

Comparative Study of EKF and UKF for SOC Estimation of Lithium-ion Batteries

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Abstract—With the expanding application of lithium-ion batteries, the state of charge (SOC) estimation based on the Kalman filter (KF) draws great attention. However, lots of KF's variants, such as extended Kalman filter (EKF) and unscented Kalman filter (UKF) are available. In order to find the suitable KF variant for SOC estimation, this paper studies the performance of EKF and UKF in three scenarios based on the two-order RC model by elaborative simulation. The accuracy and convergence of the two filters are compared. The impact of the model deviation on the estimated results is also tested to evaluate the robustness. The test results show that EKF performs much the same as UKF in its accuracy, rate of convergence as well as robustness to model errors in most cases.

Index Terms—SOC online estimation; extended Kalman filter; unscented Kalman filter; comparative study

I. INTRODUCTION

With the development of smart grids and energy internet aimed at the utilization of clean renewable energy, the demand for energy storage in power systems is increasing. On the other hand, spreading of the belief of green travel has promoted the technology progress and market-scale expanding of electric vehicles (EV) and hybrid electric vehicles (HEV). The battery has become more and more important in the field of both electrical transportation and power system [1]. Among various technologies available, lithium-ion battery has become the prior choice in EV and the demonstration projects of battery energy storage system in power grid due to its high energy density, low self-discharge rate and long life [2]. However, lithium-ion batteries are more vulnerable to overcharge or overdischarge, thus require more accurate battery manage system (BMS). State of charge (SOC) is one of the most important parameters to monitor the battery and has become the basic data in BMS to optimize the operation of batteries as well as protect them from various hazards [3].

SOC is a comprehensive variable depicting the electrochemical process that has various impact factors such as temperature, charge and discharge rates, aging and self-discharging. The complexity and nonlinearity of the electrochemical process as well as the diversity of operating conditions make SOC estimation a huge challenge. The small

voltage difference of voltage platform and limited measurement accuracy makes the SOC estimation of lithium-ion battery even more difficult. For a long time, various SOC estimation methods, such as Ampere-hour integration [4], table lookup based on internal resistance or open circuit voltage (OCV) [5]-[6], and chemical measurement method [7] have been proposed. However, Ampere-hour integration method is weak in accumulation error elimination, table lookup method is susceptible to temperature variation and requires long standing time, chemical measurement method requires to break the closure of the battery, therefore other methods such as Kalman filter (KF) algorithm based on information analysis are studied.

The KF is designed to optimally estimate the state of the dynamic system in the noisy environment and has the features of clear principle and convenient to use. It has developed many variants [8], [9] and been widely utilized in many fields such as mobile positioning [10] and navigation [11]. In recent years, KF variants, mainly including extended Kalman filter (EKF) and unscented Kalman filter (UKF), are introduced to the estimation of batteries' SOC. In [12], EKF based on a two-order RC model of battery is proposed and validated by a hardware-in-the-loop test bench. In [13], UKF was introduced to estimate the SOC according to a modified equivalent circuit model considering the impact of different current rates, temperatures and life cycle of the battery presented in the paper. In [14], the SOC estimation method based on square root unscented Kalman filter using spherical transform (Sqrt-UKFST) with unit hypersphere is proposed to improve the numerical properties of state covariance. In [15], an adaptive UKF method with the ability of adaptive adjustment of the noise covariances in the SOC estimation process was proposed to estimate SOC with better accuracy. To get correct dynamic model used by EKF or UKF to guarantee the accuracy of the SOC estimation, artificial neural network algorithm [16] and fuzzy logic algorithm [17] are utilized to predict and dynamically correct the battery's parameters in real time.

The above research mainly focuses on how to apply EKF or UKF in SOC estimation to get better-estimated accuracy, and usually draws the conclusion based on the tests of a few selected work conditions of the battery. For example, in [12]

the performance of EKF is only tested under the cycle of urban dynamometer driving schedule and highway fuel economy driving schedule. However, comprehensive comparisons under various working conditions of batteries are needed when choosing a suitable filter for SOC estimation.

