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Homework 2

***Abstract*—This document contains the solutions, explanations, comments, and related graphs and tables of the questions in the homework 2.**

I. Cameras and Sampling

A. Camera

The coordinates of the vertices can be determined by using the principal of similar triangles. The basic perspective projection in a camera is shown in Figure 1.[1]

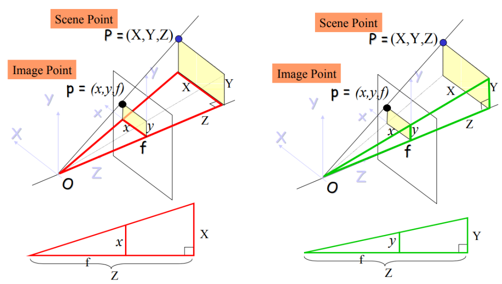

Fig. 1. The basic perspective projection in a camera[1]

$$x' = \frac{f}{Z}X \; and \; y' = \frac{f}{Z}Y \tag{1}$$

So, the coordinates of the corners can be found by using the formula 1 and they are shown in the following expression.

$$\begin{aligned} &P_1 \; x' = \frac{-1}{100}100 = -1cm \; y' = \frac{1}{100}100 = 1cm \\ &P_2 \; x' = \frac{1}{200}400 = 2cm \; y' = \frac{1}{200}200 = 1cm \\ &P_3 \; x' = \frac{-1}{300}300 = -1cm \; y' = \frac{1}{300}1200 = 4cm \end{aligned} \tag{2}$$

$$P_1' = (-1,1,1) \; P_2' = (2,1,1) \; P_3' = (1,4,1)^{2} \tag{3}$$

B. Sampling

I: The $24cos(4\pi x)$ part of the pattern governs the horizontal cycles. Therefore, in calculating the horizontal cycles per meter, this part should be considered; so, the pattern is periodic in horizontal axis with $\frac{1}{2}cm$.[3]The frequency is $2cm^{-1}$, and then the pattern has $100cm * 2cm^{-1} = 200$ many horizontal cycles per meter.
The $24cos(6\pi y)$ part of the pattern governs the vertical cycles. Therefore, in calculating the vertical cycles per meter, this part should be considered; so, the pattern is periodic in vertical axis with $\frac{1}{3}cm$.[3] The frequency is $3cm^{-1}$, and then the pattern has $100cm * 3cm^{-1} = 300$ many horizontal cycles per meter.

II: The frequencies along 0 and 90 degrees are straightforward since they correspond to horizontal and vertical frequencies, respectively. However, both horizontal and vertical frequencies contribute to the frequencies along 30 and 45 degrees. They are calculated by using the periods of the horizontal and vertical parts. In order to have a periodicity along an angle θ, the following condition should be hold,

$$tan(\theta) = \frac{T_v * n_y}{T_h * n_x} = \frac{2}{3}\frac{n_y}{n_x} \tag{4}$$

, n_x and n_y are the number of cycles (being integer) in the horizontal and vertical axis, respectively. So, the frequency along an angle θ (if a periodicity exists) can be calculated by the following expression:

$$f_\theta = \left((T_h * n_x)^2 + (T_v * n_y)^2\right)^{-\frac{1}{2}} \tag{5}$$

The pattern is not periodic along 30 degree; since no integer n_x and n_y values yield the equation 4.
The frequencies along 0, 45, and 90 are given in the following expression:

$$\begin{aligned} &f_0 = f_h = 2cm^{-1} \\ &f_{45}{}^{4} = \left((\frac{2}{2})^2 + (\frac{3}{3})^2\right)^{-\frac{1}{2}} = 0.707cm^{-1} \\ &f_{90} = f_v = 3cm^{-1} \end{aligned} \tag{6}$$

III: The pattern can be digitally represented, by using only one cycle of the pattern since it is periodic with respect to horizontal and vertical axis. And, the horizontal and vertical axis can be examined, separately since for sampling two-dimensional impulse train is used. The Nyquist rate is equal to $2 * B$, B is the bandwidth of the pattern in the frequency domain. Then, the Nyquist rate for the horizontal axis is $f_{nh} = 2 * 2 = 4cm^{-1}$, and the Nyquist rate for the vertical axis is $f_{nv} = 2*3 = 6cm^{-1}$

IV: If the pattern is sampled at Nyquist rate, then there is no space between the sampled patterns. Therefore, the side of the square on the horizontal axis contains $f_{nh} * 100cm = 400$ many samples; and the side of the square on the vertical axis contains $f_{vh} * 100cm = 600$ many samples. So, the total number samples are the multiplication of these two values 240.000.

