Physics / Mathematics Review

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August 2, 2014

Classical Physics

Rotational Motion

$$I = \sum m_i r_i^2 \qquad K = \frac{1}{2} I\omega^2 \qquad \tau = r \times F \qquad \tau = I\alpha$$

Gravitation

$F = G\frac{m_1 m_2}{r^2}$

- Shell Theorem : A uniform shell of matter exerts no net gravitational force on a particle located inside it.
- Things are heavier on the equator because, in addition to ma_g down, there is also the centripetal force of the earth turning on its axis.

Waves

$$d\sin(\theta) = m\lambda \qquad f' = f\frac{v \pm v_D}{v \pm v_S}$$

- Closed end pipe : standing waves occur when the wave hits the closed end at the middle (so there are an odd number of half periods (wave numbers)).
- Open end pipe : standing waves occur when the wave hits the open ends at the edge (so there are an even number of half periods (wave numbers)).

Heat

$$\Delta Q = cm\Delta T \qquad \text{where c is specific heat.}$$

- Two definitions for entropy: $S = \int_i^f \frac{dQ}{T}$, $S = kln(\frac{N!}{n_1!n_2!})$
- Ideal Gas Law: $PV = n\kappa T$
- Equipartition Theorem:

E & M

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s} \qquad \mathscr{E} = -\frac{d\Phi_B}{dt}$$

- Like with Gravity's shell theorem, a charged particle inside a shell of uniform charge has no net electrostatic force on it. Also, like with gravity's shell theorem, a charged particle outside the shell is attracted or repelled as if all the charge is at the center.
- Potential due to a point charge: $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$
- The electrical potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing in each charge from an infinite distance.
- An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current.
- Definition of inductance for a solenoid with N turns: L = $\frac{N\Phi_B}{i}$
- Self-induced emfs: $\mathscr{E}_L = -L\frac{di}{dt}$
- Impedance is a generalization of resistance that applies to RLC circuits.
- Magnetic Dipole moment (of a coil): $\mu = NiA\vec{n}$. So, it is normal to the coil's circle. $U = -\mu{\cdot}B$, $\tau = \mu XB$. where the torque attempts to align B and μ and the energy of the field is 'orientation energy' (potential energy stored in orientation).
- Electrical Dipole. The 'moment' vector, $\vec{p}$ points from the positive to negative point charge of the dipole. $\vec{\tau} = \vec{p}X\vec{E}$.

Maxwell's Equations

1. $\oint \vec{B} \cdot \vec{A} = \Phi_B = 0$: a mathematical statement that no magnetic monopoles exist.
2. $\oint \vec{E} \cdot \vec{A} = \frac{q_enc}{\epsilon_0}$
3. $\oint \vec{E} \cdot \vec{s} = -\frac{\Phi_B}{dt}$
4. $\oint \vec{B} \cdot \vec{s} = \mu_0\epsilon_0\frac{\Phi_E}{dt} + \mu_0 i_enc$

What is the electrical flux through? The area enclosed by the current loop.

Circuits

$q = CV$ is a property of all capacitors. The definition of a system's capicitance depends on the shape of the capacitor. Conventions: electric field lines point from + to - regions (so from one capacitor plate to another is an example).

Loop Rules

1. The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.
2. The sum of the currents entering any junction must be equal to the sum of the currents exiting the junction.

For capacitors in parallel: $C_e q = \sum_{i=0}^{n} C_i$ For capacitors in series: $\frac{1}{C_e q} = \sum \frac{1}{C_i}$ Energy stored in electric field of a capacitor: $U = \frac{q^2}{2C}$ Energy stored in the magnetic field of an inductor: $U = \frac{Li^2}{2}$ Dialetrics, physically manifested by stuff between the capacitor plates, cause any instance of ϵ_0 in an equation to $\epsilon_0 \kappa$. Definition of Resistance: $R = \rho \frac{L}{A}$ where ρ, resistivity, is a property of the material and L is length. Ohm's law is an approximation that holds well under low-heat conditions. $q_e nc = \epsilon_0 \oint \vec{E} \cdot d\vec{A}$ which is a surface integral.

RC Circuits

Charge builds up on the capacitor exponentially and decays with the 'sawtooth' pattern.

RL Circuits

When a switch is turned on, rather than the current across R approaching $\frac{\mathscr{E}}{R}$, the an opposing emf $-L\frac{i}{dt}$ is induced. As the current stops increasing, the circuit starts to act like an ordinary R circuit. The current expression involves solving the differential eqn.

