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Homework 5

EE 473
Fall 2019

Question 2

Part a) Elliptic frequency-selective filters can be used to design a bandpass filter. The Figure 1 shows the frequency response magnitude of an eighth-order elliptic filter with $\delta = 0.2$ dB ripple in the passband $0.41\pi \leq |w| \leq 0.47\pi$, and has 40 dB attenuation in the stopband.

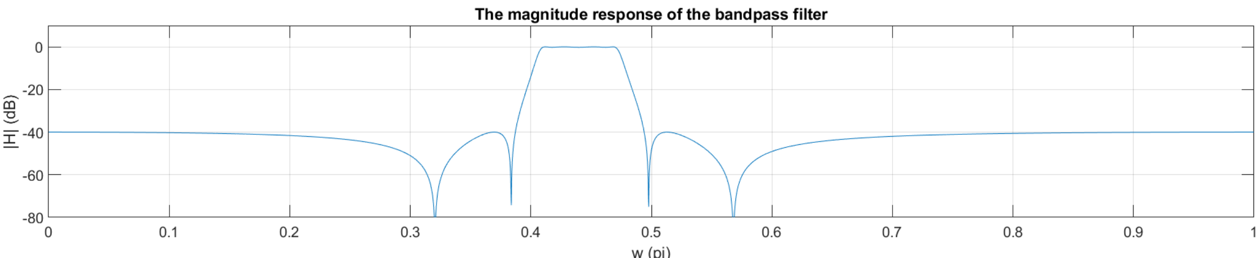

Figure 1: The frequency response magnitude of the given bandpass filter

Part b) If the coefficients of the $x[n]$'s are multiplied by $10^{\delta_1/20}$, $(20log(10^{\delta_1/20}) = \delta_1)$, the frequency response magnitude of the filter will be shift to up by δ_1 dB. The equiripple region in the passband of both original and shifted version is shown in the Figure 2. Their sizes are equal since multiplication by a constant doesn't change the structure of the filter due to the linearity.

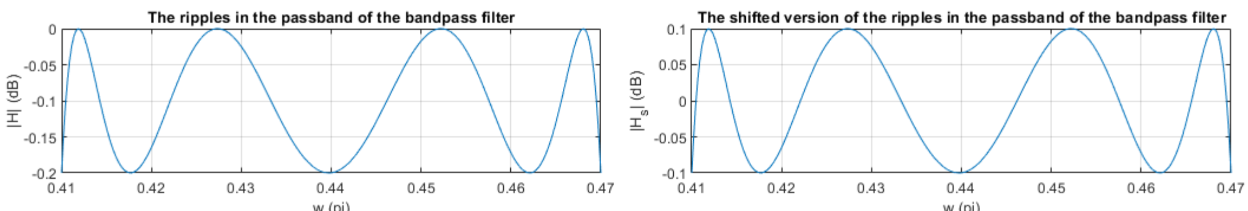

Figure 2: The equiripple region in the passband

Part c) The first 200 samples of the impulse response of the given bandpass filter is shown in the Figure 3. Since the Fourier transform is about the amplitude of the cosines in a time domain function, it can be seen from the figure that the impulse response contains cosines that have neither too large frequencies nor too small frequencies. So it can be said that it is a bandpass filter.

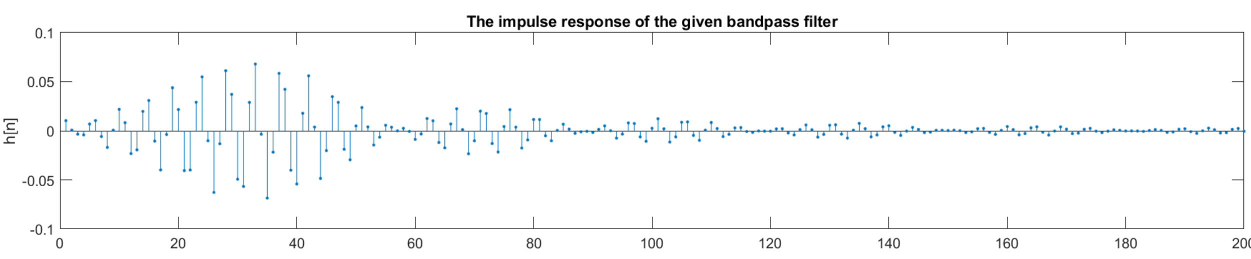

Figure 3: The impulse response of the given bandpass filter

Question 3

Part a)

