

**Math 53 (Multivariable Calculus), Section 102 & 108**

**Week 6, Wednesday**

**Sep 28, 2022**

**For the other materials: [seewoo5.github.io/teaching/2022Fall](https://seewoo5.github.io/teaching/2022Fall)**

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1. Graph the functions.

(a)  $f(x, y) = \sqrt{x^2 + y^2}$

(b)  $f(x, y) = e^{\sqrt{x^2 + y^2}}$

(c)  $f(x, y) = \ln(x^2 + y^2)$

What do they have in common? Can you also graph the following function?

$$f(x, y) = \sqrt{1 - (\sqrt{x^2 + y^2} - 2)^2}$$

2. Consider the following limit:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^3}{x^4 + y^6}$$

(a) Show that the limit along line  $y = mx$  exists and the same for all  $m$ .

(b) Show that, for any positive integer  $n$ , the limit along  $y = mx^n$  exists and the same for all  $m$ .

(c) Show that the limit does not exist.