[1]The image plane can be behind the center but this setup also gives the same coordinates with the opposite signs.
[2]They are all in cm unit.
[3]It is assumed that x and y are in cm unit.
[4]x and y values for $\theta = 45$ are 2 and 3, respectively.

II. Histogram Equalization

The histogram equalization function is implemented by using the method that is covered by the instructor. First, the probability distribution of the pixel values are calculated, and then the cumulative probabilities are assigned to the corresponding pixel values.
The function is used to equalized the histograms of the images and their histograms given by the Figures 2, 3. The output images of the function and their histograms are given in the Figures 4, 5.

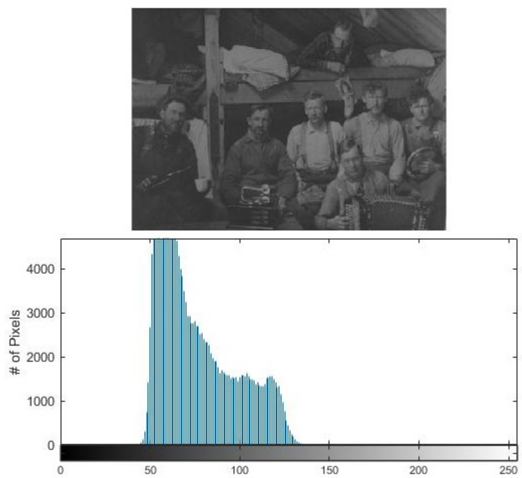

Fig. 2. The original image of Lumberjack and its histogram

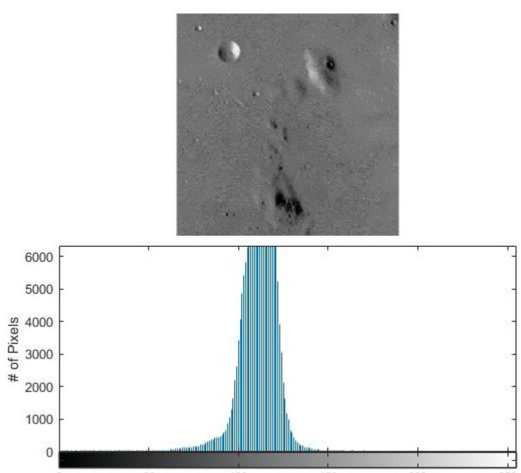

Fig. 3. The original image of moon and its histogram

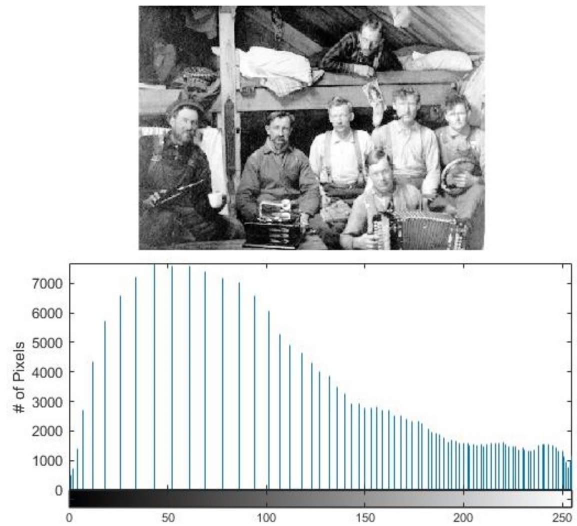

Fig. 4. The output Lumberjack image of the function and its histogram

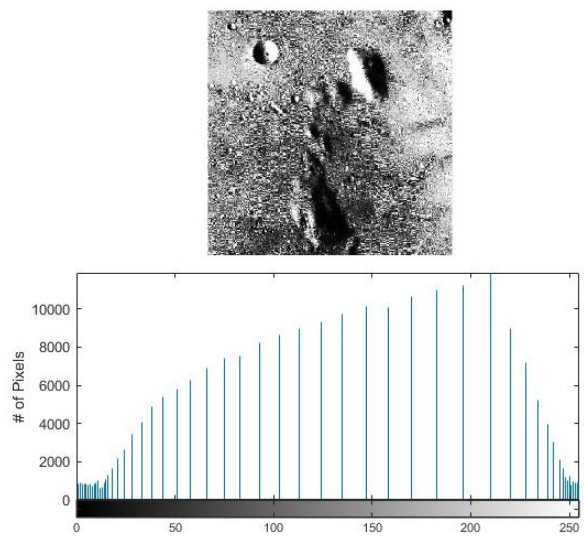

Fig. 5. The output moon image of the function and its histogram

Also, in MATLAB, there exists a function called $histeq$ which is able to equalize a given image's histogram. This function can be used to check the validity of the function that is implemented by method covered in the lecture. The output images of $histeq$ and their histograms are given in the Figures 6, 7.

