Math 53 (Multivariable Calculus), Section 102 & 108
Week 2, Friday
Sep 2, 2022
For the other materials: seewoo5.github.io/teaching/2022Fall

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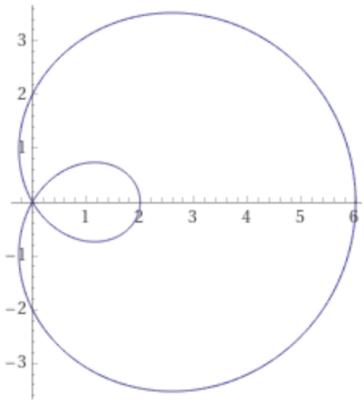
1. Find a polar equation for the following curves.
   (a) $x^2 - y^2 = 1$ (use $\cos 2\theta = \cos^2\theta - \sin^2\theta$)
   (b) $x^2 + (y - 1/2)^2 = 1/4$.
2. Assume that we have a following curve in polar equation $r = f(\theta)$.



   Sketch the curves with the following polar equations.
   (a) $r = \frac{1}{2} f(\theta)$
   (b) $r = f(\theta + \pi/2)$
   (c) $r = f(\pi - \theta)$

Solution

1. (a)

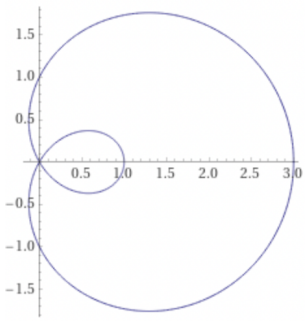
$$x^2 - y^2 = (r\cos\theta)^2 - (r\sin\theta)^2 = r^2(\cos^2\theta - \sin^2\theta) = r^2\cos 2\theta = 1 \Leftrightarrow r^2 = \frac{1}{\cos 2\theta}$$

(b)

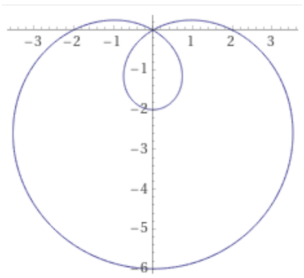
$$\begin{aligned} x^2 + \left(y - \frac{1}{2}\right)^2 &= r^2\cos^2\theta + \left(r\sin\theta - \frac{1}{2}\right)^2 \\ &= r^2\cos^2\theta + r^2\sin^2\theta - r\sin\theta + \frac{1}{4} \\ &= r^2 - r\sin\theta + \frac{1}{4} = \frac{1}{4} \\ &\Leftrightarrow r = \sin\theta \end{aligned}$$

(Compare this to the Problem 3 of the worksheet for August 22nd.)

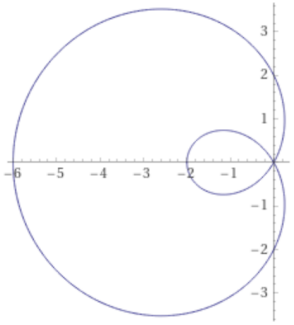
2. Note that the original curve's polar equation $r = 2 + 4\cos\theta$.
   (a) It is shrinked by half.
   (b) It is rotated by 90 degree, clockwise.
   (c) It is reflected over the y axis.



(a)



(b)



(c)