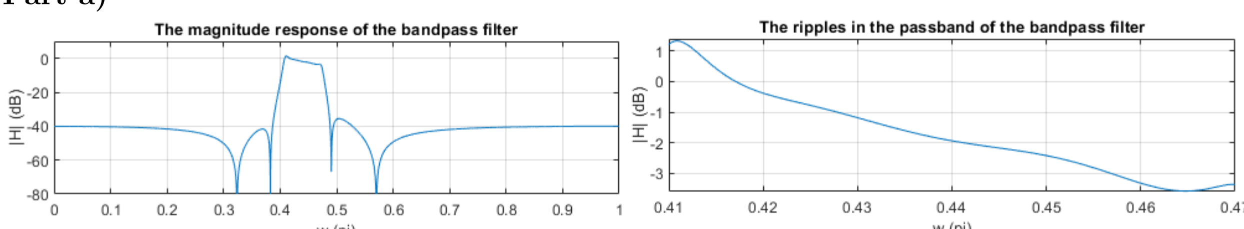

Figure 4: The frequency response magnitude of the bandpass filter that is quantized with 16 bits

When the difference equation is realized on hardware, the direct form can be sensitive to coefficient quantization. To see the impact of the quantization, the coefficients of the original filter are quantized to 16 bits. The maximum amount of change due to the quantization is $2.0133 * 10^{-4}$. The frequency response of the quantized given bandpass filter is shown in the Figure 4.

Part b) The pole-zero diagram of the filter described by the coefficients that are quantized with 16 bits is shown in the Figure 5.

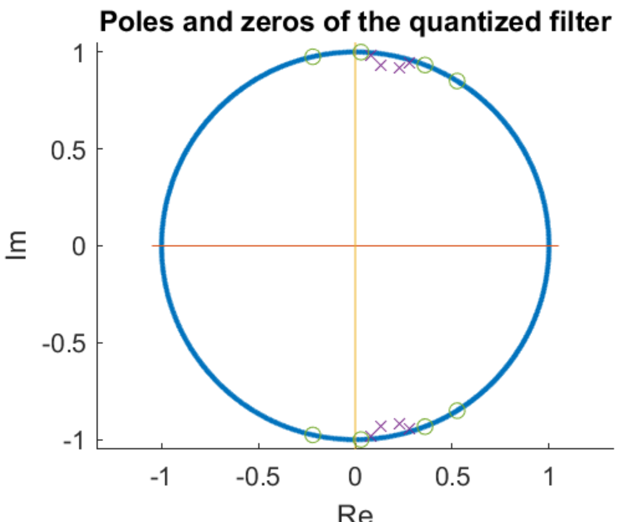

Figure 5: The 16 bit quant. filter's pole-zero diagram

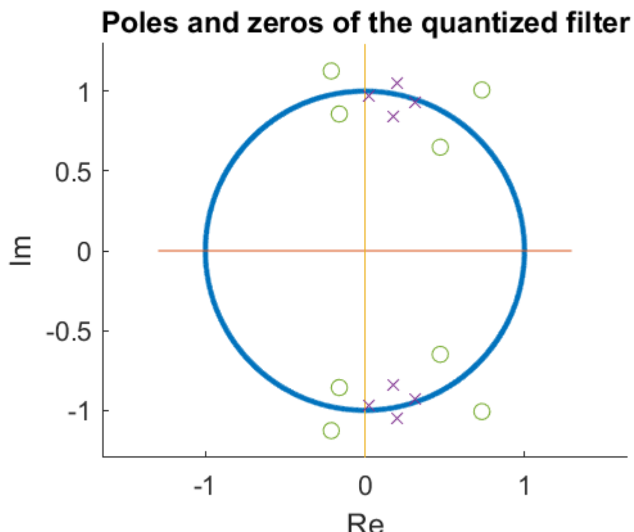

Figure 6: The 12 bit quant. filter's pole-zero diagram

Part c) The same procedure is repeated for 12 bit quantization. The maximum amount of change due to the quantization is 0.0032. The pole-zero diagram of the filter described by the coefficients that are quantized with 12 bits is shown in the Figure 6. And the frequency response of the quantized given bandpass filter is shown in the Figure 7.

