1) Areas Between Curves

1. Find the area of the region enclosed by the curves $y = x^2$ and $y = x + 6$.

2. Determine the area between $y = \sin(x)$ and $y = \cos(x)$ from $x = 0$ to $x = \frac{\pi}{2}$.

2) Average Value of a Function

3. Compute the average value of the function $f(x) = 2x^3 - 3x^2 + x$ on the interval $[1, 4]$.

4. Find the average value of $f(x) = \sqrt{x}$ on the interval $[0, 9]$.

3) Volumes of Solids

5. Use the disk method to find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = 0$ about the x-axis from $x = 0$ to $x = 2$.

6. Find the volume of the solid formed by rotating the curve $y = \sqrt{x}$, $0 \leq x \leq 4$, about the y-axis.

4) More Integration Problems

7. Find the area of the region bounded by $y = e^x$, $y = e^{-x}$, and $x = 1$.

8. Compute the volume of the solid obtained by rotating the region enclosed by $y = \ln(x)$, $y = 0$, and $x = e$ around the y-axis.

9. Determine the volume of the solid formed by rotating the region between the curves $y = x^3$ and $y = x$ around the line $y = x$.

Solutions to Worksheet on Applications of Integrals

November 3, 2023

Solutions

1) Areas Between Curves

1. The area of the region enclosed by $y = x^2$ and $y = x + 6$ is given by

$$\int_{-2}^{3} [(x+6) - x^2]\, dx = \left[\frac{x^2}{2} + 6x - \frac{x^3}{3}\right]_{-2}^{3} = \frac{125}{6}.$$

2. The area between $y = \sin(x)$ and $y = \cos(x)$ from $x = 0$ to $x = \frac{\pi}{2}$ is

$$[\sin(x) + \cos(x)]_0^{\frac{\pi}{4}} + [-\cos(x) - \sin(x)]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \sqrt{2}.$$

2) Average Value of a Function

1. The average value of $f(x) = 2x^3 - 3x^2 + x$ on $[1, 4]$ is

$$\frac{1}{4-1}\left[\frac{x^4}{2} - x^3 + \frac{x^2}{2}\right]_1^4 = \frac{125}{6}.$$

2. The average value of $f(x) = \sqrt{x}$ on $[0, 9]$ is

$$\frac{1}{9}\left[\frac{2}{3}x^{3/2}\right]_0^9 = 2.$$

3) Volumes of Solids

1. The volume of the solid obtained by rotating $y = x^2$ about the x-axis from $x = 0$ to $x = 2$ is

$$\pi\left[\frac{x^5}{5}\right]_0^2 = \frac{32\pi}{5}.$$

2. The volume of the solid formed by rotating $y = \sqrt{x}$, $0 \le x \le 4$, about the y-axis is

$$\pi\left[\frac{x^3}{3}\right]_0^4 = \frac{64\pi}{3}.$$

4) More Integration Problems

1. The area of the region bounded by $y = e^x$, $y = e^{-x}$, and $x = 1$ is

$$2\left[e^x + e^{-x}\right]_0^1 = 2(e + \frac{1}{e} - 2).$$

2. The volume of the solid obtained by rotating the region enclosed by $y = \ln(x)$, $y = 0$, and $x = e$ around the y-axis is

$$2\pi\left[\frac{x^2}{2}\ln(x) - \frac{x^2}{4}\right]_1^e = \frac{\pi e^2}{2}.$$

3. The volume of the solid formed by rotating the region between the curves $y = x^3$ and $y = x$ around the line $y = x$ is

$$\pi\left[\frac{(x^3 - x)^2}{2}\right]_0^1 = \frac{\pi}{2}.$$