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I'm planning to have a casual learning seminar on **Bruhat–Tits buildings** this semester (Spring 2025). The main goal is to understand why supercuspidal representations are "building blocks" of representations of p-adic groups, and how to construct them via Moy–Prasad filtration [13, 15], which is mainly done by Yu [1, 20, 7]. Especially, we will focus on representation theoretic perspective of the usage of the buildings. However, if time permits, we can talk about other kind of applications [10, 2, 12, 17, 14].

The only prerequisites are the representation theory of complex Lie algebra and some knowledge of local fields ($\mathbb{Q}_p$ could be enough). For people who are going to a desert for March 8-12, consider this as a $\sqrt{P}$AWS (something in between PAWS and AWS, mostly related to Fintzen's lectures).

There are several nice references, especially for beginners like me. Morris's course note [11] gives a historical introduction from a geometric point of view. I also heard that Rabinoff's senior thesis [16] is a great article in which you can learn many things (including all the preliminaries mentioned above), and also there are CDM & IHES lecture notes by Fintzen [6, 4]. We may use Kaletha–Prasad's "new approach" book as a main reference [10].

We'll meet once in every week, for maximum 1.5 hours[1]. Here is a list topics to be covered in order. Each of them would take one or two talks.

- Representation theory of $\mathrm{SL}_2(\mathbb{F}_p)$ and $\mathrm{SL}_2(\mathbb{Q}_p)$ (over $\mathbb{C}$) [9, 18]. Especially, give a classification of them. See Bump [3, Chapter 4] for a similar theory but on GL_2.

- Review prerequisites (algebraic groups, (affine) root systems, etc) [10, Part 1]

- Bruhat–Tits building of $\mathrm{SL}_n(\mathbb{Q}_p)$ [8] (maybe for $n = 2$, draw some trees... [19])

- Bruhat–Tits building of general reductive groups [10, Part 2]

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[1]I found that 1 hour is usually short for a talk.

- Moy–Prasad filtration [10, Chapter 13], originally [13, 15]

- Supercuspidal representations (Adler [1], Yu [20], *corrected* Yu [5, 7])

- Other applications

  - Classification of tori (maximal unramified / tamely ramified) [10, Chapter 16 and 17]

  - Volume formula [10, Chapter 18], originally by Prasad [14]

  - Local Langlands correspondence [2]

  - Analytic geometry [17]

  - Mostow strong rigidity theorem [12]

  - Anything else you like, but related to Bruhat–Tits bulidings

Please send me an email to seewoo5@berkeley.edu if you want to join, and also let me know if you have any suggestions (especially if you are already familiar with the materials!).

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