LC Circuits

Whereas RL and RC circuits' current grows and decays exponentially, LC circuits' current varies sinusoidally. Like a block-spring system, there is a notion of driving frequency: $\omega = \frac{1}{LC}$

RLC Circuits

RLC circuits are like LC circuits but damped. Rather than $\frac{dU}{dt} = 0$, $\frac{dU}{dt} = -i^2 R$

- If an external emf is added, damping does not occur. Most AC circuits are RLC.
- Reactance (X) - the factor $V = XI$. Resistive, Capacitave, and Inductive Reactance exist.
- $X_C = \frac{1}{\omega_d C}$
- $X_L = \omega_d L$.
- $Z = \sqrt{R^2 + (X_L - X_C)^2}$. Makes sense because X_L and $-X_C$ are colinear and R is orthogonal to that.
- $< whatever >_R MS = \frac{<whatver>_max}{\sqrt{2}}$, provied that ¡whatever¿ varies sinusoidally.
- Transformers:

Special Relativity

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \qquad \delta t = \gamma \delta t_0 \qquad L = \frac{L_0}{\gamma} E \qquad = \gamma mc^2 = mc^2 + K \qquad \vec{p} = \gamma m \vec{v}$$

- Simultinaity is relative.
- When chemical reactions occur, the Q of the reaction is equal to the mass lost times c^2.

Optics

- Plane mirrors can be thought of as spherical mirrors with an infinitely long radius of curvature.
- For plane mirrors, i = -p.
- In a concave spherical mirror, incidently parallel light rays intersect at a real focus.
- In a convex sphereical mirror, incidently parallel light rays intersect at a virtual focus.
- For a spherical mirror, f = $\frac{r}{2}$. where r is radius of curvature and f is focal distance. (this assumes that the incident rays are equidistant from radius of curvature).
- $\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$, so when p = f, the virtual image is infinitely far away. So, when p ¡ f, i is negative. So, when p ¿ f, i is positive. As p approaches infinity, i = f.
- magnification for spherical mirror: $m = -\frac{i}{p}$.
- With mirrors, the real image is on the object's side and with lenses, the virtual image is on the object's side.
- Thin lens formula: $\frac{1}{f} = (n-1)(\frac{1}{r_1} - \frac{1}{r_2})$ where r_1 is the radius of curvature of the lens half on the object's side and r_2 is that on the other side.

Fluids

$F_b = m_f g$ where $m_f$is the mass of the fluid displaced.

Parameters and Constants

$$G = \quad \mu_0 = \quad e = 1.602X10^-19C \quad k = 1.381X10^-23J/K\epsilon_0 \quad = 8.854X10^-12F/m\mu_0 = 1.257X10^-6H/m \quad h = 6.63X10^-$$

Modern

Planck's Constant

For blackbodies, $E = hf$ ($f = v/\lambda$ for waves in general). For light, $E = \frac{hc}{\lambda}$ Definition of de Broglie wavelength: $\lambda = \frac{h}{p}$. $\vec{L} = \sqrt{l(l-1)}h$ and $\mu_o rb = -\frac{e}{2m}\vec{L}$ (same units as C/sm^2).

Photoelectric Effect

Metals emit electrons when light shines on them. $K_max = h(f - f_0)$ where f is the frequency of incident frequency of photon and f_0 is the threshhold at and below which no electron is emitted. This threshold is a property of the material.

Thompson Scattering

Uncertainty Principle

$\delta x \delta p \geq \frac{h}{2\pi}$.

Pauli Exclusion Principle

No two particles may have all the same quantum numbers in the same atom. (applies to fermions, e.g. electrons). This explains why electrons are repulsed from one another and why neutrons can simply clump together.

Quantum Numbers

Bosons have integer spin quantum numbers. Fermions have half integer spin quantum numbers. Fermions obey the pauli exclusion principle. Hadrons have the strong force acting on them. Leptons do not have the strong force acting on them. $Mesons ::= bosons \cup hadrons$, $baryons ::= fermions \cup hadrons$.

What is the significance?

- Elastic Collisions - In inelastic collisions, heat loss is a big enough factor that energy is not said to be conserved.
- Degrees of Freedom
- Blackbody Radiation
- Spin (quantum)
- Antiparticles - all reversible quantum numbers are reversed.
- Phasors: A construct that represents a wave. It is a vector of length equal to the maximum magnitude of the wave. It rotates about the origin with an angular speed equal to the angular frequency of the wave. The projection of the rotating phasor onto the vertical axis is equal to the displacement of a point on the wave from the wave's axis of symmetry. Phasors are useful for thinking about how waves add (vector addition laws apply).

Analogies and Connections

- Gauss' Law ¡=¿ Surface Integrals.
- Definition of Work ¡=¿ Line Integrals.
- Taylor Series ¡=¿ Motion with arbitrary rates of change.

Open Questions

Physical Correspondence How is it that capacitors are not like closed switches in a circuit? They do not make a physical connection between the sides of the wire.

Trig Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2 theta = \sec^2 theta$$

$$\sin u \pm v = \sin u \cos v \pm \cos u \sin v \qquad \cos u \pm v = \cos u \cos v \mp \sin u \sin v$$

Unit Identities

Fundamental Units

1. meter
2. second
3. columb
4. kilogram

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} N \qquad = kg\frac{m}{s^2} J \qquad = kg\frac{m^2}{s^2} E = N/C \qquad V = EmT \qquad = \frac{kg}{sC} Wb \qquad = Tm^2$$