Y^m} = \underbrace{\begin{bmatrix} -V_{T,k-1}^m & -V_{T,k-2}^m & i_{L,k}^m & i_{L,k-1}^m & i_{L,k-2}^m \\ -V_{T,k}^m & -V_{T,k-1}^m & i_{L,k+1}^m & i_{L,k}^m & i_{L,k-1}^m \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}}_{X^m} \underbrace{\begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{c}_0 \\ \hat{c}_1 \\ \hat{c}_2 \end{bmatrix}}_{\hat{\theta}} \tag{17}$$

Using X^*, Y^* to denote the true values of voltage and current, X^m, Y^m to denote the measurements of voltage and current. ΔX and ΔY represent the measurement uncertainties, including noises and constant measurement bias, then

$$X^m = X^* + \Delta X \tag{18}$$

$$Y^m = Y^* + \Delta Y \tag{19}$$

$$Y^* = X^* \theta^* \tag{20}$$

$$\hat{\theta} = \left((X^m)^T X^m\right)^{-1} (X^m)^T Y^m \tag{21}$$

Define parameter estimation error as

$$\Delta\theta = \theta^* - \hat{\theta} \tag{22}$$

then

$$\Delta\theta = \left((X^*)^T X^*\right)^{-1} (X^*)^T (\Delta X \hat{\theta} - \Delta Y) \tag{23}$$

Define the matrix M^* consisting of the true voltage and current values:

$$M^* = \left((X^*)^T X^*\right)^{-1} (X^*)^T \tag{24}$$

The mean and variance of the parameter estimation error $E(\Delta\theta)$ and $\mathrm{var}(\Delta\theta)$ can be formulated as follows.

$$E(\Delta\theta) = M^* E(\Delta X \hat{\theta}) - M^* E(\Delta Y) \tag{25}$$

$$\mathrm{var}(\Delta\theta) = M^* [\mathrm{var}(\Delta X \hat{\theta}) + \mathrm{var}(\Delta Y)] (M^*)^T \tag{26}$$

Therefore, Sensor bias and noises can lead to biased estimates of parameters and larger variance in the estimates. $\hat{\theta}$ can be used to solve the battery parameters to update the observer in real time for SOC estimation. However, accurate SOC estimation requires a small model error. From the above analysis, it is clear that the measurement uncertainty of the sensor can lead to parameter estimation errors, causing model errors and problems for the following SOC estimation.

B. SOC estimation error

Sensor measurement uncertainties not only lead to model uncertainties, but also directly affect the SOC estimation of the observer. Next, the theoretical derivation is performed. A theoretical derivation is performed in this section.

For the observer constructed in eqn. (14), the estimated parameters $\hat{A}$, $\hat{B}$, $\hat{C}$, $\hat{D}$ may deviate from the true values A^*, B^*, C^*, D^*. Denote the parameter error by Δ. The state and output estimation errors are expressed by e_x and e_y.

$$\begin{aligned} \Delta A = A^* - \hat{A},\ \Delta B = B^* - \hat{B} \\ \Delta C = C^* - \hat{C},\ \Delta D = D^* - \hat{D} \end{aligned} \tag{27}$$

$$e_x = x^* - \hat{x},\ e_y = y^* - \hat{y} \tag{28}$$

After the Laplace transform, the observer can be written as

$$\begin{cases} s\hat{X}(s) = \hat{A}\hat{X}(s) + \hat{B}U_m(s) + L\left(Y_m(s) - \hat{Y}(s)\right) \\ \hat{Y}(s) = \hat{C}\hat{X}(s) + \hat{D}U_m(s) + \hat{b}_0 \end{cases} \tag{29}$$

And the true values of the system are

$$\begin{cases} sX^*(s) = A^*X^*(s) + B^*U^*(s) \\ Y^*(s) = C^*X^*(s) + D^*U^*(s) + {b_0}^* \end{cases} \tag{30}$$

where the current and voltage measurements include measurement uncertainties $\Delta U(s)$ and $\Delta Y(s)$.

