Signal Filter

- Signals in Time and Frequency:
  - A signal is just a value that changes over time. But looking only in terms of time hides what actually makes the shape. Every signal can also be viewed as a mixture of simpler waves/sinusoids (frequencies). This second view makes structure visible.
    - $$x[n] \leftrightarrow X[k]$$
  - The DFT turns a time signal into a list of frequency strengths:
    - $$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}$$
- Why frequency helps us understand signals
  - Sharp edges = lots of frequencies
  - Smooth waves = mostly low frequencies
  - Noise = many frequencies
    - Frequency is like seeing the ingredients of the signal, the visualiser shows these ingredients.
- Filtering as choosing what to keep
  - If a signal is made of ingredients, a filter simply just chooses which ones stay.
    - Low-pass: keep slow waves = smooth output
    - High pass: keep fast waves = highlight details
    - Band-pass: keep a specific range
  - $$Y[k] = H[k]X[k]$$
    - Where $H[k]$ is just a mask of 0s and 1s.
      - This is the entire basis of digital filtering.
- Reconstructing the filtered signal
  - Once selected, the remaining frequencies are added back together to rebuild the time signal.
    - $$y[n] = \frac{1}{N} \sum_{k=0}^{N-1} Y[k] e^{j2\pi kn/N}$$

            - The visualiser shows this happening instantly as you toggle filters.
- How sampling affects frequency
    - A digital signal is just a list of samples.
    - The sampling rate determines:
        - The maximum frequency you can represent.
        - The spacing between frequency bins.
            - This explains why the spectrum goes from 0 to $\frac{N}{2}$.
        - $$f_{\text{Nyquist}} = \frac{f_s}{2}$$
            - This is the Nyquist-Shannon sampling theorem, frequencies above f_{Nyquist} will fold back, causing aliasing.
- Why edges create high frequencies
    - Sudden jumps require rapid oscillations to approximate them.
    - Removing high frequencies smooths the signal because it removes the oscillations that try to "build" those corners.
        - This concept builds the foundations of both DSP and Fourier.
- Ideal vs Real filters
    - Real filters have limits.
    - Ideal filters instantly cut frequencies, which is perfect for demonstration but not physically possible or realisable.
        - Ideal low-pass filter's impulse response:
            - $$h[n] = \sin\mathrm{c}(x)$$
- Why this matters in engineering
    - Filtering is used in:
        - communications (noise removal, channel shaping)
        - audio processing
        - antennas and RF systems
        - ECG and biomedical filtering
        - control systems
        - data processing