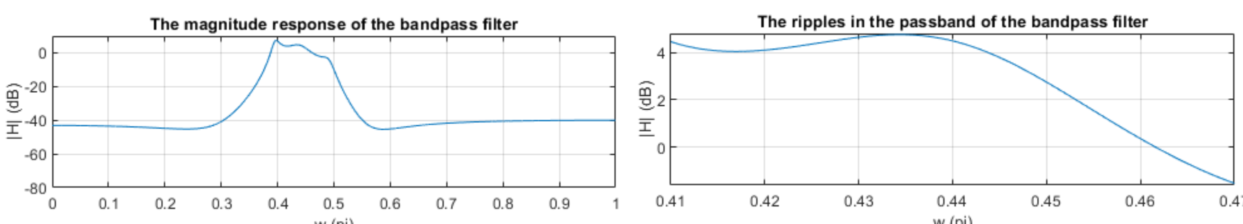

Figure 7: The frequency response magnitude of the bandpass filter that is quantized with 12 bits

Part d) In order to have both causal and stable system, the poles should be inside the unit circle. So, the bandpass filter that is quantized with 16 bits is both causal and stable; however, the one that is quantized with 12 bits is not both causal and stable, since there are 2 poles outside the unit circle.

Part e)

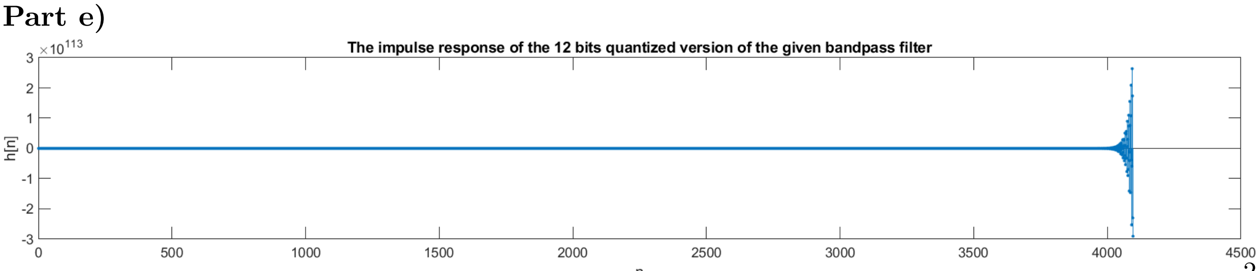

Figure 8: The impulse response of the bandpass filter that is quantized with 12 bits

The impulse response of the bandpass filter that is quantized with 12 bits is shown in the Figure 8. It can be seen from the figure that the filter is not stable since the filter is infinite impulse response and the impulse response has an increasing behaviour; so, the impulse response $h[n]$ is not absolutely summable.

Question 4

Part a) The given bandpass filter can be transformed into a cascade of second-order subsections.

Part b) The impulse response of the cascaded version of the given bandpass filter is shown in the Figure 9. And the maximum difference between the original impulse response and the cascaded version is $1.857 * 10^{-13}$; so, the cascaded version is implemented correctly.

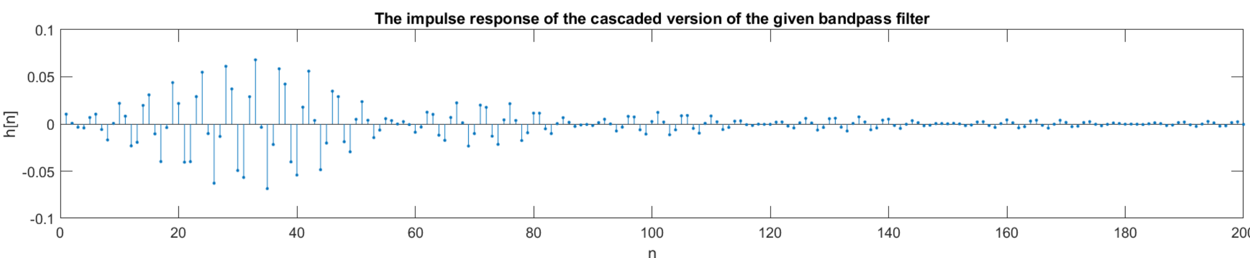

Figure 9: The impulse response of the cascaded version of the given bandpass filter

Part c) First, the coefficients of the original bandpass filter are quantized with 16 bits and then it is transformed into a cascade of second-order subsections. Its impulse response is shown in the Figure 10. And the maximum difference from the original impulse response is $1.857 * 10^{-13}$; so, the quantized-cascaded version is implemented correctly.

