

**Math 53 (Multivariable Calculus), Section 102 & 108**

**Week 12, Monday**

**Nov 7, 2022**

**For the other materials: [seewoo5.github.io/teaching/2022Fall](https://seewoo5.github.io/teaching/2022Fall)**

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1. Let  $\mathbf{F}(x, y, z) = y^2 z^3 \mathbf{i} + 2xyz^3 \mathbf{j} + 3xy^2 z^2 \mathbf{k}$ . If  $C$  is a line segment from  $(0, 0, 0)$  to  $(1, 1, 1)$ , find  $\int_C \mathbf{F} \cdot d\mathbf{r}$ .

2. Let

$$\mathbf{F}(x, y) = (2xye^{x^2y} + 1)\mathbf{i} + (x^2e^{x^2y} + 1)\mathbf{j}.$$

(a) Is  $\mathbf{F}$  conservative? If it is, find  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  such that  $\nabla f = \mathbf{F}$ .

(b) Let  $C$  be a curve parametrized by  $\mathbf{r}(t) = (\cos t, \sin t)$  for  $0 \leq t \leq \pi/2$ . Find

$$\int_C (2xye^{x^2y} + 1 - y)dx + (x^2e^{x^2y} + 1 + x)dy.$$

(Hint: express the vector field in the integral as  $\mathbf{F} + \mathbf{G}$  for some vector field  $\mathbf{G}$ .)