IE 440 Fall 2019 - Nonlinear Models in Operations Research

# Homework # 6

*due December $22^{th}$, Sunday, 23:59.*

In this homework, you will be given two data sets that includes the $x$ and $y$ values. Your task is to fit a function to the given data in the file *"training.dat"*.

1. **Least Square Method:**

   (a) Use a Linear Regression Model: $\mathbf{y} = w_1\mathbf{x} + w_0$.

   (b) Use a Polynomial Regression Model: $\mathbf{y} = w_2\mathbf{x^2} + w_1\mathbf{x} + w_0$.

   For these two methods, your task is to determine least square estimators of (a) and (b) by solving these optimization problems respectively:

$$\min_{w_0,w_1} \sum_{i=1}^{N_{tra}} (y_i - w_0 - w_1 x_i)^2 \qquad\qquad \min_{w_0,w_1,w_2} \sum_{i=1}^{N_{tra}} (y_i - w_0 - w_1 x_i - w_2 x_i^2)^2$$

   where $(x_i, y_i)$ is the $i^{th}$ data pair.

   You can use steepest descent with exact line search and thus benefit from your past programs.

2. **Nonlinear Regression:**

   (a) Design a multi-layer perceptron with one output unit, one input terminal and one hidden layer using the training data given in file *"training.dat"* and determine the number of hidden units using the test data given in file *"test.dat"*. The perceptron is illustrated in Figure 1.

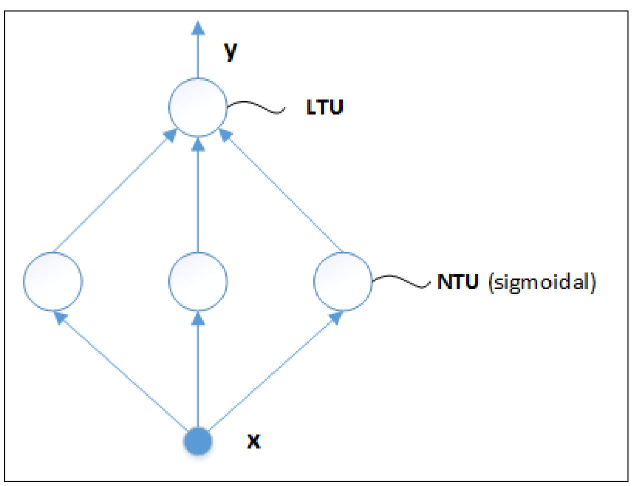


Figure 1: Sample perception for 2.a.

   (b) Design a multi-layer perceptron with one output unit, two input terminals (one for $x$ and one for $x^2$) and one hidden layer using the training data given in file *"training.dat"* and determine the number of hidden units using the test data given in file *"test.dat"*. The perceptron is illustrated for three hidden units in Figure 2.

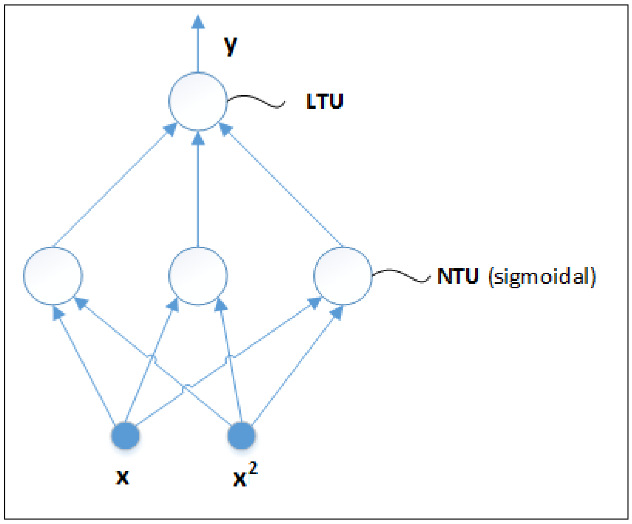


Figure 2: Sample perception for 2.b.

For both cases set $\alpha^{(0)} = 0.5, \eta = 0.9, \epsilon = 0.001, J^{(0)} = 3$ (i.e. initial number of hidden units), and assume that the output neuron is a linear threshold unit (i.e. $g(h) = h$) and each hidden neuron is a nonlinear threshold unit with sigmoidal activation factor $g(h) = \frac{1}{1+e^{-h}}$.

Report for all cases:

| Method | Training SSE | Test MSE | $s^2$ for Test MSE |
|---|---|---|---|
| 1.(a) | $\sum_{i=1}^{N_{tra}} e_i^2$ | $\frac{1}{N_{test}} \sum_{i=1}^{N_{test}} e_i^2$ | $\frac{1}{N_{test}-1} \sum_{i=1}^{N_{test}} (TestMSE - e_i^2)^2$ |
| 1.(b) | ... | ... | ... |
| 2.(a) | ... | ... | ... |
| 2.(b) | ... | ... | ... |

where $e_i^2 = (y_i - \theta_i)^2$ is the error term of the related data ($\theta_i$ is the estimated value).

- Plot the training data and the regression functions.
- Plot the test data and the regression functions.
- Comment on the results for training and test errors.

Include the screen shots of your outputs and your source codes in your report. Submit the soft copy of your report and the source code which are named as **HW6-GroupID** to moodle.