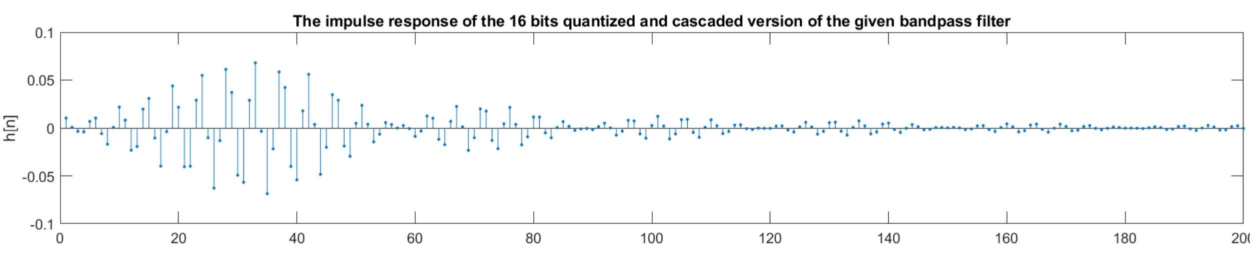

Figure 10: The impulse response of the quantized & cascaded version of the given bandpass filter

Part d) The frequency response magnitude of the bandpass filter that is cascaded and quantized with 16 bits is given in the Figure 11. They are identical with the original bandpass filter that is shown in the Figures 1, 2, yet there exist some negligible difference which is mentioned in the previous part. On the other hand, it can be seen from the comparison of the Figure 4 and 11 that the cascaded version of the quantized bandpass filter gives better results than only quantized version, since the quantization errors don't accumulate over orders in cascaded version; rather it is only two orders in the cascade case.

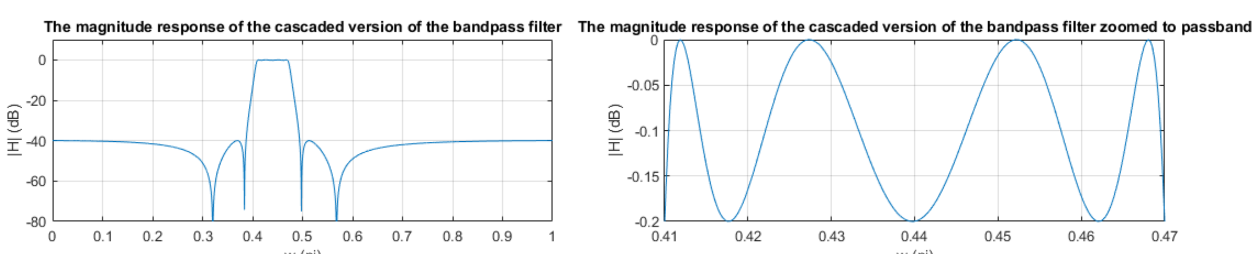

Figure 11: Frequency response magnitude of bandpass filter cascaded & quantized with 16 bits

The pole-zero diagrams of the cascaded second-order subsections are shown in the Figure 12. It can be seen from the diagrams that each of the causal subsection is also stable since poles are inside the unit circle. Each subsection's poles and zeros correspond to a complex conjugate pair of poles and zeros of original passband filter. The original bandpass filter has 8 poles and 8 zeros, so the cascaded system has 4 subsections with each of them having a complex conjugate pair of poles and zeros. So, the overall poles and zeros of subsections construct the poles and zeros of cascaded system.