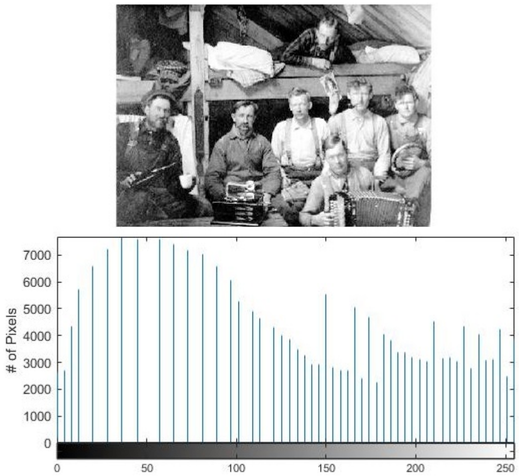

Fig. 6. The output Lumberjack image of $histeq$ and its histogram

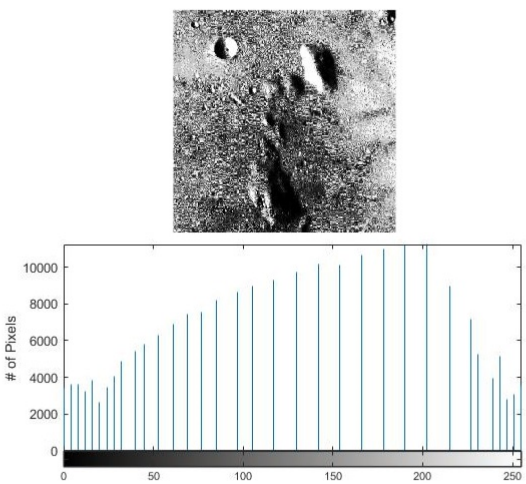

Fig. 7. The output moon image of $histeq$ and its histogram

It can be seen from the Figure 6, 7 that the implemented function works fine, except in the histogram, there is a little difference at the white region. The implemented function is not able to equally distribute the bins of the histogram. However, it can stretch the histogram of an image on the gray level. In this content, ordinary means non-existence of some instant changes in the histogram values.

For example, the moon image cannot be described as ordinary since almost all the histogram values are gathered in a few gray level. Therefore, the implemented function is not so successful in the moon image. Namely, it disrupts the histogram of the image when it tries to make it more contrast.

In order to solve this problem, an adaptive function can be implemented in a way that it does the equalization locally, to all pixels. This modification enables the function to take into consideration of the local change. Basically, it divides the image into sub-regions, and makes the equalization separately. There is a function called $adapthisteq$ in MATLAB that is able to apply the described functionality. The output images of $adapthisteq$ and their histograms are given in the Figures 8, 9.