$$U^m(s) = U^*(s) + \Delta U(s) \tag{31}$$

$$Y^m(s) = Y^*(s) + \Delta Y(s) \tag{32}$$

The estimation errors of the state and output voltage can be written as

$$\begin{aligned} sE_x = (\hat{A} - L\hat{C})E_x + (\Delta B - L\Delta D)U_* + (\Delta A - \\ L\Delta C)X - (\hat{B} - L\hat{D})\Delta U - L\left(\Delta Y + \Delta b_0\right) \end{aligned} \tag{33}$$

$$E_y = \hat{C}E_x + \Delta C X + \Delta D U^* - \hat{D}\Delta U + \Delta b_0 \tag{34}$$

The relationship of E_x and E_y with initial values, parameter mismatch and measurement uncertainties can be formulated as

$$\begin{aligned} E_x = (sI - \hat{A} + L\hat{C})^{-1} [e_{x,0} + (\Delta A - L\Delta C) \\ (sI - A^*)^{-1} x_0 + \Big((\Delta A - L\Delta C)(sI - A^*)^{-1} B^* + \\ \Delta B - L\Delta D\Big) U^* - \Big((\hat{B} - L\hat{D})\Delta U + L\left(\Delta Y + \Delta b_0\right)\Big)\Big] \end{aligned} \tag{35}$$

$$\begin{aligned} E_y = \left(1 + \hat{C}(sI - \hat{A})^{-1} L\right)^{-1} \Big[C^*(sI - A^*)^{-1} x_0 - \\ \hat{C}(sI - \hat{A})^{-1} \hat{x}_0 + \Big(C^*(sI - A^*)^{-1} B^* - \hat{C}(sI - \hat{A})^{-1} \\ \hat{B} + \Delta D\Big) U^* - \left(\hat{D}\Delta U - \Delta b_0\right) - \hat{C}(sI - \hat{A})^{-1} L\Delta Y\Big] \end{aligned} \tag{36}$$

1) Estimation errors due to parameter mismatch:

In this section, the estimation error under parameter mismatch is analyzed when the battery is charging or discharging, including mismatches in system dynamics, capacity, resistance, VOC-SOC curve slope and intercept.

For the case of system dynamics mismatch ΔA, the errors in eqn. (35) and eqn. (36) are given by

$$E_x = (sI - \hat{A} + LC^*)^{-1}\Delta A(sI - A^*)^{-1}B^*U^* \tag{37}$$

$$E_y = \frac{C^*(sI - A^*)^{-1}B^* - \hat{C}(sI - \hat{A})^{-1}\hat{B}}{1 + C^*(sI - \hat{A})^{-1}L}U^* \tag{38}$$

An unstable pole of E_x and E_y corresponds to the zero eigenvalue of A^*. Therefore, e_x and e_y is integrated over the change in current with respect to time.

For the case of capacity mismatch ΔB, E_x and E_y are

$$E_x = (sI - A^* + LC^*)^{-1}\Delta BU^* \tag{39}$$

$$E_y = \frac{C^*(sI - A^*)^{-1}\Delta B}{1 + C^*(sI - A^*)^{-1}L}U^* \tag{40}$$

Both E_x and E_y have stable poles, so the error in response to a step signal is finite for the steady state.

For the case of mismatch in VOC-SOC curve slope ΔC, E_x and E_y are

$$E_x = -(sI - A^* + L\hat{C})^{-1}L\Delta C(sI - A^*)^{-1}B^*U^* \tag{41}$$

$$E_y = \frac{\Delta C(sI - A^*)^{-1}B^*}{1 + \hat{C}(sI - A^*)^{-1}L}U^* \tag{42}$$

Since A^* has a zero eigenvalue , which corresponds to an unstable pole of E_x, e_x is the integration of current with time. All poles of E_y are stable. So the response of e_y to the step signal is finite for the steady state.

