Міністерство освіти і науки України

Національний університет «Запорізька Політехніка»

Кафедра програмних засобів

ЗВІТ

з лабораторної роботи №4

з дисципліни «Вища математика, математичний аналіз»

Виконав:

студент групи КНТ-122 О. А. Онищенко

Прийняли:

асистент: Л. М. Самсика

2023

Задача 1

$= \left(2 - \frac{2}{3}\right) - \left(-2 + \frac{2}{3}\right) = \frac{8}{3} \qquad = \frac{8}{3}$

Лобч Вадима

1. $z = \sqrt{2x^2 - y^2}, \quad 2x^2 - y^2 \geq 0, \quad y^2 \leq 2x^2$

2. а) $z = (3x^2 - 2yx)^3, \quad dz = \frac{df}{dx}dx + \frac{df}{dy}dy$

$f(x, y) = (3x^2 - 3yx)^3$

$= \frac{df}{dx} = \frac{d}{dx}(3x^2 - 3yx)^3 = 3(3x^2 - 3yx)^2 \frac{d}{dx}(3x^2 - 3yx) =$

$= 3(3x^2 - 3yx)^2(6x - 3y)$

$= \frac{df}{dy} = \frac{d}{dy}(3x^2 - 3yx)^3 = 3(3x^2 - 3yx)^2 \frac{d}{dy}(3x^2 - 3yx) =$

$= 3(3x^2 - 3yx)^2(-3x)$

$= dz = 3(3x^2 - 3yx)^2(6x - 3y)dx + 3(3x^2 - 3yx)^2(-3x)dy$

$= 3(3x^2 - 3yx)^2(6x - 3y)dx + 3(3x^2 - 3yx)^2(-3x)dy$

б) $f(x, y) = e^{\cot\left(\frac{x}{y}\right)}$

$= \frac{df}{dx} = \frac{d}{dx} e^{\cot\left(\frac{x}{y}\right)} = e^{\cot\left(\frac{x}{y}\right)} \frac{d}{dx}\cot\left(\frac{x}{y}\right) = e^{\cot\left(\frac{x}{y}\right)}\left(-\csc^2\left(\frac{x}{y}\right)\right)\frac{1}{y}$

$= \frac{df}{dy} = \frac{d}{dy} e^{\cot\left(\frac{x}{y}\right)} = e^{\cot\left(\frac{x}{y}\right)} \frac{d}{dy}\cot\left(\frac{x}{y}\right) = e^{\cot\left(\frac{x}{y}\right)}\left(-\csc^2\left(\frac{x}{y}\right)\right)\left(-\frac{x}{y^2}\right)$

$= dz = e^{\cot\left(\frac{x}{y}\right)}\left(-\csc^2\left(\frac{x}{y}\right)\right)\frac{1}{y}dx + e^{\cot\left(\frac{x}{y}\right)}\left(-\csc^2\left(\frac{x}{y}\right)\right)\left(-\frac{x}{y^2}\right)dy$

3. $z = \frac{y^2}{2x+1}, \quad x = 1 - 2t, \quad y = 1 + \arctan(t)$

$dz = \frac{df}{dx} \cdot \frac{dx}{dt} + \frac{df}{dy} \cdot \frac{dy}{dt}$

$\frac{df}{dx} = \frac{d}{dx} \cdot \frac{y^2}{2x+1} = y^2\left(-\frac{2}{(2x+1)^2}\right)$

$$\frac{df}{dy} = \frac{d}{dy} \cdot \frac{y^2}{2x+1} = \frac{2y}{2x+1}$$

$$\frac{dx}{dt} = \frac{d}{dt}(1-2t) = -2$$

$$\frac{dy}{dt} = \frac{d}{dt}(1+\arctan(t)) = \frac{1}{1+t^2}$$

$$\frac{dz}{dt} = y^2\left(-\frac{2}{(2x+1)^2}\right)(-2) + \frac{2y}{2x+1} \cdot \frac{1}{1+t^2}$$

$$t = t_0 = 0 \quad - \quad x(0) = 1-2(0) = 1,$$

$$y(0) = 1 + \arctan(0) = 1$$

$$\left.\frac{dz}{dt}\right|_{t=0} = 1^2\left(-\frac{2}{(2(1)+1)^2}\right)(-2) + \frac{2(1)}{2(1)+1} \cdot \frac{1}{1+0^2} =$$

$$= \left.\frac{dz}{dt}\right|_{t=0} = \frac{4}{9} + \frac{4}{3} = \frac{20}{9} \approx 2{,}22$$