This paper studies the performance of EKF and UKF in various scenarios by simulation. The scenarios consist of batteries with different initial SOC charged or discharged through constant current and constant voltage (CCCV), pulse and New European Driving Cycle (NEDC) process. The accuracy and convergence of the estimation by EKF and UKF with different setting are compared. The impacts of the model deviation on estimated results are also analyzed to evaluate the robustness of EKF and UKF. Through the comparative study of EKF and UKF, a relative complete evaluation of EKF and UKF in the SOC estimation are given. The rest of the paper is organized as follows: Section II summarizes the basic knowledge of KF theory, including standard Kalman filter (SKF), EKF and UKF; Section III introduces the concept of SOC estimation and the battery model used; Section IV and V give the process and the results of the simulation and conclusions are drawn in Section VI.

II. BASIC KNOWLEDGE OF EKF AND UKF

A. Standard Kalman Filter

The KF theory aims at the optimal state estimation of the dynamic system in a noisy environment and could achieve optimal in the sense of minimum variance when the estimated object is a linear system contaminated by Gaussian noise. When the noise in both inputs and outputs is considered, the model of the linear system in discrete form can be written as:

$$x(k+1) = A(k)x(k) + B[k]u(k) + w(k) \quad (1)$$

$$y(k) = C(k)x(k) + D(k)[u(k) + w(k)] + v(k) \quad (2)$$

Where $x(k)$, $u(k)$ and $y(k)$ are the variables of state, input and output; $A(k)$, $B(k)$, $C(k)$ and $D(k)$ are the matrix of state transition, input, output and feedforward; $w(k)$ and $v(k)$ denote the measurement noise of input and output. Usually, (1) is called the state equation, (2) is called the output equation and together they are called state space expression.

Generally, the measurement noise is assumed Gaussian with zero mean and uncorrelated. That is, $w(k)$ and $v(k)$ satisfy:

$$\begin{aligned} E[w(k)] &= 0 & E[w(n)w^T(k)] &= \begin{cases} Q(k) & n = k \\ 0 & n \neq k \end{cases} \\ E[v(k)] &= 0 & E[v(n)v^T(k)] &= \begin{cases} R(k) & n = k \\ 0 & n \neq k \end{cases} \end{aligned} \quad (3)$$

The original Kalman filter is designed for the linear system and is usually called SKF in some kinds of literature. The framework of SKF can be conceptualized as two phases: prediction and correction.

Prediction is a priori estimation that calculates the expected value of the states and the predicted error covariance based on the information until the previous time-step as:

$$\hat{x}(k|k-1) = A(k-1)\hat{x}(k-1|k-1) + B(k-1)u(k-1) \quad (4)$$

$$\begin{aligned} P(k|k-1) &= A(k-1)P(k-1|k-1)A^T(k-1) \\ &\quad + B(k-1)Q(k-1)B^T(k-1) \end{aligned} \quad (5)$$

Correction is a posteriori estimation that updates the expected value of the states and the error covariance according to the latest output measurements as:

$$\hat{x}(k|k) = \hat{x}(k|k-1) + K(k)[y(k) - \hat{y}(k|k-1)] \quad (6)$$

$$\begin{aligned} P(k|k) &= [I - K(k)C(k)]P(k|k-1)[I - K(k)C(k)]^T \\ &\quad + K(k)[D(k)Q(k)D^T(k) + R(k)]K^T(k) \end{aligned} \quad (7)$$

Where $K(k)$ is the optimal Kalman gain that represents relative weight between predicted states and innovation (error between measured and predicted outputs). $K(k)$ could be calculated as:

$$K(k) = (P(k|k-1)C^T(k) + B(k)Q(k)D^T(k))S^{-1}(k) \quad (8)$$

$$S(k) = C(k)P(k|k-1)C^T(k) + D(k)Q(k)D^T(k) + R(k) \quad (9)$$

B. Extended Kalman Filter

As designed for the linear system, SKF could not be applied in the nonlinear system as below.

$$x(k+1) = f[x(k), u(k) + w(k)] \quad (10)$$

$$y(k) = h[x(k), u(k) + w(k)] + v(k) \quad (11)$$

Where $f[x(k), u(k)]$ is the general form of the state equation, and $h[x(k), u(k)]$ is the general form of the output equation.