For the case of mismatch in VOC-SOC curve intercept Δb_0, E_x and E_y are

$$E_x = -(sI - A^* + LC^*)^{-1}L\Delta b_0 \tag{43}$$

$$E_y = \frac{\Delta b_0}{1 + C^*(sI - A^*)^{-1}L} \tag{44}$$

Since the poles of E_x and E_y are the eigenvalues of the observer, it is shown that the estimation errors e_x and e_y can converge to zero.

For the case of mismatch in resistance ΔD:

$$E_x = -(sI - A^* + LC^*)^{-1}L\Delta DU^* \tag{45}$$

$$E_y = \frac{\Delta D}{1 + C^*(sI - A^*)^{-1}L}U^* \tag{46}$$

The finite steady-state estimation errors e_x and e_y can be obtained since the poles of E_x and E_y are stable.

2) Estimation errors due to measurement uncertainties:

Estimation errors due to measurement uncertainties ΔU and ΔY are as follows.

$$E_x = -(sI - \hat{A} + L\hat{C})^{-1}\left[(\hat{B} - L\hat{D})\Delta U + L\Delta Y\right] \tag{47}$$

$$E_y = \frac{-\hat{D}\Delta U - \hat{C}(sI - \hat{A})^{-1}L\Delta Y}{1 + \hat{C}(sI - \hat{A})^{-1}L} \tag{48}$$

Since the eigenvalues of the observer correspond to the poles of E_x, finite steady state estimation error e_x. The unstable pole of $\hat{A}$ could lead to a non-zero steady state e_y since the eigenvalues of $\hat{A}$ are the poles of E_y.

3) Estimation errors caused by unknown initial conditions:

The part of the estimation errors associated with the unknown initial values can be written as

$$E_x = (sI - \hat{A} + L\hat{C})^{-1}\left[e_{x,0} + (\Delta A - L\Delta C)(sI - A^*)^{-1}x_0\right] \tag{49}$$

$$E_y = \frac{C^*(sI - A^*)^{-1}x_0}{1 + \hat{C}(sI - \hat{A})^{-1}L} - \frac{\hat{C}(sI - \hat{A})^{-1}\hat{x}_0}{1 + \hat{C}(sI - \hat{A})^{-1}L} \tag{50}$$

The zero eigenvalue of A^* is the unstable pole of E_x and E_y, which leads to non-zero steady states e_x and e_y.

The above analysis shows that, on the one hand, measurement uncertainties will cause model uncertainty and affect the stability and convergence of the SOC estimation algorithm, on the other hand, measurement uncertainties can directly cause the SOC estimates to deviate from the true values.

To cope with the effect of measurement uncertainty on SOC estimation, a widely adopted approach is to consider the possible measurement bias as one of the states with an augmented observer. however, this approach relies on an accurate model and cannot overcome the problem caused by the model error due to measurement uncertainties in algorithm stability and convergence.

IV. Proposed Robust SOC Estimation Algorithm

In this section, a robust adaptive observer algorithm is proposed to perform SOC estimation against the effect of measurement uncertainties on SOC estimation with strict stability and convergence guarantees.

A. Robust adaptive observer

In our previous work, we proposed a moving-window adaptive observer approach to simultaneously estimate the parameters and states for battery [16]. In this paper, in order to improve the robustness of the adaptive observer, the possible measurement bias is also taken into account as an unknown variable, which is estimated simultaneously with the model parameters and states. The proposed robust adaptive observer has a similar structure to the moving window adaptive observer including three subsystems. For the case with output measurement uncertainty d_y, the robust adaptive observer is designed as follows

1) Residual generator:

$$\hat{z}_{k+1} = A_{or}\hat{z}_k + \left(L_r + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) y_k + Q(u_k, y_k)\hat{\theta}_k + \begin{bmatrix} 1 \\ -1 \end{bmatrix} \hat{\alpha}_k \hat{d}_{y,k} + V_{k+1}(\hat{\theta}_{k+1} - \hat{\theta}_k) \tag{51}$$

$$r_k = y_k - C_o\hat{z}_k + C_r(u_k)\hat{\theta}_k \tag{52}$$

$$Q = \begin{bmatrix} u_k & 0 & -y_k & -l_{r1}u_k & -l_{r1} \\ 0 & u_k & y_k & -l_{r2}u_k & 2 - l_{r2} \end{bmatrix} \tag{53}$$

