1. (a) How many 4 digit numbers that starts with 3?
   (b) How many ways are there to put 10 indistinguishable balls into 5 distinguishable boxes?
   (c) You randomly shuffle a standard deck of 52 poker cards, and draw 4 cards from it. What is the probability that you get all 4 aces?
   (d) How many integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ where $x_1 \geq 1$, $x_2 \geq 2$, $x_3 \geq 3$, $x_4 \geq 4$, and $x_5 \geq 5$?
   (e) What is the probability that when you roll a fair die 8 times, you never get a multiple of 3?
   (f) How many numbers must be selected from the set $\{1,2,3,4,5,6,7,8,9,10\}$ to guarantee that at least one pair of these numbers add up to 7?

2. (a) How many strings of length 10 with five upper case letters and five lower case letters, where no two upper case letters are adjacent and no two lower case letters are adjacent?
   (b) How many nonnegative integer solutions for the inequality $x_1 + x_2 + x_3 + x_4 \leq 12$?
   (c) Roll a die three times. What is the probability to get the total outcome of 7?
   (d) Show that whenever 5 cats and 5 dogs are seated around a circular table there is always an animal both of whose neighbors are cats.

3. Suppose that I flip a coin six times.
   (a) What is the likelihood that I receive exactly two heads?
   (b) Is it more likely to receive exactly two heads if the first flip is heads, or if it is tails?
   (c) Explain intuitively why the one you chose is more likely.

4. (a) Let there be two independent events E and F, with probabilities $P(E) = 0.5$ and $P(F) = 0.4$. What is $P(E \cup F)$?
   (b) If E and F are not necessarily independent, what is the possible range of $P(E \cup F)$?