

# State of Charge Estimation of Lithium-Ion Battery Based on Second-order Extended Kalman Filter

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**Abstract**—This paper presents an approach to state of charge (SOC) estimation of lithium-ion battery based on second-order extended Kalman filter (EKF). Aiming at the problem of inaccurate estimation of the SOC during the operation of lithium-ion battery, the second-order EKF approach is chosen to reduce estimation error. Firstly, the internal characteristics of lithium-ion battery are fully considered, and the equivalent circuit model is established. The parameters are identified according to battery charging and discharging. Then the state space model of the battery is built. Finally, the simulation is carried out under the discharging condition of lithium-ion battery. The simulation results demonstrate that the second-order EKF method can improve estimation effect compared with the first-order EKF method.

**Keywords**—lithium-ion battery; equivalent circuit model; SOC estimation; extended Kalman filter

## I. INTRODUCTION

Lithium-ion batteries have become the first choice for electric vehicle manufacturers at home and abroad due to their high single-cell voltage, high energy density and long cycle life. The battery management system (BMS) can monitor battery status, prevent overcharging and overdischarging of the battery and extend battery life. The SOC<sup>[1]</sup> of the lithium-ion battery can indicate the remaining capacity of the battery pack, which plays an important role in the BMS.

In recent years, existing works mainly include three major research methods for battery SOC. The first method is based on the electrochemical properties of the battery including ampere-hour (AH) integration method and open circuit voltage (OCV) method (see, e.g., [2], [3]). The two methods are combined to estimate SOC of the battery. This kind of method is simple and easy to be implemented, but it does not have real-time correction ability that may lead to the error accumulation of SOC estimation under the variable working condition. The second category is mainly based on the emerging intelligent prediction algorithm such as the artificial neural network (see, e.g., [4], [5]). However, the neural network estimation method is greatly influenced by the sample data size and the training algorithm rules. In addition, the calculation amount is large, which will increase the online cost. The third type of method is the Kalman filter algorithm based on the battery model (see, e.g., [6], [7], [8]). The SOC of the battery is estimated based on the electrochemical model of the battery in combination with the extended Kalman filter. The equivalent circuit model is built after full consideration of the characteristics of lithium-

ion battery. Compared with other SOC estimation methods, the first-order EKF algorithm not only has online estimation capability but also is suitable for various types of lithium-ion batteries. However, since the first-order EKF algorithm ignores higher-order terms above the second order in the estimation process, the accuracy is not high and stable. Therefore, based on the first-order EKF, the second-order EKF algorithm is proposed to realize the SOC estimation of the battery.

## II. BATTERY MODELING

EKF is a model-driven approach that requires the establishment of a battery equivalent circuit model. The equivalent voltage source model and the equivalent impedance model are established based on the hysteresis characteristics and rebound characteristics of the battery. In addition, in the process of modeling, the relationship between the remaining battery SOC of the battery and the battery itself is also taken into consideration, which is in favor of simulation analysis of the circuit. Therefore, the equivalent circuit model<sup>[9]</sup> established in this paper is mainly composed of SOC calculation model, equivalent voltage source model and equivalent impedance model. The equivalent circuit model is shown in Fig. 1.

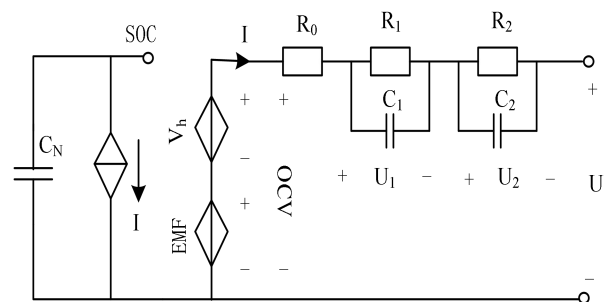


Fig.1. Equivalent circuit model

$C_N$  represents the rated capacity of the battery, current  $I$  is the charging and discharging current flowing through the battery, SOC represents the remaining capacity of the battery and the value of SOC is between 0 and 1. EMF is the equilibrium potential of the battery (refers to the open circuit voltage when the battery is stopped charging and discharging for 30 minutes). It is controlled by the battery SOC and is a function of the SOC.  $V_h$  is the hysteresis voltage of the battery and also a function of SOC.  $R_0$  is an ohmic resistor, and  $R_1$ ,  $R_2$

and  $C_1$ ,  $C_2$  are polarization resistances and polarization capacitances.