$$C_r = \begin{bmatrix} 0 & 0 & 0 & -u_k & -1 \end{bmatrix} \tag{54}$$

$$\hat{\theta}_k = \begin{bmatrix} \hat{\beta}_{2,k} & \hat{\beta}_{1,k} & \hat{\alpha}_k & \hat{R}_{0,k} & \hat{d}_{y,k} \end{bmatrix}^T \tag{55}$$

where $L_r = \begin{bmatrix} l_{r1} \\ l_{r2} \end{bmatrix}$ is the observer gain. Q and C_r are information matrices composed of measurement data. $\hat{\theta}_k$ consists of unknown model parameters and measurement uncertainty and is estimated in the parameter estimator.

2) Auxiliary filter:

Variables for correction of parameter and state estimates are generated in the auxiliary filter.

$$V_{k+1} = A_{or}V_k + Q \tag{56}$$

$$\varphi_k = C_oV_k - C_r \tag{57}$$

3) Parameter estimator:

In the parameter estimator, the parameter estimate $hat\theta_k$ is updated iteratively every n samples. n is the window size. when $k = nj, j = 0, 1, 2, \cdots$

$$\hat{\theta}_{k+1} = \hat{\theta}_k + \gamma_k\bar{\varphi}_k^T\bar{r}_k \tag{58}$$

$$\gamma_k = \mu/n(\sigma + \bar{\varphi}_k^T\bar{\varphi}_k)^{-1} \tag{59}$$

where

$$\bar{\varphi}_k = \begin{bmatrix} \varphi_{k-n+1} & \varphi_{k-n+2} & \cdots & \varphi_k \end{bmatrix}^T \tag{60}$$

$$\bar{r}_k = \begin{bmatrix} r_{k-n+1} & r_{k-n+2} & \cdots & r_k \end{bmatrix}^T \tag{61}$$

if $k \neq np, p = 0, 1, 2, \cdots$

$$\hat{\theta}_{k+1} = \hat{\theta}_k \tag{62}$$

According to the linear transformation relationship between V_{OC} and observer state z, the estimation of V_{OC} is:

$$\hat{V}_{OC} = \frac{\begin{bmatrix} 1 & 1 \end{bmatrix}\hat{z}}{\hat{\alpha} + 1} \tag{63}$$

Then the estimation $S\hat{O}C$ can be obtained from $\hat{V}_{OC}$ with a known VOC-SOC look-up table. For the case with input current measurement uncertainty, the robust SOC estimation can also be achieved by considering the measurement uncertainty into the unknown parameter vector $\hat{\theta}$ according to the above structure of the robust adaptive observer.

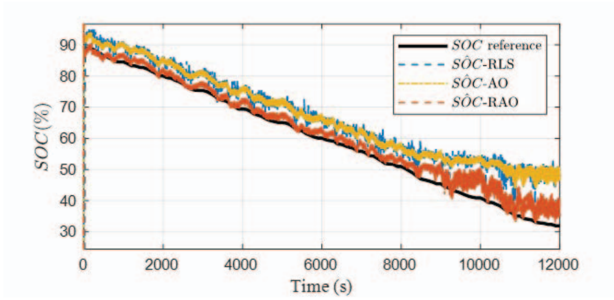

Fig. 2. SOC estimates for RLS-EKF, moving window adaptive observer and robust adaptive observer

B. Stability and convergence analysis

According to the theorems and derivation of the estimation error for the adaptive observer in [16] and [20], it is clear that if

1) Select L_r to make A_{or} stable.
2) $0 < \mu < 2nand\sigma \geq 0$.
3) There exist positive constants l_1, l_2, and integer Π such that for all j:

$$0 < l_1I \leq \sum_{i=k}^{k+\Pi-1} \bar{\varphi}_i^T\bar{\varphi}_i \leq l_2I < \infty \tag{64}$$

Then both the parameter and state estimation errors are bounded.

