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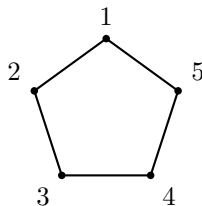
- All the group actions in this worksheet are *left* actions, i.e. it is a map $G \times X \rightarrow X$, not $X \times G \rightarrow X$. In particular, dihedral group D_n acts on a regular n -gon by left action; if r is a rotation and s is a reflection, then rs means we first reflect, then rotate. I'll update the previous worksheets accordingly.

1. Let G be a finite group and H be a normal subgroup of G . What is a relation between the order of $g \in G$ and the order of the element $gH \in G/H$? Try to play with $G = \mathbb{Z}_4 \times \mathbb{Z}_6$ and $H = \langle (1, 3) \rangle$.
2. The symmetric group S_4 acts on the set $X = \{1, 2, 3, 4\}$ by permuting the elements.
 - (a) Let $g = (23)$. Which element of X fixed by g ?
 - (b) Let $x = 1$. What is the stabilizer (isotropy) subgroup G_x ?
 - (c) What is the orbit of x ? Check that the orbit-stabilizer theorem holds.
 - (d) What can you say about S_n acting on $\{1, 2, \dots, n\}$?
3. Find the order of the left coset σA_{100} in S_{100}/A_{100} , where

$$\sigma = (1\ 2\ 3\ 4\ 5)(6\ 7\ 8)(9\ 10)(11\ 12\ 13\ 14)(15\ 16\ 17\ 18\ 19\ 20).$$

(Hint: I'm not a mean person.)

4. Consider D_5 , the dihedral group of order 10. Here is a picture of a regular pentagon.



Let r be the counterclockwise rotation by 72° . Let s be the reflection about the vertical axis.

- (a) Write r , s , and rs as a permutation of the vertices $\{1, 2, 3, 4, 5\}$.
 - (b) What is the stabilizer group of vertex 1?
 - (c) What is the orbit of vertex 1?
 - (d) Check that the orbit-stabilizer theorem holds.
 - (e) Show that $\langle r \rangle$ is a normal subgroup of D_5 .
 - (f) What can you say about D_n acting on a regular n -gon?
5. (*) Consider a solid cube. Let G be the group of rotational symmetries of the cube (i.e. no reflections), which can be regarded as a subgroup of S_8 by considering the action of G on the set of vertices of the cube. What is the order of G ? In fact, it is isomorphic to a group you already know. Can you find it?