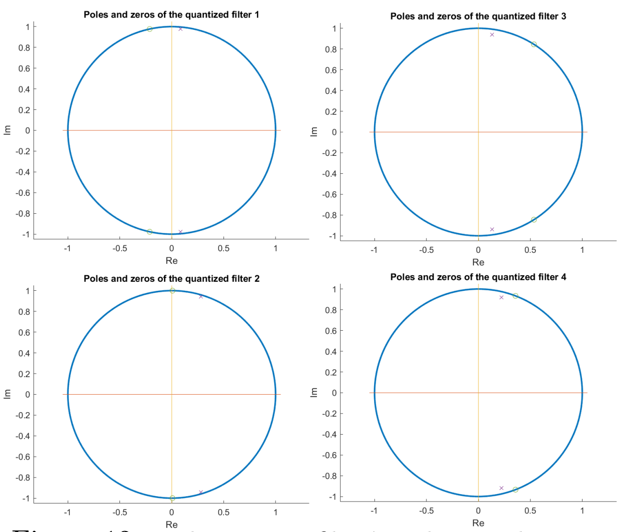

Figure 12: 16 bit quant. filter's pole-zero diagrams

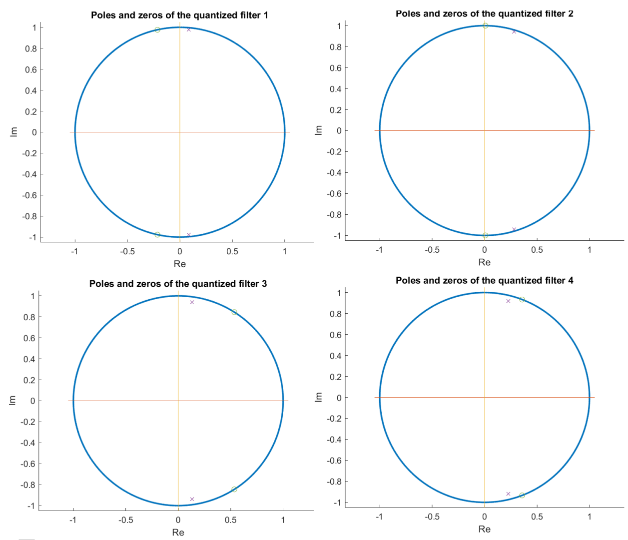

Figure 13: 12 bit quant. filter's pole-zero diagrams

Part e) The same procedure is repeated for 12 bit quantization. The pole-zero diagrams of the cascaded second-order subsections are shown in the Figure 13. The same interpretations as in the 16 bits quantization are valid for 12 bits quantization, since the poles of the subsections are inside the unit circle, too. So, the subsections are both causal and stable.

Its impulse response is shown in the Figure 14. And the maximum difference from the original impulse response is $1.857 * 10^{-13}$; so, the quantized-cascaded version is implemented correctly, as opposed to only quantized with 12 bits.

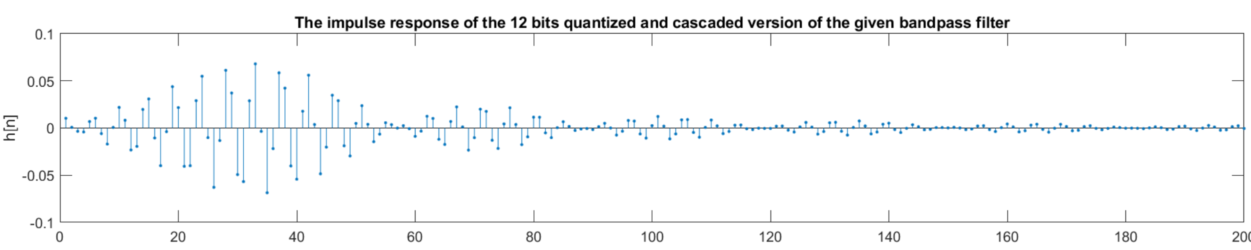

Figure 14: The impulse response of the quantized & cascaded version of the given bandpass filter

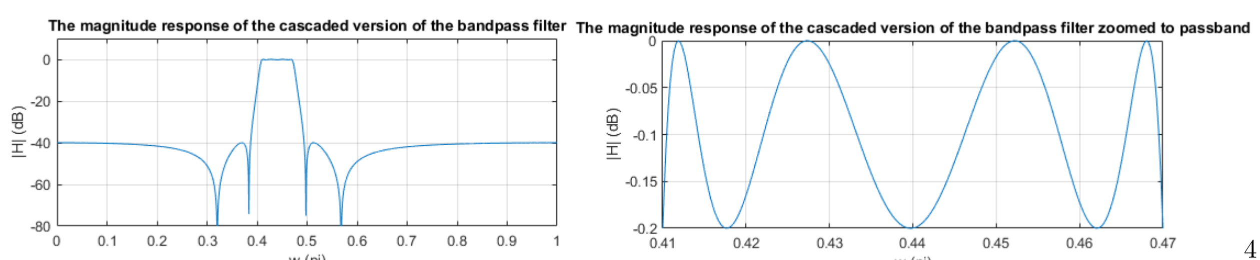

Figure 15: Frequency response magnitude of the bandpass filter cascaded & quantized with 12 bits

Its frequency response magnitude is given in the Figure 15. It is identical with the original bandpass filter that is shown in the Figures 1, 2, yet there exist some negligible difference which is mentioned above. On the other hand, it can be seen from the comparison of the Figure 7 and 15 that the cascaded version of the quantized bandpass filter is implemented correctly, whereas only quantized version fails, since the quantization errors accumulate over orders in only quantized version; and this causes bigger errors and the system doesn't function as desired.