The open circuit voltage corresponding to the battery at different SOC points can be obtained by the ampere-hour integration method and the open circuit voltage method. Since the charging balance potential of the battery is higher than the discharging balance potential at the same SOC point, the balance potential is identified according to the average value of charging and discharging value. The hysteresis voltage is identified according to the mean value of the charging-discharging difference. As shown in (1)

$$\begin{cases} EMF = \frac{(EMF_c + EMF_d)}{2} \\ V_h = \frac{(EMF_c - EMF_d)}{2} \end{cases} \quad (1)$$

where  $EMF_c$  represents the value in charging condition and  $EMF_d$  represents the value in discharging condition. The equivalent voltage source for the battery is as (2):

$$U_{OCV}(SOC) = EMF(SOC) + V_h(SOC) \quad (2)$$

The battery was pulse-charged and discharged at each SOC point, and the HPPC experimental data in one cycle was selected for description when SOC value is 90%. The HPPC voltage curve is shown in Fig. 2.

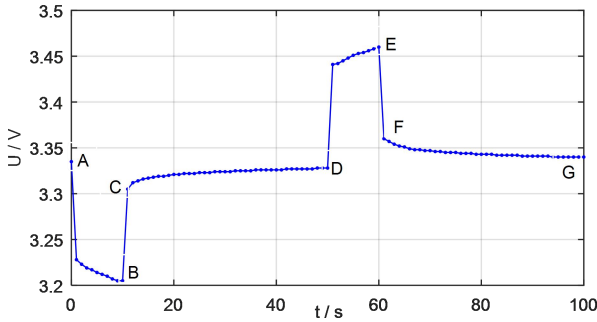


Fig. 2. HPPC voltage curve

As shown in Fig. 2, the ohmic resistance inside the battery is the main cause of the sudden change of the voltage, such as the sudden voltage change of B~C and the change of E~F. Therefore, the charging and discharging ohmic resistances can be shown as (3).

$$\begin{cases} R_c = \frac{U_E - U_F}{I} \\ R_d = \frac{U_C - U_B}{I} \end{cases} \quad (3)$$

The slow rise of the voltage during the pulse discharging is mainly affected by the polarization resistance and polarization capacitance existing inside the battery. The process is reflected in the C~D phase in Fig. 2. In this process, the circuit has zero input response, and the operating voltage of the battery at any time is (4):

$$U = U_{OCV(D)} - IR_1 e^{-\tau_1 t} - IR_2 e^{-\tau_2 t} \quad (4)$$

where  $\tau_1$  and  $\tau_2$  are time constant. The parameters of the model can be obtained by voltage curve as (5). Combining (4) and (5), we can get the value of resistances and capacitances as (6).

$$f(t) = A - B * \exp(-at) - C * \exp(-bt) \quad (5)$$

$$\begin{cases} R_1 = \frac{B}{I} \\ R_2 = \frac{C}{I} \\ C_1 = \frac{I}{aB} \\ C_2 = \frac{I}{bB} \end{cases} \quad (6)$$

### III. SOC ESTIMATION USING SECOND-ORDER EKF ALGORITHM

#### A. Battery state space model

The relationship between the SOC value, current value and battery capacity is shown as (7)

$$SOC_{k+1} = SOC_k - \frac{I\Delta t}{C_N} \quad (7)$$

where  $C_N$  is the battery capacity,  $I$  is the current, and  $\Delta t$  is the sample time. According to Kirchhoff's law, the equivalent circuit model can be written as follows

$$\dot{U}_1 = -\frac{U_1}{R_1 C_1} + \frac{I}{C_1} \quad (8)$$

$$\dot{U}_2 = -\frac{U_2}{R_2 C_2} + \frac{I}{C_2} \quad (9)$$

Taking the SOC of the battery, the voltages  $U_1$  and  $U_2$  as the state variables of the system, the current  $I$  is the input, and the terminal voltage  $U$  is the output. The state space equation of the battery can be described as (10)

$$\begin{bmatrix} SOC(k+1) \\ U_1(k+1) \\ U_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-\Delta t / R_1 C_1} & 0 \\ 0 & 0 & e^{-\Delta t / R_2 C_2} \end{bmatrix} \begin{bmatrix} SOC(k) \\ U_1(k) \\ U_2(k) \end{bmatrix} + \begin{bmatrix} -\Delta t / C_N \\ R_1(1 - e^{-\Delta t / R_1 C_1}) \\ R_2(1 - e^{-\Delta t / R_2 C_2}) \end{bmatrix} I(k) \quad (10)$$

the output equation is as (11).