4. $x\tan(y) + y\cos(z) + \tan(x) = 5$

$$F(x,y,z) = x\tan(y) + y\cos(z) + \tan(x) - 5$$

$$F(x,y,z) = 0 \qquad z = z(x,y)$$

$$z_x = -\frac{F_x}{F_z} = -\frac{\frac{dF}{dx}}{\frac{dF}{dz}} = -\frac{\tan y + \sec^2 x}{-y\sin z} = \frac{\tan y + \sec^2 x}{y\sin z}$$

$$z_y = -\frac{F_y}{F_z} = -\frac{\frac{dF}{dy}}{\frac{dF}{dz}} = -\frac{x\cos y + \cos z}{-y\sin z} = \frac{x\cos y + \cos z}{y\sin z}$$

$$z_{xy} = \frac{dz_x}{dy} = \frac{d}{dy}\left(\frac{\tan y + \sec^2 x}{y\sin z}\right) = \frac{(y\sin z)(\sec^2 y) - (\tan y + \sec^2 x)(y\sin z)_y}{(y\sin z)^2}$$

$$(y\sin z)_y = y(\sin z)_y = y(\cos z)\,z_y = y(\cos z)\left(\frac{x\cos y + \cos z}{y\sin z}\right)$$

$$z_{xy} = \frac{(y\sin z)(\sec^2 y) - (\tan y + \sec^2 x)(x(\cos y\cos z) + \cos^2 z)}{(y^2)(\sin^2 z)}$$

$$z_{xy} = \frac{(y\sin z\cos y) - (\sin y + \sec^2 x)(x(\cos y\cos z) + (\cos^2 z)}{(y^2)(\sin^2 z)}$$

$$z_{yx} = \frac{dz_y}{dx} = \frac{d}{dx}\left(\frac{x\cos y + \cos z}{y\sin z}\right) = \frac{(y\sin z)(\cos y) - (x\cos y + \cos z)(y\sin z)_x}{(y\sin z)^2}$$

$$(y\sin z)_x = y(\sin z)_x = y(\cos z)z_x = y(\cos z)\left(\frac{\sin y + \sec^2 x}{y\sin z}\right)$$

$$z_{yx} = \frac{(y\sin z\cos y) - (x\cos y + \cos z)(y(\cos z))\left(\frac{\sin y + \sec^2 x}{y\sin z}\right)}{(y^2)(\sin^2 z)} =$$

$$= \frac{(y\sin z)(\cos y) - (x\cos y + \cos z)(x(\sin y)\cos z) + (\sec^2 x\cos z)}{(y^2)(\sin^2 z)}$$

5. $S: x^2+y^2 = z+x+y,\ M_0(1, -3, 12)$

$F(x,y,z) = x^2+y^2-z-x-y$

$F(x,y,z) = 0 \qquad \nabla F(x,y,z) = (2x-1, 2y-1, -1)$

$\nabla F(1,-3,12) = (2(1)-1, 2(-3)-1, -1) = (1, -7, -1)$

$M_0 = (1, -7, -1)$

$F_x(M_0)(x-x_0) + F_y(M_0)(y-y_0) + F_z(M_0)(z-z_0) = 0$

$(1)(x-1) + (-7)(y+3) + (-1)(z-12) = 0 =$

$= x - 7y - z - 32 = 0$

6. $z = e^{2x^2+y^2}$

$$f_{xx} = \frac{d^2f}{dx^2} = \frac{d}{dx}\left(\frac{df}{dx}\right) = \frac{d}{dx}(4xe^{2x^2+y^2}) = 4e^{2x^2+y^2} + 16x^2e^{2x^2+y^2}$$

$$f_{yy} = \frac{d^2f}{dy^2} = \frac{d}{dy}\left(\frac{df}{dy}\right) = \frac{d}{dy}(2ye^{2x^2+y^2}) = 2e^{2x^2+y^2} + 4y^2e^{2x^2+y^2}$$

$$f_{xy} = \frac{d^2f}{dxdy} = \frac{d}{dx}\left(\frac{df}{dy}\right) = \frac{d}{dx}(2ye^{2x^2+y^2}) = 8xye^{2x^2+y^2}$$

