

Math 53 (Multivariable Calculus), Section 102 & 108

Week 11, Wednesday

Nov 2, 2022

For the other materials: seewoo5.github.io/teaching/2022Fall

1. Find the gradient vector field ∇f of f and sketch it.

(a) $f(x, y) = \frac{1}{2}(x^2 - y^2)$

(b) $f(x, y) = \ln \sqrt{x^2 + y^2}$

2. Compute the following line integrals.

(a) $\int_C y ds$, where $C : (x(t), y(t)) = (t^2, 2t)$, $0 \leq t \leq 3$.

(b) $\int_C \mathbf{F} \cdot d\mathbf{r}$, where \mathbf{F} is a vector field

$$\mathbf{F}(x, y) = \left(-\frac{y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right)$$

and C is a circle $x^2 + y^2 = a^2$ with parametrization given by $\mathbf{r}(t) = (a \cos t, a \sin t)$, $0 \leq t \leq 2\pi$.