$$U(k) = U_{OCV}(SOC(k)) - U_1(k) - U_2(k) - R_0 I(k) \quad (11)$$

#### B. Second-order EKF algorithm

The standard linear Kalman filter algorithm is the basis of the extended Kalman filter algorithm. The standard KF algorithm is mainly composed of the state equation and the output equation of the estimated system. Among them, the state equation is used to describe the relationship between the state

quantity and the input quantity of the system, and can reflect the mathematical relationship between the adjacent time. The output equation is used to describe the relationship between the estimated state quantity, input quantity and observation quantity. The main idea of the algorithm is utilizing actual state observation data to update and correct the state variables of the system under the principle of minimum mean square error in the condition of known system state space model, noise statistical characteristics and state initial value. Combined with the SOC estimation of lithium-ion battery, it is known that the nonlinear filtering method is needed. The first-order EKF and second-order EKF methods are used under the Kalman filter framework. The core idea of extended Kalman filter is adopting Taylor expansion at the estimated point to convert nonlinear problems into linear problems and the high-order terms is neglected. However, ignoring high-order terms may introduce large errors or even cause the filter to diverge. The second-order EKF preserves the second-order term on the basis of the first-order, thereby reducing the error caused by the estimation process. The state space equation for the equivalent circuit model of a lithium-ion battery can be expressed as (12):

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + w_k \\ z_k = h(x_k, u_k) + v_k \end{cases} \quad (12)$$

where  $x_k$  represents the state variable at time  $k$ ,  $u_k$  represents the control input of the system, and  $z_k$  represents the output at time  $k$ .  $A$  is state transition matrix and  $B$  is control matrix.  $h$  is nonlinear function.  $w_k$  and  $v_k$  are the process noise and measurement noise with mean zero and are uncorrelated.  $w_k \sim N(0, Q_k)$ ,  $v_k \sim N(0, R_k)$ , where  $Q_k$  is the covariance matrix of the process noise,  $R_k$  is the covariance matrix of the observation noise.

The recursive flow of the first-order EKF algorithm is:

### 1. Initialization

$$\hat{x}_0 = E[x_0], P_0 = E\{(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T\} \quad (13)$$

### 2. Prediction

$$\hat{x}_{k|k-1} = A\hat{x}_{k-1} + Bu_{k-1} \quad (14)$$

error covariance matrix for prediction estimation

$$P_{k|k-1} = AP_{k-1}A^T + Q_{k-1} \quad (15)$$

### 3. Correction

Kalman filter gain

$$K_k = P_{k|k-1}(h_k^x)^T (h_k^x P_{k|k-1}(h_k^x)^T + R_k)^{-1} \quad (16)$$

corrected estimation of state variables

$$\hat{x}_k = \hat{x}_{k|k-1} + K_k[z_k - h(\hat{x}_{k|k-1}, u_k)] \quad (17)$$

the error covariance matrix of the state variables

$$P_k = (I - K_k h_k^x)P_{k|k-1} \quad (18)$$

where  $h_k^x$  is first-order derivative.

$$h_k^x = \left. \frac{\partial h_k(x_k, u_k)}{\partial x_k} \right|_{x_k = \hat{x}_{k|k-1}} \quad (19)$$

It can be seen from the established state space equation that the system state equation is linear and the observation equation is nonlinear. When extended Kalman filter is used, only the observation equation needs to be linearized. Matrix  $A$  and  $B$  are as follows

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-\Delta t / R_1 C_1} & 0 \\ 0 & 0 & e^{-\Delta t / R_2 C_2} \end{bmatrix}, \quad B = \begin{bmatrix} -\Delta t / C_N \\ R_1(1 - e^{-\Delta t / R_1 C_1}) \\ R_2(1 - e^{-\Delta t / R_2 C_2}) \end{bmatrix}$$

The recursive flow of the second-order EKF algorithm is:

### 1. Initialization

$$\hat{x}_0 = E[x_0], P_0 = E\{(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T\} \quad (20)$$

