

# State of Charge Estimation of Lithium-Ion Battery Using State Augmented Cubature Kalman Filter in Presence of Uncharacterized Coloured Noise

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**Abstract**—This paper addresses the State of Charge (SOC) estimation problem for Li-ion battery with the measurement perturbed with uncharacterised colour noise. Accurate estimation of the state of charge is a crucial task for the Battery Management System (BMS) to monitor the health of the battery. Often, SOC is estimated by Bayesian estimators using noisy measurements with the assumption of Gaussian white noise. For this assumption, the estimation results would degrade substantially in the presence of colour noise in the measurement and the situation worsens when the auto correlation coefficient of the noise remains unknown. This paper presents a solution method based on Cubature Kalman filter (CKF) with the noise augmented state vector which demonstrates superior estimation performance over the standard CKF. The augmented CKF is also validated in simulation using the current signals from realistic cell tests consisting of urban dynamometer driving schedule (UDDS) cycles. The results demonstrate the suitability of the proposed state augmented CKF where the measurement is perturbed with uncharacterised colour noise.

**Index Terms**—Battery model, cubature Kalman Filter, extended Kalman Filter (EKF), lithium-ion battery, open circuit voltage (OCV), state augmented cubature kalman filter (SA-CKF), State of charge.

## I. INTRODUCTION

In modern transportation technology electrical vehicles (EVs) and hybrid vehicles are preferred over standard internal combustion (IC) engine vehicle towards transportation of goods and human beings essentially for reduced emission of greenhouse gases along with the advancement of battery technology. Usage of Li-ion battery is becoming more and more prevalent because of their key features such as long life, high energy density, low self-discharge rate, less weight and safety with in permissible limits of charging or discharging [1]. Battery Management System (BMS) will distribute and make sure the utilization of power among different components of EVs in an efficient way. BMS will take the control over managing batteries charging, discharging and balancing battery cells, ensures the safety and also extend the battery life. The performance of the EV is dependent on the battery characteristics, which depends on many factors like operating temperature, discharge and charge cycles, rate of charge and discharge, and aging of the battery [2]. So, accurate algorithms

plays a significant role to monitor the battery conditions i.e. state of charge (SOC) and state of health (SOH), etc., so that, the performance of battery remains satisfactory.

Amount of charge available in the battery, i.e., state of charge is a significant parameter which must be known to BMS for the battery management and to predict how long a vehicle can travel further. But SOC of the battery cannot be measured directly because it is associated with many parameters like temperature, discharge/charge cycles as mentioned before. So, SOC has to be estimated with an accurate estimation algorithm so that over charging or over discharging of batteries can be controlled with an accurately estimated SOC. Otherwise, over charging or discharging has detrimental effect on Li ion batteries. It is desired that, to maintain the integrity of battery SOC must be maintained within limits ( $20\% \leq S(t) \leq 95\%$ ) as mentioned in [1].

Fast developing artificial intelligence-based, machine learning, and support vector machine, multivariate adaptive regression splines can estimate SOC accurately without prior knowledge of electrochemistry of battery, but involves complex training to meet all the conditions that a battery can have over lifespan. Hence, trained methods are not suggested to estimate the online SOC because actual conditions of battery are unpredictable or uncertain [3].

Model based estimation methods have become more popular because they are quite easy to implement and to estimate online SOC without knowing initial conditions accurately. Here, measured current will be an input signal to the model and compute the output with the help of present and/or past states [2]. Deviation of the calculated value from measured value will be an input to the filter and states will be corrected to new values. Some of such model based methods of SOC estimation includes the estimations such as Luenberger observer, Kalman filter (KF), Extended Kalman Filter (EKF), Cubature Kalman filter (CKF), and Unscented Kalman filter (UKF) which are from the family of Kalman filter and its variants. Lot of research is going on battery SOC estimation using UKF [4], [5] and CKF [3] with the help of sigma and cubature points. But none of them has incorporated the colour noise in the measurements. This review motivated the present work to