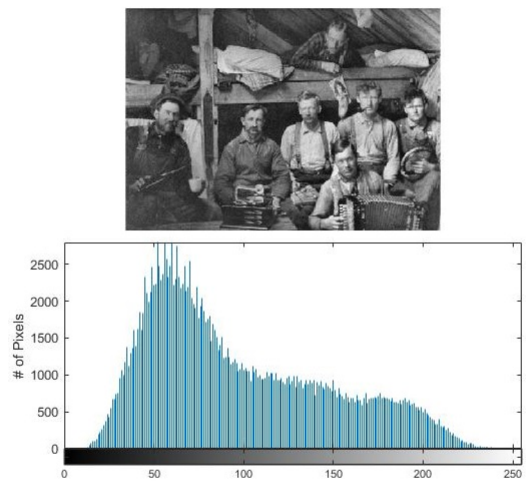

Fig. 8. The output Lumberjack image of $adapthisteq$ and its histogram

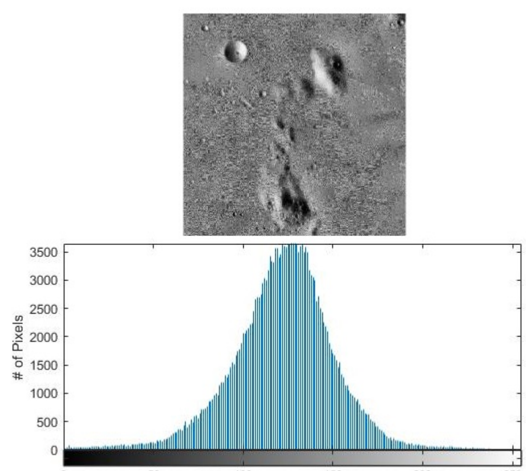

Fig. 9. The output moon image of $adapthisteq$ and its histogram

It is obvious that the adaptive function gives better results. The improvement in the Lumberjack image and the moon image can be observed by just looking. The

reason for the improvement is that it deals with the image's histogram locally, the chance of obtaining an instant change in the histogram is so low in the sub regions. Therefore, it can achieve better results.
However, there is a drawback of this modification. It can overlook the image as a whole; namely, it is not able to consider the histogram of the whole image. An example for this failure can be an image of chess board. The algorithm would fail if the sub-regions are being overlapped with black and white squares. Also, it can amplify the noise in the sub-regions whose pixel values are close to each other. Namely, it can amplify the noise in the homogeneous sub-regions.

III. Histogram Flattening and Matching

The equalization of the four face images that is given in the Figure 10 is done by using the function that is implemented by previous part. However, there is some modification in the function; since the images are colored, the RGB components of the images are equalized separately. And also, in formation of the histogram, the background (having value 255 in the gray level) and black line (having value 0 in the gray level) are ignored. Namely, they are excluded in the calculation of the cumulative probabilities; at the end of the process, the values of the background are made equal to 255 and the values of the black regions are made equal to 0.
The equalized images of the faces are given in the Figure 11.

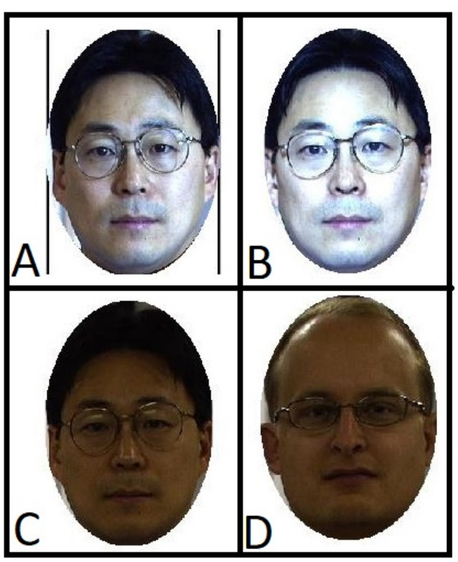

Fig. 10. The original images of the four faces

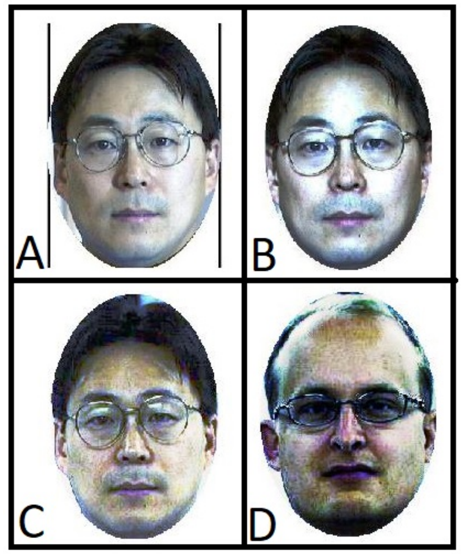

Fig. 11. The equalized images of the four faces

In order to match the histograms of the face images with image A, the $\tilde{J}$ matrices of the images (It is the matrix that is obtained before the last step of equalization; it contains the cumulative probabilities of each pixel) are formed; and they are mapped into the inverse cumulative probability of image A, using the method covered in the lecture.
The matched images of four faces with image A using their histogram are given in the Figure 12.