V. Simulation Results and Analysis

To test the proposed robust adaptive observer, comparison is carried out among the simulation results of the traditional RLS-EKF scheme, the moving window adaptive observer and the proposed algorithm for SOC estimation with measurement noises and sensor bias.

The simulation Lithium battery model is based on the electric circuit model. It can provide the parameters of the battery and the SOC reference value. A constant value is added to the battery measurements as sensor bias and a white noise signal is added to simulate the measurement noise. The RLS-EKF algorithm, moving window adaptive observer and the proposed robust adaptive observer are used for SOC estimation. The SOC estimation results and errors are shown in Fig. 2 and Fig. 3, respectively.

It can be seen that although the RLS-EKF algorithm and the moving window adaptive observer have the ability to cope with measurement noise, they cannot handle the problem of SOC estimation bias due to measurement bias. The proposed robust adaptive observer can deal with the measurement uncertainties including measurement bias and measurement noise, so the SOC estimation accuracy is effectively improved.

The SOC estimation results of the three algorithms can be compared with three metrics: mean absolute error, maximum absolute error, and standard deviation, as shown in TABLE I. Mean absolute error and maximum absolute error denote the mean and maximum absolute value of the SOC estimation error, respectively. The standard deviation can measure the

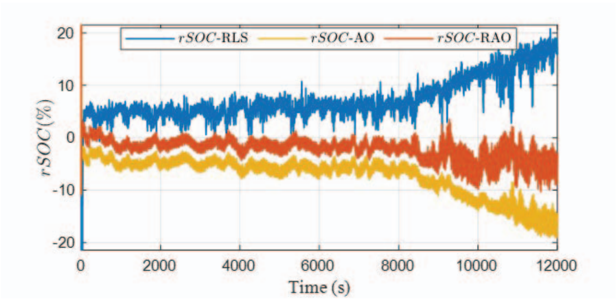

Fig. 3. SOC estimation errors for RLS-EKF, moving window adaptive observer, and robust adaptive observer

variation from the mean exists. The comparison shows that the SOC estimation performance of the robust adaptive observer is greatly improved under measurement uncertainties.

TABLE I
SOC ESTIMATION ERROR EVALUATION

Algorithms	SOC estimation error		
	Mean absolute error	Maximum absolute error	Standard deviation
RLS-EKF	7.3566	20.7610	3.7820
Moving window adaptive observer	7.3521	19.0118	3.7287
Robust adaptive observer	2.2641	10.3164	1.8055

VI. Conclusion

This paper derives the effects of sensor measurement bias and noise on least-squares-based battery parameter estimation and observer-based SOC estimation. The stability and convergence problems that may be caused by measurement uncertainty and its induced model errors are analyzed. In addition, a robust adaptive observer algorithm for SOC estimation of lithium-ion batteries with strict stability and convergence guarantees is proposed. The algorithm can realize accurate SOC estimation under the effect of measurement bias and noises. Simulation results show the performance of the algorithm.

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References

[1] H. Rahimi-Eichi, U. Ojha, F. Baronti, and M.-Y. Chow, "Battery management system: An overview of its application in the smart grid and electric vehicles," *IEEE Industrial Electronics Magazine*, vol. 7, no. 2, pp. 4–16, 2013.

[2] J. Zhang, Y. Jiang, X. Li, M. Huo, H. Luo, and S. Yin, "An adaptive remaining useful life prediction approach for single battery with unlabeled small sample data and parameter uncertainty," *Reliability Engineering System Safety*, vol. 222, p. 108357, 2022. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0951832022000369

[3] J. Zhang, Y. Jiang, X. Li, H. Luo, S. Yin, and O. Kaynak, "Remaining useful life prediction of lithium-ion battery with adaptive noise estimation and capacity regeneration detection," *IEEE/ASME Transactions on Mechatronics*, pp. 1–12, 2022.