investigate further in this particular domain. In this paper, the battery model parameters developed from the experimental data and are taken from [2]. Two basic modelling parts of the battery are considered, (1) nonlinear relation between open circuit voltage (OCV) vs SOC and (2) second order capacitance-resistance (RC) circuit, which reflects the battery dynamics. In most of the papers reviewed above the measurement noise is assumed to be Gaussian white noise. Therefore, the filtering algorithms can be used as they are proposed in seminal papers. However, in the present work it is assumed that the measurement noise is a coloured noise instead of white noise. In literature very few works [6], [7] exist where the authors have addressed this problem. The authors of [7] employed well known innovation based Adaptive EKF in presence of coloured noise. However, authors do not modify the algorithm of AEKF for coloured noise although the filter demonstrates improved estimation result over its non-adaptive version in presence of coloured noise. In [6] the authors propose augmented matrix EKF where the measurement model is augmented with the state vector so that the coloured noise sequence can be estimated as a state. In this case the authors assume that the auto correlation coefficient of the coloured noise is accurately known to the designer. The novelty of the present paper lies in addressing the problem in this work where the coloured noise that perturbs the measurement is considered to be uncharacterised. In other words, we can say that auto correlation coefficient of the noise remains unknown to the designer. The authors propose a solution method based on state augmented filter in this paper which, although apparently similar to [6], is substantially different from this work. The present authors use CKF as an underlying framework instead of EKF due its improved estimation accuracy and better convergence. The state augmentation was proposed where the coloured noise and its unknown auto correlation coefficient are modelled as states. This increases the order of the system by twice the size of the measurement vector. The proposed filter can successfully estimate the unknown auto correlation coefficient which enables the designer to avoid large number of hit and trial cases with the assumed value of unknown auto correlation coefficient. Due to state augmentation the effective system dynamics becomes significantly nonlinear. Therefore, the scope of employing improved nonlinear filters remains open for this problem. For the present study the authors uses CKF as an underlying framework. The rest of the paper is coordinated as follows: Section II depicts the battery model with corresponding state space equations and about coloured noise. In section III, solution method for SOC estimation is explained. Finally, Simulation results and the conclusion is given in section IV and V respectively.

## II. PROBLEM FORMULATION

### A. Battery Model

As Li-ion battery is a electrochemical and high non linear system, it is difficult to obtain battery model [8] Researchers have developed battery models such as Rint Model , 1<sup>st</sup> order RC model [9], [10], 2<sup>nd</sup> order RC model [11]. Estimation

will be accurate if the model can reflect the physical battery characteristics. Battery model computation will be complex if order of the RC model increase. Considering model complication and accuracy of the model second order RC model is considered in this paper. The equivalent circuit model of 2<sup>nd</sup> order RC model depicted in Fig.1 consists of three parts. First one, the battery internal open circuit voltage  $E_o$ , which is function of state of charge of the battery and is denoted by  $S$ . Second, the internal resistances, and capacitances. Internal resistance include ohmic resistance of the battery ( $R_i$ ), electrochemical polarization resistance ( $R_{pe}$ ), and concentration

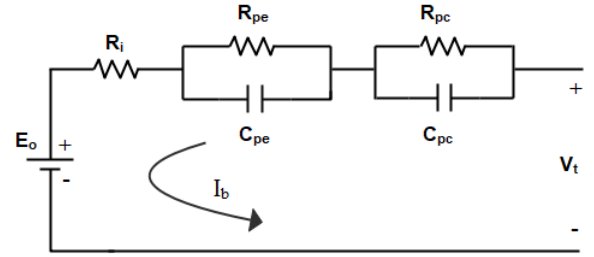


Fig. 1. Battery model.

