**Міністерство освіти і науки України**

**Національний університет «Запорізька Політехніка»**

Кафедра програмних засобів

# ЗВІТ

з лабораторної роботи №3

з дисципліни «Вища математика, математичний аналіз»

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2023

**Задача 1**

1\. а) $\int \frac{3x^4 \sqrt[3]{x^2}+1}{x^2}\,dx,\quad u = x^2 \quad du = 2x\,dx \quad dx = \frac{du}{2x}$ [illegible] ВИМА

$\int \frac{3u^2 \sqrt[3]{u}+1}{u}\,\frac{du}{2x} = \frac{1}{2}\int (3u\sqrt[3]{u}+1)\,du = \frac{1}{2}\int (3u^{\frac{4}{3}}+1)\,du =$

$= \frac{1}{2}\left(\frac{9}{7}u^{\frac{7}{3}} + u + C\right),\quad u = x^2 = \frac{1}{2}\left(\frac{9}{7}x^{\frac{14}{3}} + \frac{1}{2}x^2 + C\right) =$

$= \frac{9}{14}x^{\frac{14}{3}} + \frac{1}{2}x^2 + C$

б) $\int \sin(3-11x)\,dx,\quad u = 3-11x \quad du = -11\,dx$

$= \int \frac{\sin(u)}{-11}\,du = -\frac{1}{11}\int \sin u\,du = -\frac{1}{11}(-\cos u) + C =$

$= -\frac{1}{11}(-\cos(3-11x)) + C = \frac{\cos(3-11x)}{11} + C$

2\. $\int \frac{dx}{\sqrt{1-x^2}\,\arcsin(x)} = \quad u = \arcsin(x),\quad du = \frac{1}{\sqrt{1-x^2}}\,dx =$

$= \int \frac{du}{u} = \ln|u| + C = \ln|\arcsin(x)| + C$

3\. $\int \frac{2x-1}{x^2-x+1}\,dx$

1) $x^2 - x + 1 = \left(x-\frac{1}{2}\right)^2 + \frac{3}{4}$

$\int \frac{2x-1}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}\,dx \quad u = x - \frac{1}{2} \quad du = dx \quad =$

$= \int \frac{2\left(u+\frac{1}{2}\right)-1}{u^2+\frac{3}{4}}\,du = \int \frac{2u+1}{u^2+\frac{3}{4}}\,du$

$= \int \frac{2u+1}{u^2+\frac{3}{4}}\,du = 2\int \frac{u}{u^2+\frac{3}{4}}\,du + \int \frac{1}{u^2+\frac{3}{4}}\,du$

1) $v = u^2 + \frac{3}{4} \quad dv = 2u\,du$

$2\int \frac{u}{u^2+\frac{3}{4}}\,du = 2\int \frac{dv}{v} = 2\ln|v| + C_1 = 2\ln\left|u^2+\frac{3}{4}\right| + C_1$

2) $\int \frac{dx}{x^2+a^2} \quad \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C \quad a = \sqrt{\frac{3}{4}} =$

$= \int \frac{1}{u^2+\frac{3}{4}}\,du = \sqrt{\frac{4}{3}}\arctan\left(\sqrt{\frac{4}{3}}\,u\right) + C_2$

$\sqrt{\frac{4}{3}}\ \operatorname{arctan}\left(\sqrt{\frac{4}{3}}\left(x-\frac{1}{2}\right)\right)+C_2$

$\int\frac{2x-1}{x^2-x+1} = 2\ln\left|\left(x-\frac{1}{2}\right)^2+\frac{3}{4}\right| + \sqrt{\frac{4}{3}}\ \operatorname{arctan}\left(\sqrt{\frac{4}{3}}\left(x+\frac{1}{2}\right)\right)+C$

4. $\int \ln\frac{2-x}{2+x}dx \quad u=\ln\frac{2-x}{2+x} \quad v=x \quad dv=dx \quad du=\frac{-1}{2-x}-\frac{1}{2+x}$

$= x\ln\frac{2-x}{2+x} - \int x\left(\frac{-1}{2-x}-\frac{1}{2+x}\right)dx$

$= \int x\left(\frac{-1}{2-x}-\frac{1}{2+x}\right)dx = -\int\frac{x}{2-x}dx - \int\frac{x}{2+x}dx$

1) $u=2-x \quad du=-dx$

$= -\int\frac{x}{2-x}dx = -\int\frac{2-u}{u}(-du) = \int(2-u)\frac{du}{u} =$

$= 2\int\frac{du}{u} - \int du = 2\ln|u| - u + C_1 = 2\ln|2-x| -$

$-(2-x)+C_1$

2) $v=2+x \quad dv=dx$

$= -\int\frac{x}{2+x}dx = -\int\frac{v-2}{v}dv = -\int dv + 2\int\frac{dv}{v} =$

$= -v+2\ln|v|+C_2 = -(2+x)+2\ln|2+x|+C_2$

$= x\ln\frac{2-x}{2+x} - \left(-(x-4)+4\ln|x-4|+C\right)$

5. $\int\frac{\sqrt{x}}{3x+\sqrt[3]{x^3}}dx \quad u=\sqrt[3]{x^3} \quad du\ \sqrt[3]{x^3}\ dx$