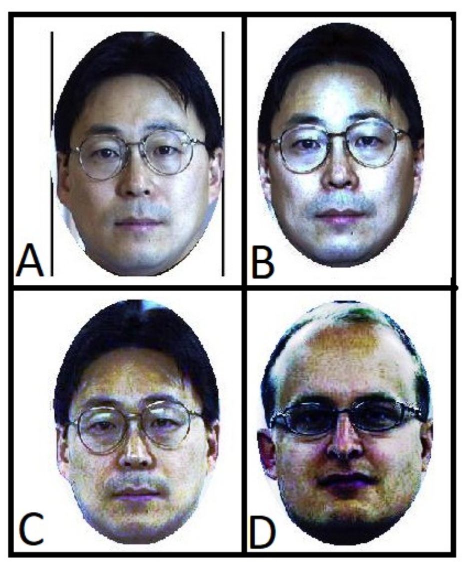

Fig. 12. The matched images of the four faces with image A

The histogram distances can be used to determine how well the algorithm accomplished histogram matching. The two of the histogram distances are the chi-square histogram distance given in the equation 7 and the Kullback-Leibler distance given in the equation 8.[5]

$$\chi_{p,q} = \sqrt{\frac{1}{2}\sum_{i=0}^{L-1}\frac{|p(i)-q(i)|^2}{p(i)+q(i)}} \tag{7}$$

$$KL_{pq} = \frac{1}{2}\left[\sum_{i=0}^{L-1} p(i)log_2(\frac{p(i)}{q(i)}) + q(i)log_2(\frac{q(i)}{p(i)})\right] \tag{8}$$

Since the images are colored, the distances are calculated as the average value of the each RGB component distance.
The distance values between original and equalized images, original and matched images are given in the following tables.

The chi-square distance with image A				
Images	A	B	C	D
Original	0	0.31	0.64	0.62
Equalized	0.42	0.46	0.72	0.70
Matched	0.18	0.40	0.67	0.63

The Kullback-Leibler distance with image A				
Images	A	B	C	D
Original	0	0.35	2.04	1.74
Equalized	1.21	1.38	3.27	2.89
Matched	0.18	1.00	2.85	2.38

It can be seen from the tables that the matched images are closer to the image A than its equalized version image. Since the equalization causes a major change in the image histogram; and the matching process tries to make the image histogram to the reference image.
Another observation can be that the original images are closer to image A than the matched images. This is due to the fact that when implementing the equalization, the image histogram becomes so much farther to the image A.

IV. Histogram Equalization of Color Images

A. The RGB Eqaualization

The histogram equalization can be applied to a color image by equalizing the RGB components of the image separately. This implementation is applied to the kugu and beach images 13, 14. And the equalized images are given in the Figure 15, 16.
The equalization gives good results for the images since the details in the images are more visible such as the background in the beach image and the leafs on the right-top in the kugu image. It can be seen that contrast of the images are increased. However, the equalized images look eye-straining.

Fig. 13. The original kugu image

Fig. 14. The original beach image

Fig. 15. The equalized kugu image

Fig. 16. The equalized beach image

B. The Intensity Equalization

Another equalization can be done to color images such as the RGB component (Red Green Blue) of images can be converted to HSV (Hue Saturation Value) component, and made the equalization to one of the HSV components. In this part, only the intensity component is equalized while freezing the hue and saturation components. The intensity equalized images are given in the Figure 17, 18.

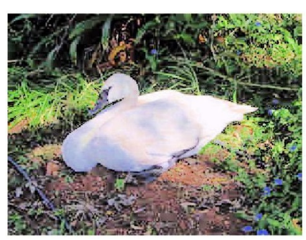

Fig. 17. The intensity equalized kugu image

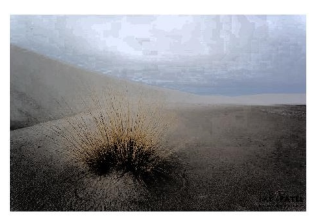

Fig. 18. The intensity equalized beach image

[5] $L-1$ is the highest gray value; p and q are the probability densities of the histogram.

C. The Saturation Equalization

In this part, only the saturation component is equalized while freezing the hue and intensity components. The saturation equalized images are given in the Figure 19, 20.

Fig. 19. The saturation equalized kugu image

Fig. 20. The saturation equalized beach image

D. Comments about the three Equalization

The RGB histogram equalization can be used to determine the details namely, to increase the contrast of the image. However, it doesn't give very presentable image. The intensity equalization can be used to better contrast images in comparison to the RGB equalization, it makes the color change more smoothly; however, if the colors of the pixels are so close to each other, then the equalization might give poor results. The saturation equalization can be used to determine the boundaries of the different color in the image. For example, in the beach image, this equalization can differentiate the boundary between the sea and the beach.

References

[1] Szeliski, R. (2019). [online] Available at: https://www.iitr.ac.in/departments/MA/uploads/Unit%202%20lec-1.pdf [Accessed 12 Oct. 2019].