polarization resistance ( $R_{pc}$ ). Whereas, capacitance include, electrochemical polarization capacitance ( $C_{pe}$ ), and concentration polarization capacitance ( $C_{pc}$ ), will describe the transient responses during charging/discharging transits. Battery current is denoted by  $I_b$ .  $V_t$  is the battery terminal voltage. The electrical behavior of the battery can be represented in terms of differential equations by writing kirchhoff's law and can be expressed as follows:

$$\begin{cases} \dot{V}_{pe} = \frac{-V_{pe}}{R_{pe} C_{pe}} + \frac{I_b}{C_{pe}} \\ \dot{V}_{pc} = \frac{-V_{pc}}{R_{pc} C_{pc}} + \frac{I_b}{C_{pc}} \\ V_t = E_o + V_{pe} + V_{pc} + I_b R_i \end{cases} \quad (1)$$

where,  $V_{pe}$  and  $V_{pc}$  are the voltages on  $C_{pe}$  and  $C_{pc}$  respectively. Battery SOC can be obtained with the help Coulomb counting method by integrating the measured battery terminal current, which is given below.

$$S = S_0 - \eta \int_0^t I_b(t) dt \quad (2)$$

where,  $\eta = 1/(3600C_b)$ ,  $S_0$  is the initial SOC, and  $C_b$  is the capacity of the battery in ampere-hours. As the measurement includes errors and measurement noise, Coulomb counting ( $CC$ ) method method is not suggested for SOC estimation. To apply  $CC$  method initial SOC should be known beforehand, which may not be known in practice and is the drawback of this method.

For further analysis voltage drop on capacitors along with SOC are chosen as the state variables. The state space equation can be represented as

$$\dot{x} = \begin{bmatrix} -\frac{1}{R_{pe} C_{pe}} & 0 & 0 \\ 0 & -\frac{1}{R_{pc} C_{pc}} & 0 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} \frac{1}{C_{pe}} \\ \frac{1}{C_{pc}} \\ -\eta \end{bmatrix} I_b \quad (3)$$

where,  $\chi = [V_{pe} \ V_{pc} \ S]^T$ , and  $S$  is the battery SOC. We can represent (1) and (2) in standard state space form as

$$\begin{aligned} \dot{x} &= A x + B u \\ y &= C x + D u \end{aligned} \quad (4)$$

where

$$A = \begin{bmatrix} -\frac{1}{R_{pe}C_{pe}} & 0 & 0 \\ 0 & -\frac{1}{R_{pc}C_{pc}} & 0 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{C_{pe}} \\ \frac{1}{C_{pc}} \\ -\eta \end{bmatrix}, \quad C = [1 \ 1 \ \frac{\partial E_0}{\partial S}], \text{ and } D = R_i \quad (5)$$

after discretizing (3), we get

$$\begin{aligned} \dot{x}_k &= \begin{bmatrix} -\frac{\tau}{R_{pe}C_{pe}} + 1 & 0 & 0 \\ 0 & -\frac{\tau}{R_{pc}C_{pc}} + 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot x_{k-1} \\ &+ \begin{bmatrix} \frac{\tau}{C_{pe}} \\ \frac{\tau}{C_{pc}} \\ -\eta \cdot \tau \end{bmatrix} I_b \end{aligned} \quad (6)$$

where,  $x_k$  and  $x_{k-1}$  denotes the state vector at time step  $k$  and  $k-1$  respectively, and  $\tau$  is the time interval. from (6), it is clear that the state variables at time step  $k$  will be calculated based on the earlier time step  $k-1$ , and is not dependent on the history of the states. Terminal voltage ( $V_t$ ) is calculated only when the states are available.  $V_t$  is the function of  $E_0$ , which is the function of SOC.