EKF is an SKF's nonlinear variant that linearizes nonlinear system shown in (10) - (11) by Taylor series expansion successively for each state estimation. It keeps the first-order differentiation and ignores the higher order terms, therefore could be applied to the nonlinear system at the expense of losing some accuracy. Actually, EKF follows the same framework as SKF except that the states and outputs are predicted by (10) - (11) and the matrixes in (1) - (2) are the matrixes of partial derivatives (the Jacobian) computed as:

$$\text{Transition matrix: } A(k-1) = \left. \frac{\partial f[x(k), u(k)]}{\partial x(k)} \right|_{\hat{x}(k-1|k-1), u(k-1)}$$

$$\text{Input matrix: } B(k-1) = \left. \frac{\partial f[x(k), u(k)]}{\partial u(k)} \right|_{\hat{x}(k-1|k-1), u(k-1)}$$

$$\text{Output matrix: } C(k) = \left. \frac{\partial h[x(k), u(k)]}{\partial x(k)} \right|_{\hat{x}(k|k-1), u(k)}$$

$$\text{Feedforward matrix: } D(k) = \left. \frac{\partial h[x(k), u(k)]}{\partial u(k)} \right|_{\hat{x}(k|k-1), u(k)}$$

C. Unscented Kalman Filter

UKF is another nonlinear variant of SKF applicable to the nonlinear system shown in (10) - (11). It follows the similar framework of SKF but utilizes the unscented transformation (UT) proposed by Julier [8] as a tool to propagate mean and covariance information. The UT is a method that could calculate the projected mean and covariance of a nonlinear function with the second order accuracy based on a set of sigma points which encodes the mean and covariance correctly. The process of estimation by UKF is as follows.

In the prediction phase, a set of sigma points representing augmented variables in state equation (10) is firstly chosen with the following mean and covariance:

$$\hat{x}_a(k-1) = [\hat{x}(k-1|k-1) \quad u(k-1)]^T \quad (12)$$

$$P_a(k-1|k-1) = \text{diag}(P(k-1|k-1), Q(k-1)) \quad (13)$$

Then, transform the sigma points by nonlinear function (10) to get the transformed sigma points:

$$\hat{x}^{(i)}(k|k-1) = f[\hat{x}_a^{(i)}(k-1|k-1)] \quad (14)$$

Where $\hat{x}_a^{(i)}(k-1|k-1)$ is the i -th point of the set of sigma points chosen.

Next, the predicted states and corresponding covariance are calculated by the weighted average and outer product of the transformed points as:

$$\hat{x}(k|k-1) = \sum_{i=0}^N W^{(i)} \hat{x}^{(i)}(k|k-1) \quad (15)$$

$$P(k|k-1) = \sum_{i=0}^N W^{(i)} [\hat{x}^{(i)}(k|k-1) - \hat{x}(k|k-1)][\cdot]^T \quad (16)$$

Where N is the number of points in the set of sigma points, $W^{(i)}$ is the weight of the i -th point and $[V][\cdot]^T$ equals $V \times V^T$.

