

**Math 53 (Multivariable Calculus), Section 102 & 108**

**Week 10, Monday**

**Oct 24, 2022**

**For the other materials: [seewoo5.github.io/teaching/2022Fall](https://seewoo5.github.io/teaching/2022Fall)**

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1. Let  $D$  be a rectangle defined by the inequalities  $0 \leq x \leq a$  and  $0 \leq y \leq b$ . Assume that it has a uniform density  $\rho(x, y) = 1$ .

- (a) Guess the center of the mass of  $D$ . Check that your guess is correct.
- (b) Find the moments of inertia  $I_x$ ,  $I_y$ , and  $I_0$ . Compare them.
- (c) For given  $h$  and  $k$ , let  $D'$  be the translated rectangle

$$D' = \{(x, y) : -h \leq x \leq a - h, -k \leq y \leq b - k\}.$$

Find the moment of inertia  $I = I(h, k)$  about the origin of  $D$ , as a function in  $h$  and  $k$ . (Hint: you can directly compute it, or you can use the result from (b).)

- (d) When  $I(h, k)$  is minimized? Could you guess it before do the computation?