### B. OCV and SOC Relation

In order to estimate the SOC, first parameters of the model mentioned in (1) are to be estimated and must be validated experimentally. Battery SOC has strong nonlinear relationship with OCV. The relation has to be find out by performing charge and discharge tests on the battery. For charge test, battery has to be charged for 5 to 10 minutes at certain rate and let the terminals remain open for the rest period of 1 to 2 hours, so that, battery will come to an equilibrium condition then OCV can be measured at the terminals of the battery. This process should be repeated from 0% SOC to 100% SOC with an increment of 5% SOC. For discharge test, similarly discharge battery from 100% to 0% at certain rate. Relationship between OCV and SOC is taken from [2] and the corresponding seventh-order polynomial equation of OCV as a function of SOC is shown in (7)

$$E_0 = a_1 S^7 + a_2 S^6 + a_3 S^5 + a_4 S^4 + a_5 S^3 + a_6 S^2 + a_7 S + a_8 \quad (7)$$

where,  $S$  is SOC,  $a_1$  to  $a_8$  are the coefficients.  $a_1=8.4073$ ,  $a_2=-19.892$ ,  $a_3=11.497$ ,  $a_4=4.161$ ,  $a_5=-4.5533$ ,  $a_6=0.34365$ ,  $a_7=0.64685$ , and  $a_8=3.5016$ .

### C. SOC Estimation problem with colour Noise

Battery SOC is direct integral of current, and is not sensitive to the white noise. And white noise can not meet the complexity of working conditions of EVs. So, in order to estimate SOC accurately, coloured noise in the terminal voltage measurement should be taken. State augmented CKF (SA-CKF) is proposed in this paper to estimate SOC accurately even in presence of colour noise. Dimension of state vector increased with the inclusion of colour noise as state. New state space equations can be represented below as per [6]

$$\begin{aligned} \chi_{k+1} &= \Phi_k \chi_k + \mathcal{W}_k \\ z_k &= H_k \chi_k + \Pi_k \mathcal{V}_k \end{aligned} \quad (8)$$

where,  $\mathcal{V}_k$  and  $\mathcal{W}_k$  are the coloured noise and white noise respectively. And  $\Pi$  is colour noise coefficient. Colour noise can be represented as:

$$\mathcal{V}_{k+1} = \Psi_{k,k} \mathcal{V}_k + \Theta_k \quad (9)$$

where,  $\Theta$  is a zero mean white noise and  $\Psi$  is the auto correlation coefficient. The augmented state vector with 4<sup>th</sup> order is  $\chi = [V_{pe} \ V_{pc} \ S \ \mathcal{V}]^T$  and corresponding  $A$ ,  $B$ , and  $C$  are

$$A = \begin{bmatrix} -\frac{1}{R_{pe}C_{pe}} & 0 & 0 & 0 \\ 0 & -\frac{1}{R_{pc}C_{pc}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Psi \end{bmatrix}, B = \begin{bmatrix} \frac{1}{C_{pe}} \\ \frac{1}{C_{pc}} \\ -\eta \\ 0 \end{bmatrix}, \quad \text{and } C = [1 \ 1 \ \frac{\partial E_0}{\partial S} \ 1]$$

In this paper, we have considered unknown colour noise coefficient (CNC) and is estimated by considering CNC as the state. So that the 5<sup>th</sup> order state vector is  $\chi = [V_{pe} \ V_{pc} \ S \ \mathcal{V} \ \Psi]^T$ , and  $A$ ,  $B$ , and  $C$  are

$$A = \begin{bmatrix} -\frac{1}{R_{pe}C_{pe}} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_{pc}C_{pc}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Psi & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{C_{pe}} \\ \frac{1}{C_{pc}} \\ -\eta \\ 0 \\ 0 \end{bmatrix}, \quad \text{and } C = [1 \ 1 \ \frac{\partial E_0}{\partial S} \ 1 \ 0]$$

### III. CUBATURE KALMAN FILTER

CKF basically uses multivariate moment integrals in the nonlinear Bayesian filter with the help of spherical radial cubature rule [12]. Number of cubature points in linearly proportional to the dimension ( $\mathbf{n}$ ) of the state vector.