In the correction phase, a set of sigma points representing augmented variables in output equation (11) is again chosen with the following mean and covariance:

$$\hat{x}_{a2}(k-1) = [\hat{x}(k|k-1) \quad u(k) \quad 0]^T \quad (17)$$

$$P_{a2}(k|k-1) = \text{diag}(P(k|k-1), Q(k), R(k)) \quad (18)$$

The sigma points are also transformed by (11) as:

$$y^{(i)}(k) = h[\hat{x}_{a2}^{(i)}(k|k-1)] \quad (19)$$

Then, the predicted mean, covariance of the outputs and the cross-covariance matrix between states and outputs are calculated as:

$$\hat{y}(k) = \sum_{i=0}^N W^{(i)} \hat{y}^{(i)}(k) \quad (20)$$

$$P_{yy}(k) = \sum_{i=0}^N W^{(i)} [\hat{y}^{(i)}(k) - \hat{y}(k)][\cdot]^T \quad (21)$$

$$P_{xy}(k) = \sum_{i=0}^N W^{(i)} [\hat{y}^{(i)}(k) - \hat{y}(k)] [\hat{x}^{(i)}(k|k-1) - \hat{x}(k|k-1)]^T \quad (22)$$

Finally, the predicted states and covariance are updated as:

$$\hat{x}(k|k) = \hat{x}(k|k-1) + K(k)[y(k) - \hat{y}(k|k-1)] \quad (23)$$

$$P(k|k) = P(k|k-1) - K(k)P_{yy}K^T(k) \quad (24)$$

Where the Kalman gain is calculated as:

$$K(k) = P_{xy}P_{yy}^{-1} \quad (25)$$

Various choices of the sigma points set are available as summarized in [9] and it is not possible to determine which will be more accurate than the others if only mean and variance are known. Therefore, the commonly used symmetric sampling method proposed in [8] is adopted in this paper and the vectors $x^{(i)}$ and their associated weights $W^{(i)}$ are as follows:

$$\begin{aligned} x^{(0)} &= \bar{x}, x^{(i)} = \bar{x} + \tilde{x}^{(i)}, i = 1, 2, \dots, 2n \\ \tilde{x}^{(i)} &= \left(\sqrt{(n+\lambda)P_{xx}} \right)_i^T, i = 1, 2, \dots, n \\ \tilde{x}^{(n+i)} &= -\left(\sqrt{(n+\lambda)P_{xx}} \right)_i^T, i = 1, 2, \dots, n \\ W_0^{(m)} &= \frac{\lambda}{n+\lambda}, W_0^{(c)} = \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta) \\ W_i^{(m)} &= W_i^{(c)} = \frac{1}{2(n+\lambda)}, i = 1, \dots, 2n \end{aligned} \quad (26)$$

Where n is the demission of x and λ is the scale parameter that equals $\alpha^2(n + \kappa) - n$. The choosing principle of α , β and κ may refer to [9].

III. SOC ESTIMATION AND BATTERY MODEL

SOC represents the ratio of available capacity and total capacity in the battery. Theoretically, it can be obtained by the Ampere-hour integration method that integrates the measured battery current as:

$$s = s_0 + \frac{\eta}{C_r} \int_{t_1}^{t_2} I(t) dt \quad (28)$$

Where, s is SOC; C_r , η and I is the rated capacity, Coulomb efficiency and charge current of the battery.

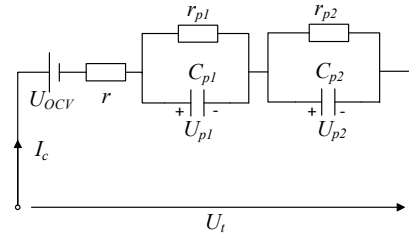


Figure 1. Two-order RC model of the battery

To realize accurate SOC estimation based on KF theory, a dynamic model of the battery is needed. The dynamic model depicts the external characteristics of the battery as well as the variation rule of the related variables such as SOC, OCV. The

model consists of a two-order RC circuit, an ohmic resistance and a potential source representing OCV is shown in Fig.1 and has been adopted and verified by many scholars [12]-[16]. In this paper, we also base on this model to generate simulated data and to construct EKF and UKF.