[4] K. S. Ng, C.-S. Moo, Y.-P. Chen, and Y.-C. Hsieh, "Enhanced coulomb counting method for estimating state-of-charge and state-of-health of lithium-ion batteries," *Applied Energy*, vol. 86, no. 9, pp. 1506–1511, 2009. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0306261908003061

[5] C.-S. Huang and M.-Y. Chow, "Accurate thevenin's circuit-based battery model parameter identification," in *2016 IEEE 25th International Symposium on Industrial Electronics (ISIE)*, 2016, pp. 274–279.

[6] J. Li, J. Klee Barillas, C. Guenther, and M. A. Danzer, "A comparative study of state of charge estimation algorithms for lifepo4 batteries used in electric vehicles," *Journal of Power Sources*, vol. 230, pp. 244–250, 2013. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0378775312019039

[7] M. Charkhgard and M. H. Zarif, "Design of adaptive h filter for implementing on state-of-charge estimation based on battery state-of-charge-varying modelling," *Iet Power Electronics*, vol. 8, pp. 1825–1833, 2015.

[8] H. Rahimi-Eichi, F. Baronti, and M.-Y. Chow, "Online adaptive parameter identification and state-of-charge coestimation for lithium-polymer battery cells," *IEEE Transactions on Industrial Electronics*, vol. 61, no. 4, pp. 2053–2061, 2014.

[9] X. Guo, L. Kang, Y. Yao, Z. Huang, and W. Li, "Joint estimation of the electric vehicle power battery state of charge based on the least squares method and the kalman filter algorithm," *Energies*, vol. 9, p. 100, 02 2016.

[10] M. Li, Y. Zhang, Z. Hu, Y. Zhang, and J. Zhang, "A battery soc estimation method based on affrls-ekf," *Sensors*, vol. 21, p. 5698, 08 2021.

[11] S. Zhao, S. R. Duncan, and D. A. Howey, "Observability analysis and state estimation of lithium-ion batteries in the presence of sensor biases," *IEEE Transactions on Control Systems Technology*, vol. 25, no. 1, pp. 326–333, 2017.

[12] Z. Liu, Q. Ahmed, J. Zhang, G. Rizzoni, and H. He, "Structural analysis based sensors fault detection and isolation of cylindrical lithium-ion batteries in automotive applications," *Control Engineering Practice*, vol. 52, pp. 46–58, 2016. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0967066116300569

[13] X. Lin, "Analytic derivation of battery soc estimation error under sensor noises," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 2175–2180, 2017, 20th IFAC World Congress. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S2405896317305943

[14] S. Yuan, H. Wu, X. Ma, and C. Yin, "Stability analysis for li-ion battery model parameters and state of charge estimation by measurement uncertainty consideration," *Energies*, vol. 8, pp. 7729–7751, 07 2015.

[15] X. Lin, A. Stefanopoulou, P. Laskowsky, J. Freudenberg, Y. Li, and D. Anderson, "State of charge estimation error due to parameter mismatch in a generalized explicit lithium ion battery model," vol. 1, 01 2011.

[16] X. Xu, H. Luo, H. Wang, M.-Y. Chow, and C.-S. Huang, "Data-driven soc estimation with moving window adaptive residual generator for li-ion battery," in *2021 CAA Symposium on Fault Detection, Supervision, and Safety for Technical Processes (SAFEPROCESS)*, 2021, pp. 1–6.

[17] C.-S. Huang, T. W.-S. Chow, and M.-Y. Chow, "Li-ion battery parameter identification with low pass filter for measurement noise rejection," *2017 IEEE 26th International Symposium on Industrial Electronics (ISIE)*, pp. 2075–2080, 2017.

[18] X. Lin, "Analytic derivation of battery soc estimation error under sensor noises," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 2175–2180, 2017.

[19] H. Rahimi-Eichi, F. Baronti, and M.-Y. Chow, "Modeling and online parameter identification of li-polymer battery cells for soc estimation," *2012 IEEE International Symposium on Industrial Electronics*, pp. 1336–1341, 2012.

[20] Q. Zhang, "Adaptive observer for multiple-input-multiple-output (mimo) linear time-varying systems," *IEEE Transactions on Automatic Control*, vol. 47, no. 3, pp. 525–529, 2002.