A discrete-time nonlinear dynamical system with process and measurement noise is as shown in (10) as

$$\begin{aligned} \chi_{k+1} &= \mathbf{f}(\chi_k, \mathbf{u}_k) + \mathbf{w}_k \\ \mathbf{z}_{k+1} &= \mathbf{g}(\chi_k, \mathbf{u}_k) + \mathbf{v}_k \end{aligned} \quad (10)$$

where  $\chi_k$  is the state vector at time step  $k$ ;  $\mathbf{z}_k$  is the measurement vector at time step  $k$ ;  $\mathbf{f}(\cdot)$  and  $\mathbf{g}(\cdot)$  are nonlinear functions of system dynamics and measurement;  $\mathbf{w}_k$  and  $\mathbf{v}_k$  are independent Gaussian white process and measurement noise with covariance  $\mathbf{Q}_k$  and  $\mathbf{R}_k$ , respectively. Let  $\hat{\chi}_{k-1|k-1}$  and

$\mathbf{P}_{k-1|k-1}$  are the initial values of the state vector and state covariance matrix respectively.  $\xi$  consists cubature points ( $\rho = 2n$ ), then CKF algorithm for battery SOC estimation is summarized in as follows.

#### Time Update

- 1) At time step  $k$  that the posterior density function  $p(x_{k-1}|D_{k-1}) = \mathcal{N}(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{P}_{k-1|k-1})$  is known. Obtain  $\bar{\mathbf{S}}_{k-1|k-1}$  such that

$$\mathbf{P}_{k-1|k-1} = \bar{\mathbf{S}}_{k-1|k-1} \bar{\mathbf{S}}_{k-1|k-1}^T. \quad (11)$$

- 2) Compute the cubature points ( $j = 1, 2, \dots, \rho$ )

$$\zeta_{j,k-1|k-1} = \bar{\mathbf{S}}_{k-1|k-1} \xi_j + \hat{\mathbf{x}}_{k-1|k-1}. \quad (12)$$

- 3) After propagating through  $\mathbf{f}(\cdot)$ , cubature points becomes ( $j = 1, 2, \dots, \rho$ )

$$\tilde{\zeta}_{j,k|k-1} = \mathbf{f}(\zeta_{j,k-1|k-1}, \mathbf{u}_{k-1}). \quad (13)$$

- 4) Predicted estimate of the state is obtained as

$$\check{\mathbf{x}}_{k|k-1} = \frac{1}{\rho} \sum_{j=1}^{\rho} \tilde{\zeta}_{j,k|k-1}. \quad (14)$$

- 5) Corresponding predicted error covariance can be obtained as

$$\begin{aligned} \mathbf{P}_{k|k-1} &= \frac{1}{\rho} \sum_{j=1}^{\rho} \tilde{\zeta}_{j,k|k-1} \tilde{\zeta}_{j,k|k-1}^T \\ &\quad - \check{\mathbf{x}}_{k|k-1} \check{\mathbf{x}}_{k|k-1}^T + \mathbf{Q}_{k-1}. \end{aligned} \quad (15)$$

#### Measurement Update

- 1) Obtain  $\check{\mathbf{S}}$  such that

$$\mathbf{P}_{k|k-1} = \check{\mathbf{S}}_{k|k-1} \check{\mathbf{S}}_{k|k-1}^T. \quad (16)$$

- 2) Compute cubature points ( $j = 1, 2, \dots, \rho$ )

$$\zeta_{j,k|k-1} = \check{\mathbf{S}}_{k|k-1} \xi_j + \check{\mathbf{x}}_{k|k-1}. \quad (17)$$

- 3) After passing through  $\mathbf{g}(\cdot)$ , cubature points becomes ( $j = 1, 2, \dots, \rho$ )

$$\dot{\zeta}_{j,k|k-1} = \mathbf{g}(\zeta_{j,k|k-1}, \mathbf{u}_k). \quad (18)$$

- 4) Predicted estimate of the measurement is obtained as

$$\check{\mathbf{z}}_{k|k-1} = \frac{1}{\rho} \sum_{j=1}^{\rho} \dot{\zeta}_{j,k|k-1}. \quad (19)$$