In Fig.1, U_t , I_c and U_{OCV} denote the terminal voltage, charging current and OCV of the battery; r , r_{p1} and r_{p2} denote the ohmic resistance and the polarization resistance; C_{p1} and C_{p2} denote the polarization capacitance. Based on the theory of circuit and discretization, we could get the discretized state space expression of the model in Fig.1 as:

$$x(k+1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 - \frac{\Delta T}{r_{p1}C_{p1}} & 0 \\ 0 & 0 & 1 - \frac{\Delta T}{r_{p2}C_{p2}} \end{bmatrix} x(k) + \begin{pmatrix} \frac{\eta\Delta T}{C_r} \\ \frac{\Delta T}{C_{p1}} \\ \frac{\Delta T}{C_{p2}} \end{pmatrix} u(k) \quad (29)$$

$$y(k) = [0 \quad 1 \quad 1]x(k) + ru(k) + f_{SOC-OCV}(s) \quad (30)$$

where, $x = [s \quad U_{p1} \quad U_{p2}]^T$, $u = I_c$ and $y = U_t$; ΔT is the time interval of discretization and $f_{SOC-OCV}(s)$ is the function mapping s to OCV. It is obvious that the state equation (29) is linear while the output equation (30) is nonlinear, as the mapping relationship between SOC and OCV depicted by $f_{SOC-OCV}(s)$ is generally nonlinear.

Based on the structure of the model in (29) - (30), parameters such as r , r_{p1} , r_{p2} , C_{p1} , C_{p2} and $f_{SOC-OCV}(s)$ could be determined offline or identified online from the measured data of the elaborate experiments as in [12] - [15]. Then, based on the model established, EKF and UKF could be used to estimate SOC.

IV. PROCESS OF SIMULATION ANALYSIS

This paper focuses on the comparative analysis of the performance of EKF and UKF with different start conditions and underlying model when they are used to estimate SOC of the batteries that are charged or discharged in different ways from various initial states. The analysis consists of two parts: accuracy analysis and robustness evaluation. The former tests the performance of EKF and UKF when the underlying model is correct and the latter tests the impact the biased underlying model on the estimation accuracy.

The flow of the analysis is shown in Fig.2. In order to separate the estimation error caused by model error and that caused by filtering method, simulated data are utilized. Firstly, the electrical variables as well as SOC are simulated based on the two-order RC model for batteries with different initial SOC charged or discharged in CCCV, pulse or NEDC work condition. Then, noise is injected into the simulated data to mimic the process of measurement. Next, the contaminated data are processed by EKF and UKF that is based on the correct or biased model to estimate SOC. Finally, the actual SOC and the estimated SOC or the estimated SOC based on the correct model and that based on the biased model are compared to evaluate the accuracy and the robustness.

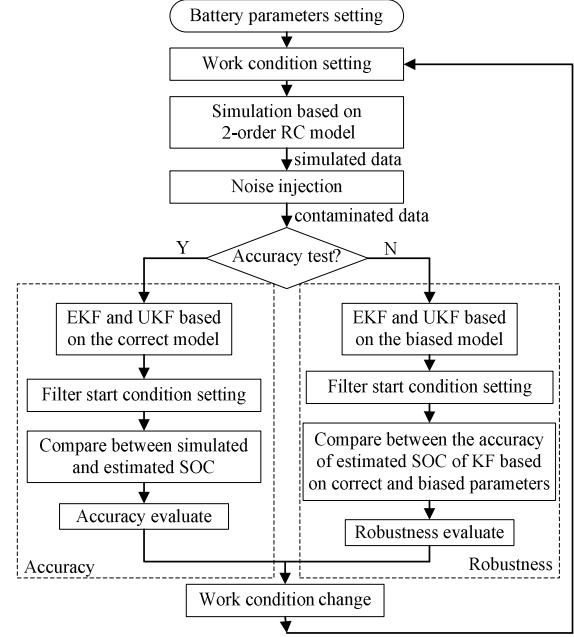


Figure 2. Flow of the test of EKF and UKF based on simulation

V. RESULTS AND DISCUSSION

A lithium-ion battery with the rated voltage of 3.2V and the rated capacity of 11.5Ah is used. The program of the battery simulation, EKF and UKF are all implemented in Matlab. The other battery parameters are shown in Tab. 1.