### 2. Prediction

$$\hat{x}_{k|k-1} = A\hat{x}_{k-1} + Bu_{k-1} \quad (21)$$

error covariance matrix for prediction estimation

$$P_{k|k-1} = AP_{k-1}A^T + Q_{k-1} \quad (22)$$

### 3. Correction

Kalman filter gain

$$K_k = P_{k|k-1}(h_k^x)^T (h_k^x P_{k|k-1}(h_k^x)^T + R_k)^{-1} \quad (23)$$

corrected estimation of state variables

$$\hat{x}_k = \hat{x}_{k|k-1} + K_k[z_k - h(\hat{x}_{k|k-1}, u_k) - \varphi] \quad (24)$$

the error covariance matrix of the state variables

$$P_k = (I - K_k h_k^x)P_{k|k-1} \quad (25)$$

The second-order EKF increases the correction term, and the increase of the second-order correction term reduces the error caused by the Taylor expansion and improves the estimation accuracy.

$$\varphi_k = \frac{1}{2} \sum_{j=1}^m e_j \text{tr}(h_{j,k}^{xx} P_{k|k-1}) \quad (26)$$

$$h_{j,k}^{xx} = \left. \frac{\partial}{\partial x_k} \left[ \frac{\partial h_{j,k}(x_k, u_k)}{\partial x_k} \right]^T \right|_{x_k = \hat{x}_{k|k-1}} \quad (27)$$

where state transition  $h_{j,k}^{xx}$  is the second-order derivative of the function,  $e_j$  is the standard base vector, and  $\text{tr}$  is the trace of the matrix.

#### IV. SIMULATION

In this paper, the 18650 lithium-ion battery produced by Tianjin Lishen was selected as the experimental object. The main performance parameters of the battery are shown in Table I.

TABLE I. MAIN PARAMETERS

Nominal voltage(V)	3.2	Maximum charging voltage(V)	3.65
Nominal capacity(AH)	1.35	Minimum discharging voltage(V)	2.0
Battery Life	≥1000	Maximum charging and discharging current(C)	2

The DC power is the IT6942A power produced by Itech. The discharging equipment is the IT8512C+ series programmable electronic load produced by Itech. The battery was subjected to a constant current discharging experiment using a 1C discharging rate. Fig. 3 is a graph that shows the SOC estimation of constant current discharging.

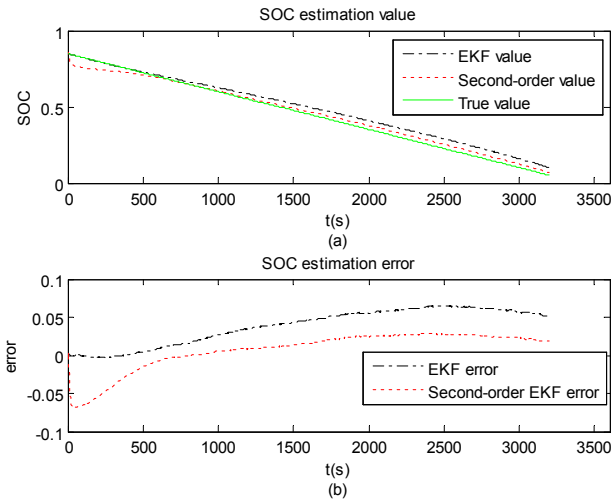


Fig. 3. SOC estimation result. (a) SOC estimation value, (b) Estimation error.

It can be seen from the Fig. 3 that the second-order EKF estimation error is larger than that of the first-order EKF in the initial stage of SOC estimation. After a period of time, it can be seen that the second-order EKF can achieve more accurate effect. The error of the two methods and the time used are shown in TABLE II.

TABLE II. ERROR AND RUNNING TIME

	average error	time(s)
EKF	3.87%	0.039
Second-order EKF	2.17%	0.079

It can be seen from Table I. that the average SOC estimation error of the first-order extended Kalman filter is 3.87%, and the running time is 0.039s. Although the second-order EKF takes more time, it reduces the error to 2.17% compared with the first-order EKF.

#### V. CONCLUSION

In this paper, a equivalent circuit model with hysteresis voltage is established and the parameters are identified by analyzing the characteristics of lithium-ion battery. Based on the equivalent model, the second-order EKF algorithm is used to estimate the SOC of the battery. The simulation accuracy is verified by simulating the discharging condition of the battery. The simulation results show that the second-order EKF algorithm can obtain a good estimation effect in lithium-ion battery SOC estimation.

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