- 5) Corresponding the innovation covariance matrix is obtained as

$$\begin{aligned} \tilde{\mathbf{P}}_{zz,k|k-1} &= \frac{1}{\rho} \sum_{j=1}^{\rho} \dot{\zeta}_{j,k|k-1} \dot{\zeta}_{j,k|k-1}^T \\ &\quad - \check{\mathbf{z}}_{k|k-1} \check{\mathbf{z}}_{k|k-1}^T + \mathbf{R}_k. \end{aligned} \quad (20)$$

- 6) The cross covariance matrix is obtained as

$$\begin{aligned} \bar{\mathbf{P}}_{xz,k|k-1} &= \frac{1}{\rho} \sum_{j=1}^{\rho} \zeta_{j,k|k-1} \dot{\zeta}_{j,k|k-1}^T \\ &\quad - \check{\mathbf{x}}_{k|k-1} \check{\mathbf{z}}_{k|k-1}^T \end{aligned} \quad (21)$$

- 7) The kalman gain is obtained as

$$\mathbf{K}_{G,k} = \bar{\mathbf{P}}_{xz,k|k-1} \tilde{\mathbf{P}}_{zz,k|k-1}^{-1}. \quad (22)$$

- 8) Updated state estimate becomes

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_{G,k} (\mathbf{z}_k - \check{\mathbf{z}}_{k|k-1}) \quad (23)$$

- 9) The updated error covariance becomes

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}_{G,k} (\tilde{\mathbf{P}}_{zz,k|k-1}) \mathbf{K}_{G,k}^T \quad (24)$$

#### IV. SIMULATION RESULTS

In this section significant results obtained from the simulation studies have been presented. Fig. 5 demonstrates the superiority of CKF over EKF in SOC estimation in presence of white noise. This study has been carried out to demonstrate the suitability of CKF algorithm for SOC estimation and its superiority over the EKF algorithm. Therefore, SA-CKF algorithm instead of SA-EKF has been proposed in this paper for uncharacterized colour noise. The simulation study is carried out with the colour noise in the measurement where the auto-correlation coefficient is unknown to the designer. For estimation of SOC different versions of CKF algorithms, viz., SA-CKF with 4<sup>th</sup> and 5<sup>th</sup> order and non-augmented CKF (3<sup>rd</sup> order) have been employed. To generate the true state trajectories battery is considered initially charged to 100%. For all the estimators initial SOC is chosen as 0.5 and the parameters of the model shown in Fig. 1 are taken from [2] are given in table II. The significant parameters for the filter initialization are given as:  $\hat{\mathbf{x}} = [0.023 \ 0.0135 \ 0.8 \ 0.1 \ 0.1]^T$ ,  $\mathbf{P}_0 = \text{diag}([1 \ 1 \ 1 \ 1 \ 0.0235])$ ,  $\mathbf{Q} = \text{diag}(10^{-5} * [1 \ 1 \ 1 \ 0.1 \ 10^{-20}])$  and  $\mathbf{R} = 10^{-13}$ . The true measurement noise is considered to be a coloured noise with auto correlation coefficient of 0.9 and a covariance of 0.0235. As for the estimators the noise remains uncharacterised. Therefore, the auto correlation factor is assumed arbitrarily (0.1 for this study).

Simulations are carried out for different loading conditions i.e., with constant current of 8A and a real current sequence based on UDDS [13] which is shown in Fig.2 respectively. Sampling period  $\tau$  was set to 1s. Fig. 6 and 7 demonstrate the efficacy of the proposed 5<sup>th</sup> order SA-CKF over 4<sup>th</sup> order SA-CKF and the non-augmented CKF for a representative run. Table I indicates that the standard deviation for 5<sup>th</sup> order SA-CKF is lowest for each case compared to 4<sup>th</sup> order SA-CKF and non-augmented CKF. For a typical run the estimated auto-correlation coefficient is illustrated in Fig. 4. It is observed that although initialized with an arbitrary choice of the value 0.1 the estimated coefficient has been quickly converged to the true value (which is 0.9) and continues to track it closely. This enables to proposed 5<sup>th</sup> order SA-CKF to estimate with improved accuracy. To check the consistency of the proposed 5<sup>th</sup> order SA-CKF Monte Carlo (MC) simulation has been performed for 500 runs wherein different noise sequence has been considered for each MC run. For the performance comparison of the proposed 5<sup>th</sup> order SA-CKF with 4<sup>th</sup> order SA-CKF and non-augmented CKF from the MC simulation, same noise sequence as of 5<sup>th</sup> order SA-CKF has been used