Question 5

Part a) The given bandpass filter can be transformed into a parallel form of second-order subsections that a complex conjugate poles and zeros pair. First, the system function is factored into its partial fraction expansion, and the complex conjugate pairs are gathered into one subsection, then all the subsections are connected parallel to obtain the original system.

Part b) The impulse response of the parallel form of the given bandpass filter is shown in the Figure 16. And the maximum difference between the original impulse response and the cascaded version is 2.48810^{-13}; so, the parallel form is implemented correctly.

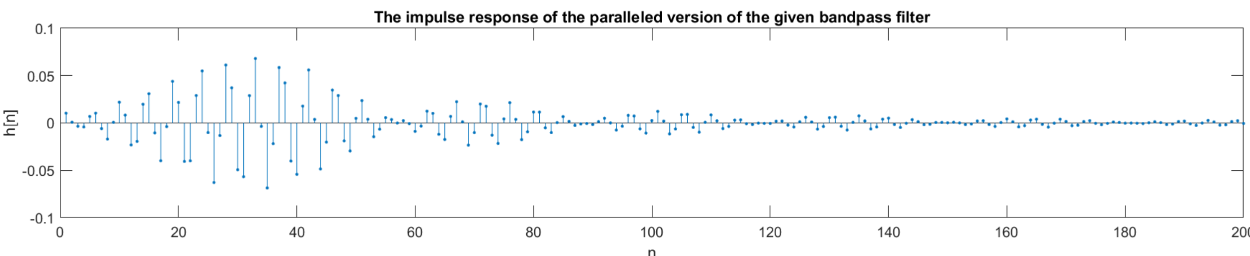

Figure 16: The impulse response of the parallel form of the given bandpass filter

Part c) The coefficients of the parallel subsections of the system can be quantized with 16 bits for the hardware purposes. After quantization, each subsection's partial fraction expansion (PFE) is computed to combine them into a single equation. Then, the system function is obtained by converting the combined PFE into difference equation coefficients. Note that although the system function is given as a single difference equation, the coefficients were quantized with the system in parallel form.

Part d) The frequency response magnitude of the parallel form that is quantized with 16 bits which is formed by adding the impulse responses of the subsection systems is shown in the Figure 17 on left. In addition, the frequency response magnitude of the system function which is described in the previous part is shown in the Figure 17 on right. So, it can be seen from the figure that the described system in the previous part is indeed the same as that of the quantized parallel form system.

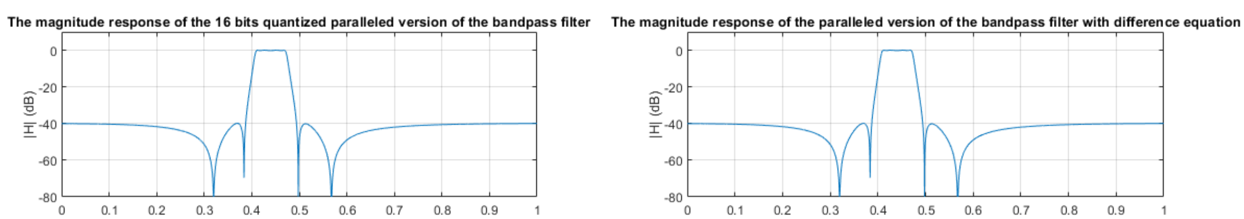

Figure 17: The frequency response magnitude of the quantized & parallel form of the given bandpass filter

Part e) The pole-zero diagram of the filter described in the part c is shown in the Figure 18. Since all the poles are inside the unit circle, the parallel form that is quantized with 16 bits is both causal and stable.

It can be seen from the Figure 17 that the parallel form that is quantized with 16 bits gives the same frequency response magnitude as the filter which is cascaded and quantized with 16 bits shown in the Figure 11 and the direct form implementation shown in the Figure 1.