$z = f(x,y) = e^{2x^2+y^2}$; $f_{xx} = 4e^{2x^2+y^2} + 16x^2e^{2x^2+y^2}$

$f_{yy} = 2e^{2x^2+y^2} + 4y^2e^{2x^2+y^2} \qquad f_{xy} = 8xye$

7. $z = 5x^2 - 2xy + y^2$, $M_0(1,1)$, $\bar{a} = 2\bar{i} - \bar{j}$

$f(x,y) = 5x^2 - 2xy + y^2$, $M_0(1,1)$, $\nabla f(x,y) = (10x - 2y, -2x+2y)$

1) $\bar{a} = 2\bar{i} - \bar{j}$

$D_{\bar{a}} f(M_0) = \nabla f(M_0) \cdot \frac{\bar{a}}{\|\bar{a}\|}$

$\nabla f(M_0) = \nabla f(1,1) = (10(1) - 2(1), -2(1)+2(1)) = (8,0)$

$\frac{\bar{a}}{\|\bar{a}\|} = \frac{2\bar{i} - \bar{j}}{\sqrt{(2)^2+(-1)^2}} = \frac{2}{\sqrt{5}}\bar{i} - \frac{1}{\sqrt{5}}\bar{j}$

$D_{\bar{a}} f(M_0) = (8,0) \cdot \left(\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right) = \frac{16}{\sqrt{5}}$

2) $z = f(x,y)$, $M_0(1,1)$, $\nabla f(M_0) = (8,0)$

8. $z = x^2 - xy + y^2 + 9x - 6y + 20$

$f_x = 2x - y + 9$ $\quad f_y = 2y - x - 6$

$\begin{cases} 2x - y + 9 = 0 \\ 2y - x - 6 = 0 \end{cases}$

$(x,y) = (-3,-6)$

$f_{xx} = 2$ $\quad f_{yy} = 2$ $\quad f_{xy} = -1$

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2 = 4 - 1 = 3$

$D > 0$, $f_{xx} > 0$ — локальний мінімум

$(x,y) = (-3,-6)$

9. $z = x^2 + y^2$ $\quad x - y = 1$

$z = f(x,y) = x^2 + y^2$ $\quad g(x,y) = x - y - 1$

$L(x,y,\lambda) = f(x,y) + \lambda g(x,y)$

$\frac{\partial L}{\partial x} = 2x + \lambda = 0$

$\frac{\partial L}{\partial y} = 2y - \lambda = 0$

$\frac{\partial L}{\partial \lambda} = x - y - 1 = 0$

$\lambda = -2x \quad \lambda = 2y \quad x - y - 1 = 0 \quad x = y + 1 \quad \lambda = -2x$

$\lambda = -2(y+1) \quad \lambda = 2y \quad , \quad -2(y+1) = 2y \quad y = -\frac{1}{2}$

$x = y + 1 \quad x = -\frac{1}{2} + 1 = \frac{1}{2}$

$= (x,y) = (\frac{1}{2}, -\frac{1}{2}) \quad z = f(\frac{1}{2}, -\frac{1}{2}) = (\frac{1}{2})^2 + (-\frac{1}{2})^2 = \frac{1}{2}$

10. $z = 2xy - 4x - 2y \quad D: -3 \leq x \leq 0; 0 \leq y \leq -3x$

$z = f(x,y) = 2xy - 4x - 2y$

$f_x(x,y) = 2y - 4 \quad f_y(x,y) = 2x - 2$

$2y - 4 = 0 \Rightarrow y = 2$

$2x - 2 = 0 \Rightarrow x = 1$

$L_1: x = -3, 0 \leq y \leq 9, \ L_2: y = 0, -3 \leq x \leq 0, \ L_3: y = -3x, -3 \leq x \leq 0$

$L_1 = f(-3,y) = 2(-3)y - 4(-3) - 2y = -8y + 12$

$f(-3,0) = 12 \quad , \quad f(-3,9) = -42$

$L_2 = f(x,0) = 2x(0) - 4x - 2(0) = -4x$

$f(-3,0) = 12 \quad f(0,0) = 0$

$L_3 = f(x, -3x) = 2x(-3x) - 4x - 2(-3x) = -6x^2$

$\frac{-b}{2a} = \frac{-0}{2(-6)} = 0 \qquad f(0, -3(0)) = 0$

$\min(f(-3, -3(-3)), f(0, -3(0))) = \min(18, 0) = 0$

$\max(12, 12, 0) = 12$, $\min(-42, 0, 0) = -42$

$= \max\limits_{(x,y)\in D} f(x, y) = 12$

$\min\limits_{(x,y)\in D} f(x, y) = -42$