$= \int\frac{u}{3+u}du = \int du - 3\int\frac{du}{3+u}$

1) $= u + C_1$

2) $-3\int\frac{du}{3+u} = -3\int\frac{dv}{v} = -3\ln|v|+C_2 = -3\ln|3+u|+C_2$

$= u - 3\ln|3+u| + C$

6. $\int \frac{x^2+3x+2}{x^3-1} dx$

$x^3-1=(x-1)(x^2+x+1)$

$= \frac{x^2+3x+2}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} \quad \Big| \cdot (x-1)(x^2+x+1)$

$= x^2+3x+2 = A(x^2+x+1) + (Bx+C)(x-1)$

$= 6 = 3A \Rightarrow A=2 \qquad x=1$

$= 2 = -C \Rightarrow C=-2 \qquad x=0$

$= 0 = -A+B-C \Rightarrow B=0 \qquad x=-1$

$= \frac{x^2+3x+2}{(x-1)(x^2+x+1)} = \frac{2}{x-1} + \frac{-2}{x^2+x+1}$

$= \int \frac{x^3+3x+2}{(x-1)(x^2+x+1)} dx = 2\int\frac{dx}{x-1} - 2\int\frac{dx}{x^2+x+1}$

1) $= 2\ln|x-1| + C_1$

2) $u = 2ax+b \quad du = 2a\,dx$

$-2\int\frac{dx}{x^2+x+1} = -\int\frac{du}{u^2+u} = -\frac{1}{2}\arctan\left(\frac{u}{2}\right)+C_2 =$

$= -\arctan\left(x+\frac{1}{2}\right)+C_2$

$= -\arctan\left(x+\frac{1}{2}\right)+C_2$

7. $\int \frac{\cos^2 x}{1+\sin^2 x} dx \qquad u=\tan(x) \quad du = \sec^2(x)\,dx$

$= \int \frac{1}{1+\tan^2(x)} + \frac{1}{\sin^2(x)} \sec^2(x)\,dx = \int \frac{du}{1+u^2}$

$= \int\frac{du}{1+u^2} = \arctan(u)+C$

$= \arctan(\tan(x))+C$

8. $\int_0^1 x^2 e^{-\frac{x^3}{3}} dx \quad u = -\frac{x^3}{3}, \quad du = -x^2 dx$

$= -\int_0^{-\frac{1}{3}} e^u du = -e^u \Big|_0^{-\frac{1}{3}} = -e^{-\frac{1}{3}} + e^0 = 1 - e^{-\frac{1}{3}}$

9. a) $\int_1^\infty \frac{x^3+1}{x^4} dx$

$= \int_1^\infty \frac{x^3}{x^4} dx + \int_1^\infty \frac{1}{x^4} dx = \int_1^\infty \frac{dx}{x} + \int_1^\infty x^{-4} dx$

$= \int_1^\infty x^{-4} dx = -\frac{x^{-3}}{3} \Big|_1^\infty = -\frac{1}{3x^3}\Big|_1^\infty = 0 - \left(-\frac{1}{3}\right) = \frac{1}{3}$

Перший = розбіжний => весь інтеграл розбіжн

б) $\int_0^5 \frac{dx}{\sqrt[3]{x-4}} \qquad x=0, \; x=4$

$= \int_0^4 \frac{dx}{\sqrt[3]{x-4}} + \int_4^5 \frac{dx}{\sqrt[3]{x-4}}$

$= \int_0^4 \frac{dx}{\sqrt[3]{x-4}} = \lim_{t \to 4^-} \int_0^t \frac{dx}{\sqrt[3]{x-4}} \qquad u = x-4 \quad du = dx$

$= \lim_{t \to 4^-} \int_0^t \frac{dx}{\sqrt[3]{x-4}} = \lim_{t \to 4^-} \int_{-4}^{t-4} \frac{du}{\sqrt[3]{u}} = \lim_{t \to 4^-} (3u^{\frac{2}{3}}) \Big|_{-4}^{t-4} =$

$= \lim_{t \to 4^-} (3(t-4)^{\frac{2}{3}} - (-32)) \qquad t \to 4^-, \; (t-4)^{\frac{2}{3}} \to 0 =$

$= \lim_{t \to 4^-} (3(t-4)^{\frac{2}{3}} - (-32)) = -32$

$= \int_4^5 \frac{dx}{\sqrt[3]{x-4}} = \lim_{t \to 0^+} \int_t^5 \frac{dx}{\sqrt[3]{x-4}}$

$= \lim_{t \to 0^+} \int_t^5 \frac{dx}{\sqrt[3]{x-4}} = \lim_{t \to 0^+} \int_{t-4}^1 \frac{du}{\sqrt[3]{u}} = \lim_{t \to 0^+} (3u^{\frac{2}{3}}) \Big|_{t-4}^1 =$

$= (3 - (-32)) = 35$

$= \int_0^5 \frac{dx}{\sqrt[3]{x-4}} = -32 + 35 = 3$

10. $y = x^2 \quad y = 2 - x^2$

$= x^2 = 2 - x^2, \quad 2x^2 = 2, \quad x^2 = 1 \quad \Rightarrow \quad x = -1, \; x = 1$

$= A = \int_{-1}^1 (2 - x^2) - x^2 dx \quad \int_{-1}^1 (2 - 2x^2) dx = 2x - \frac{2}{3}x^3 \Big|_{-1}^1 =$

$= A = \int_{-1}^1 (2 - x^2) - x^2 dx = \int_{-1}^1$

$$= \left(2 - \frac{2}{3}\right) - \left(-2 + \frac{2}{3}\right) = \frac{8}{3} \qquad = \frac{8}{3}$$