TABLE I. PARAMETERS OF THE BATTERY

η (%)	r (ohm)	r_{p1} (ohm)	C_{p1} (F)	r_{p2} (ohm)	C_{p2} (F)
98	0.028	0.00667	62891	0.00786	4490

A. Accuracy and Convergency Analysis

The accuracy as well as the convergency of EKF and UKF are tested in three scenarios. The first manner is common in the charge of the ordinary battery, the second is typical in the application of batteries in power grid such as peak-load shaving and the third one is the strand process for the power battery test. EKF and UKF with different start conditions including initial SOC x_0 and corresponding covariance P_0 are utilized. For the generality, two initial values of SOC equal 0.2 and 0.5 are used in each scenario. The time step of simulation and SOC estimation are 0.001h and 0.01h, respectively.

The simulated results of each scenario are shown in Fig. 3 - Fig. 5. The legend in the figure of estimated SOC and its error has the form as "filter name- x_0 ". For example, "EKF-0.5" means estimation by EKF with x_0 equals 0.5. From the results, we could see that the estimated SOC by both EKF and UKF converges to the actual SOC in all work conditions, whether the initial SOC is 0.2 or 0.5. The estimated error decreases from an initial value determined by x_0 sharply to a value less than 5% in the first few steps and then oscillating attenuates to a small steady value of about 1%. When x_0 equals the initial SOC of the battery, the estimated error of both EKF and UKF is small and varies around zero with 0.1% amplitude. EKF performs as good as UKF except x_0 equals 0, in which case the estimated error of EKF converges slowly. The results of the

three scenarios show an obvious consistency and demonstrate that both EKF and UKF are effective in the SOC estimation when the dynamic model based is in accordance with the actual battery model. Besides, the start condition x_0 of EKF should keep away from zero to avoid slow convergence. For other cases, EKF performs as good as UKF.

