Math 53 (Multivariable Calculus), Section 102 & 108
Week 11, Friday
Nov 4, 2022
For the other materials: seewoo5.github.io/teaching/2022Fall

---

1. Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.
   (a) $\mathbf{F}(x, y) = y^2 e^{xy}\mathbf{i} + (1 + xy)e^{xy}\mathbf{j}$
   (b) $\mathbf{F}(x, y) = \sin(x + y)\mathbf{i} + \cos(x - y)\mathbf{j}$

2. (a) Let
$$\mathbf{F}(x, y) = (\cos(xy) - xy\sin(xy))\mathbf{i} + (-x^2\sin(xy))\mathbf{j}.$$
   Find $f(x, y)$ such that $\mathbf{F} = \nabla f$.
   (b) Using (a), compute
$$\int_C (\cos(xy) - xy\sin(xy) + x)dx + (-x^2\sin(xy) + y)dy$$
   where C is an arc from $(-1, 0)$ to $(1, 0)$, along the unit circle $x^2 + y^2 = 1$. (Hint: decompose the integral into sum of two line integrals.)