for the competing filters. It is observed from Fig.7 that, when coloured noise is unknown, mean square error ( $MSE$ ) for  $5^{th}$  order SA-CKF has been settled to a lower value compared to  $4^{th}$  order SA-CKF and non-augmented CKF. However, initially the MSE of  $5^{th}$  order SA-CKF is more as auto-correlation coefficient try to converge to its truth value during this time interval the estimated. Author has observed a very few divergences with  $5^{th}$  order SA-CKF and is under investigation.

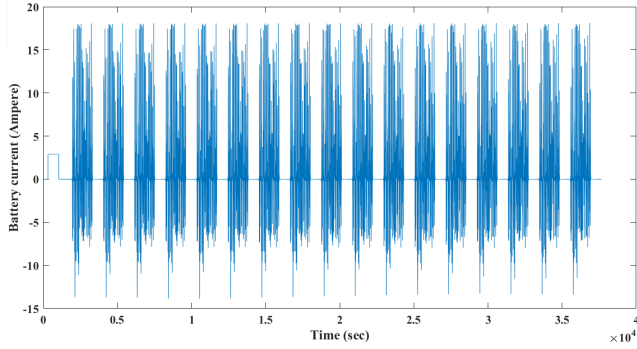


Fig. 2. Battery current based on UDDS drive cycle.

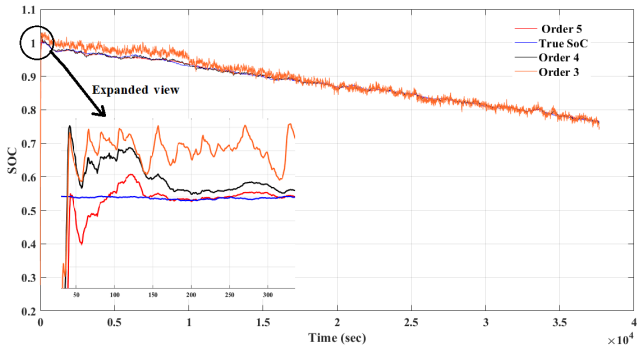


Fig. 3. SOC comparison with real current .

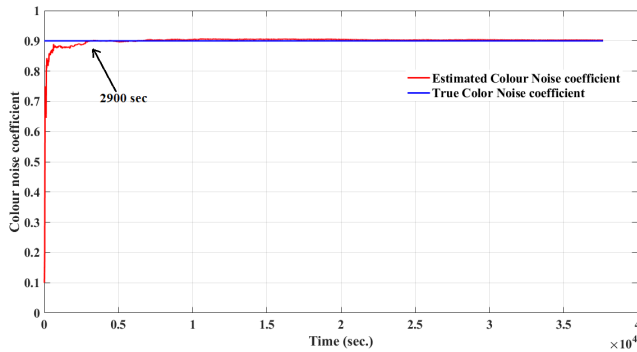


Fig. 4. Convergence of colour noise coefficient

## V. CONCLUSIONS

This paper has addressed the problem of the State of Charge (SOC) estimation using measurement perturbed with uncharacterized coloured noise in the terminal voltage measurement