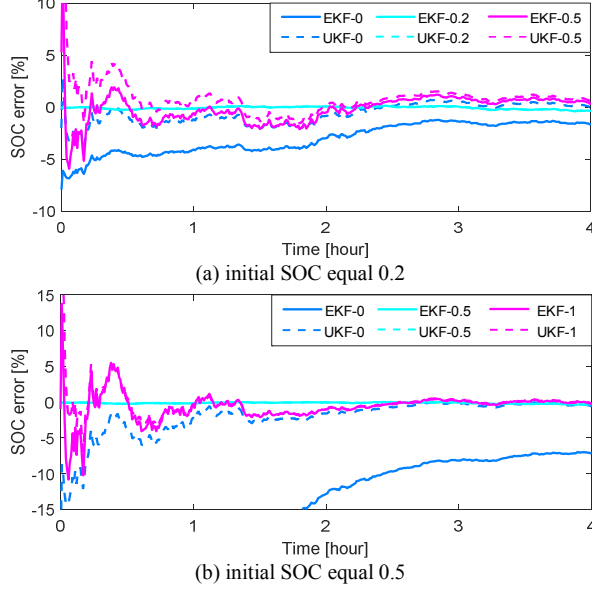


Figure 3. SOC estimated error of scenario 1

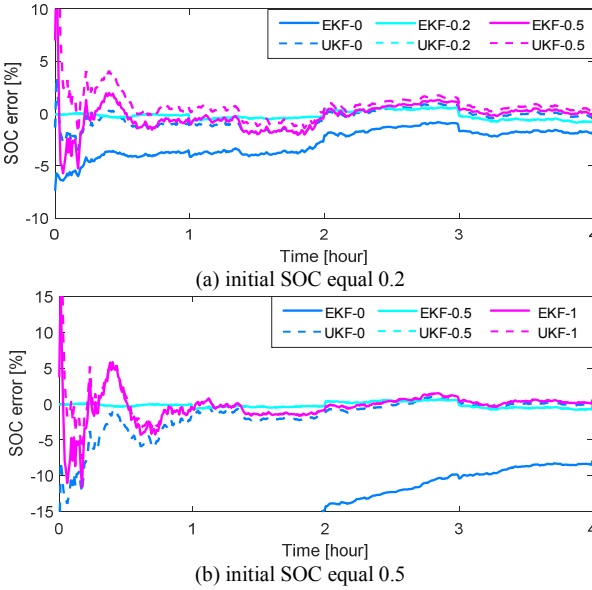


Figure 4. SOC estimated error of scenario 2

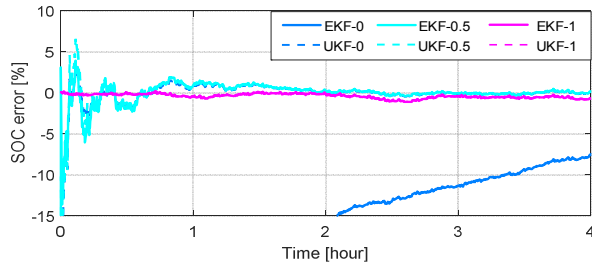


Figure 5. Results of scenario 3 with initial SOC equal 1

B. Robustness Evaluation

In practical application, it is impossible to get the exact parameters of the battery as they may be influenced by factors such as temperature, SOC, charge rates and so on. Therefore the robustness of EKF and UKF in the above three scenarios is tested by deviating the parameters of the model. The capacity and ohmic resistance are biased and the estimated SOC error is given in Fig. 6 and Fig. 7. The legend has the “filter name- x_0 -SOC0” form. For example, “EKF-0.5-0.2” means estimated by EKF with x_0 equals 0.5 when SOC of the battery equals 0.2.

Comparing Fig. 6, Fig. 7 with Fig. 3 - Fig.5, it is obvious that the estimated accuracy of both EKF and UKF decreases when there is a deviation in capacity or ohmic resistance and the degree of the degradation depends on work condition, initial SOC of the battery and start conditions of the filter. Fig. 6 shows that the deviation of capacity mainly affects the steady-state accuracy and error accumulates with proportional to battery current. When x_0 equals the initial SOC, the estimated error increases much quicker than other cases. Fig. 7 shows that the deviation of ohmic resistance affects both the convergence of the initial SOC error and the steady state error, it prolongs the initial convergence process as well as increases the steady state error. Besides, the closer is x_0 to the initial SOC, the smaller estimated error is, which is opposite to that for capacity deviation.

