

1. (Conditional probabilities) Consider a standard deck of 52 cards. Shuffle the deck and draw two cards from it.
 - (a) What is the probability of having two Queens?
 - (b) What is the probability of having two Queens, given that the first card is a queen?
 - (c) What is the probability of having two Queens, given that the second card is a queen?
 - (d) What is the probability of having two Queens, given that the two cards have the same number/alphabet?
 - (e) What is the probability of having two Queens, given that the first card has an alphabet on it (one of A, J, Q, K)?

2. (Bayes rule) Suppose that 4% of the patients tested in a clinic are infected with avian influenza. Furthermore, suppose that when a blood test for avian influenza is given, 97% of the patients infected with avian influenza test positive and that 2% of the patients not infected with avian influenza test positive. What is the probability that
 - (a) a patient testing positive for avian influenza with this test is infected with it?
 - (b) a patient testing positive for avian influenza with this test is not infected with it?
 - (c) a patient testing negative for avian influenza with this test is infected with it?
 - (d) a patient testing negative for avian influenza with this test is not infected with it?

3. (Bayesian learning) You have a coin, possibly unfair. It lands with head with probability $0 < p < 1$, and our goal is to estimate the probability from 100 flips.
- (a) Assume that we get 65 heads among 100 flips. What is a likelihood, as a function in p ?
 - (b) If you need to guess p , what would be your choice?
 - (c) Let $f(p)$ be the function in (a). Find the derivative $f'(p)$ (Hint: you may remember keywords from 10A like product rule, logarithmic derivative, ...).
 - (d) Find p with $f'(p) = 0$, which maximizes $f(p)$.
 - (e) Compare the answer for (d) with with your guess in (b).