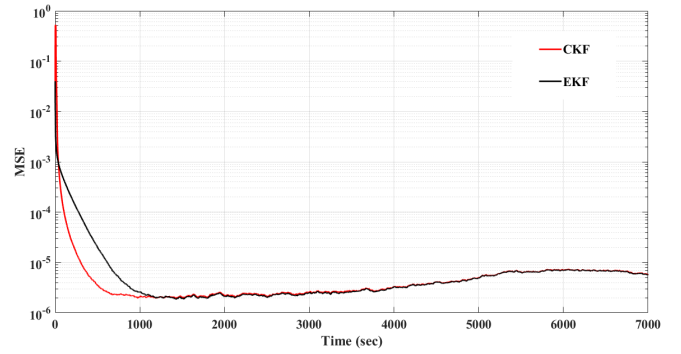


Fig. 5. MSE comparison between EKF and non-augmented CKF in presence of white noise.

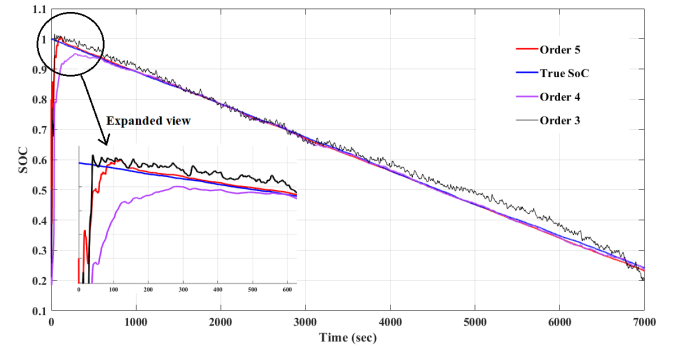


Fig. 6. SOC estimation comparison with constant current for a single representative run

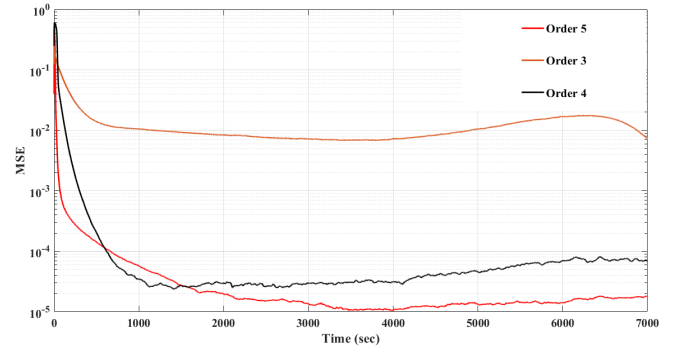


Fig. 7. MSE comparison with constant current

TABLE I  
SPECIFICATIONS AND PARAMETERS OF THE BATTERY MODEL

$R_{pc}$ ( $\Omega$ )	$R_{pe}$ ( $\Omega$ )	$C_{pc}$ (F)	$C_{pe}$ (F)	$R_i$ ( $\Omega$ )	$C$ (F)
0.0042	0.0024	17111	440.57	0.0013	20AH

TABLE II  
COMPARISON TABLE

Order of the CKF algorithm	$3^{rd}$	$4^{th}$	$5^{th}$
Standard deviation with real current	0.0732	0.0648	0.0640
Standard deviation with constant current	0.2226	0.2206	0.0337

and provides a satisfactory solution method using higher order State Augmented Cubature Kalman filter (SA-CKF). The problem of SOC estimation becomes critical when the auto-correlation coefficient remains unknown to the designer which is successfully taken care of by the proposed 5<sup>th</sup> order SA-CKF as the SOC estimates illustrate its superiority over the 4<sup>th</sup> order SA-CKF with arbitrarily assumed auto-correlation coefficient and non-augmented CKF with the assumption of white noise. The unknown auto correlation coefficient has been observed to be appropriately estimated by the proposed filter starting from a value arbitrarily assigned by the designer. Monte Carlo simulation demonstrates the consistent efficacy of the proposed filter over its close competitors and also indicates its suitability for SOC estimation with nonlinear filters with uncharacterized colour noise.

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