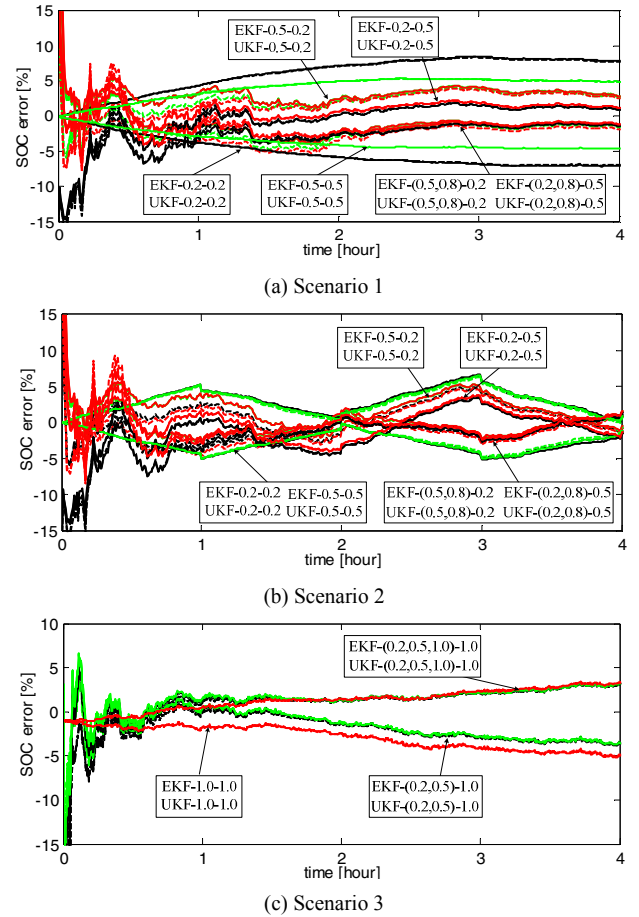


Figure 6. Estimated error when capacity has $\pm 10\%$ deviation

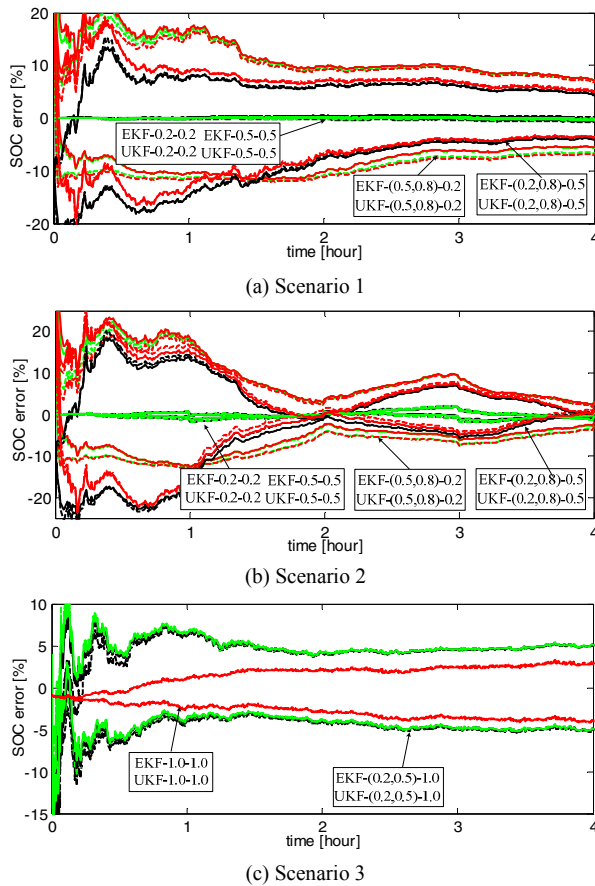


Figure 7. Estimated error when ohmic resistance has $\pm 30\%$ deviation

The estimated error of EKF and UKF denoting by the dashed line and solid lines are almost coincident in Fig. 6 and Fig. 7, indicating that the robustness of the two filters is close to each other. Besides the steady state error in Fig. 6 and Fig. 7 is about 5%, the maximum value commonly accepted in BMS, therefore it is significant to calculate or identify the capacity and the ohmic resistance of the battery with the accuracy within 10% and 30%, respectively.

VI. CONCLUSION

In this paper, the performance of EKF and UKF with different start condition and underlying model in SOC estimation are studied by simulation. The simulation results of the three scenarios that batteries charged by CCCV, pulse and NEDC process show that both EKF and UKF are effective in the SOC estimation when they base on the correct model. EKF performs as good as UKF when its initial SOC is far from zero. The robustness of EKF and UKF are almost same but relative weak: $\pm 10\%$ deviation in capacity or $\pm 30\%$ deviation in ohmic resistance could cause the steady error increase from 1% to over 5%, therefore it is significant to improve the accuracy of the model as much as possible. It is worth mentioning that the nonlinearity considered in the paper mainly caused by SOC-OCV mapping and this may limit the universality of the conclusions obtained.

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