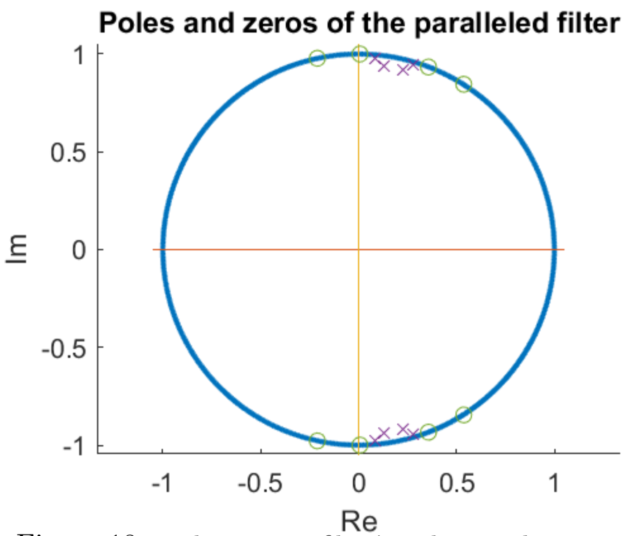

Figure 18: 16 bit quant. filter's pole-zero diagram

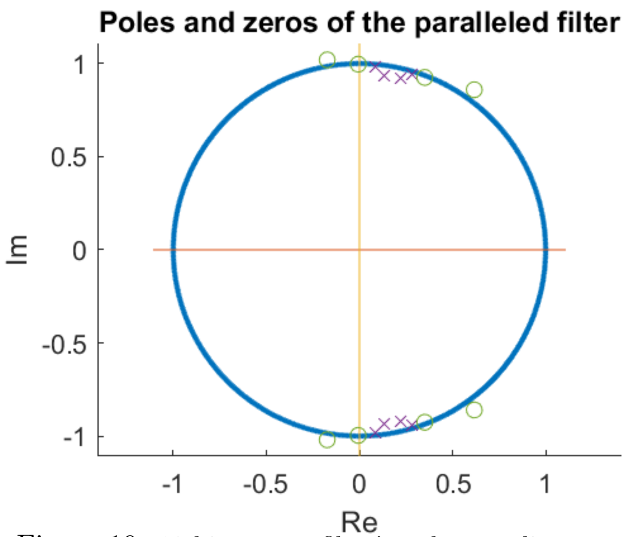

Figure 19: 12 bit quant. filter's pole-zero diagram

Part f) The same procedure is repeated for 12 bit quantization. The pole-zero diagram of the filter described in the part c for 12 bits quantization is shown in the Figure 19. Since all the poles are inside the unit circle, the parallel form that is quantized with 12 bits is both causal and stable.

The frequency response magnitude of the parallel form that is quantized with 12 bits which is formed by adding the impulse responses of the subsection systems is shown in the Figure 20 on left. In addition, the frequency response magnitude of the system function which is described in the part c for 12 bits quantization is shown in the Figure 20 on right. So, it can be seen from the figure that the described system in the part c is indeed the same as that of the quantized parallel form system.

It can be seen from the Figure 20 that the parallel form that is quantized with 12 bits gives the same frequency response magnitude as the filter which is cascaded and quantized with 12 bits shown in the Figure 15 and the direct form implementation shown in the Figure 1.

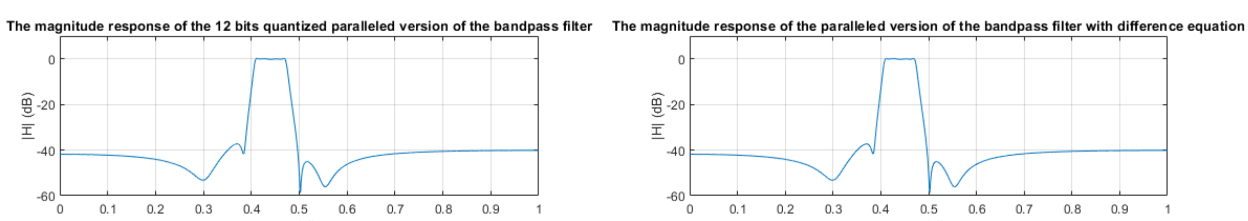

Figure 20: The frequency response magnitude of the quantized & parallel form of the given bandpass filter