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Homework 2

EE 477
Fall 2019

Introduction

Monte Carlo Simulation is done in order to analyze the performance of modulation types, QPSK, BPSK, 8PSK, 4PAM, 16QAM, BFSK. There are 2 simulation parameters that determine how many trials for one SNR value should run until the simulation stops. The first one is *ferlim* which determines the maximum number of the trials that have an error in the detection; the second one is *max-nframes* which determines the maximum number of trials. So, if these values are increased, then the simulation will run more trials and it is expected to obtain the simulation results closer to the theoretical results; or visa verse.

1 QPSK

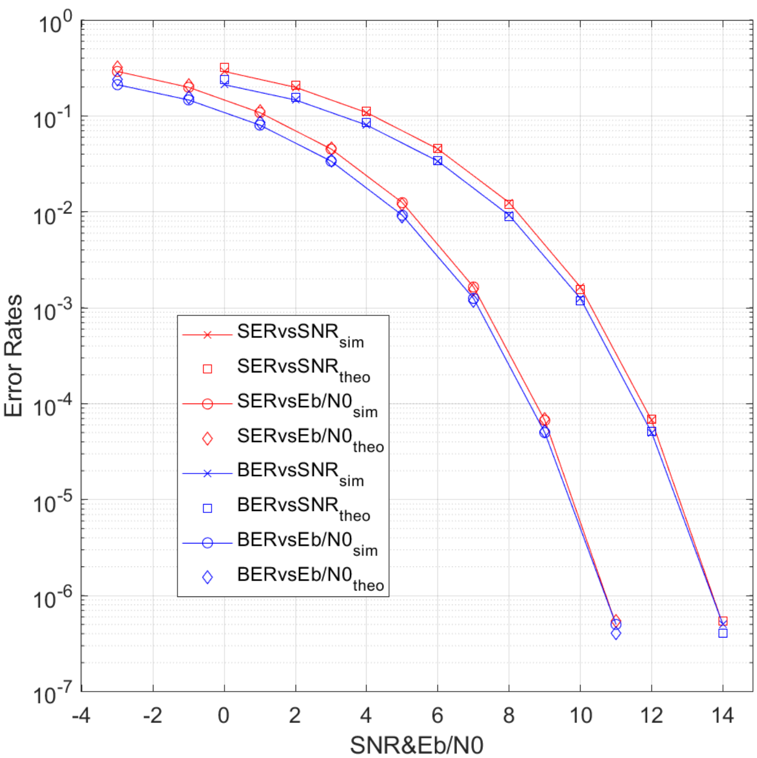

Figure 1: The simulation and theoretical values of SER and BER for QPSK

The simulation detects both bit error rate (BER), and symbol error rate (SER). Since in QPSK, a symbol contains two bits, and uniform constellation diagram is applied in mapping; it is expected to observe more bit errors than symbol errors. Namely, in uniform constellation diagram, one symbol error can lead to more than one bit errors.
The SER and BER values can be plotted as a function of both SNR (E_s/N_0, where E_s is the symbol energy, N_0 is noise energy) and E_b/N_0, where E_b is the bit energy. Since in QPSK, one symbol consists of two bits, E_b is equal to half of E_s. Therefore, the plot as a function of E_b/N_0 is the 3dB left shifted version of the plot as a function of SNR.
The theoretical values of SER for QPSK is shown in the following expression 1.

$$P_{SER} = 2Q(\sqrt{SNR}) - Q^2(\sqrt{SNR}) \approx 2Q(\sqrt{SNR}) \quad (1)$$

The theoretical values of BER for QPSK can be calculated by using P_{SER} and the constellation diagram that determines the average number of bit errors for one symbol error that can

Regarding the graphs: Since the errors drop down to 0 after some point and log scale is used for plots, they are not shown after these points due to the $log(0)$ is $-\infty$, namely it is not defined at 0.

be determined by using nearest neighbors. (For example, 00 symbol has 2 nearest neighbors, 01 and 11; average bit errors for 00 is 3/2. And this calculation is considered separately for each symbols.) The following expression 2 shows the derivation of SER value.

$$
\begin{aligned}
P_{SER} &= \frac{\text{symbol errors}}{\text{total symbols}},\ P_{BER} = \frac{\text{bit errors}}{\text{total bits}},\ \text{total bits} = 2\text{total symbols} \\
e_b &= e_{\text{average bit error}} e_s,\ e_{\text{average bit error}} \approx \frac{1}{4}\left(\frac{3}{2}+\frac{3}{2}+\frac{3}{2}+\frac{3}{2}\right) = \frac{3}{2} \\
P_{BER} &\approx \frac{3}{4} P_{SER}
\end{aligned}
\tag{2}
$$

The resulting graphs are plotted BER and SER as a function of SNR and E_b/N_0. They are shown in the Figure 1. So, it can be seen from the figure that the simulation results almost match with the theoretical ones. As expected, the BER curves are below the SER curves since the probability of making bit errors is smaller than the probability of making symbol errors since the number of total bits is twice the total symbols yet the average bit errors per symbol is 3/2. So, BER is 3/4 of SER.

2 BPSK

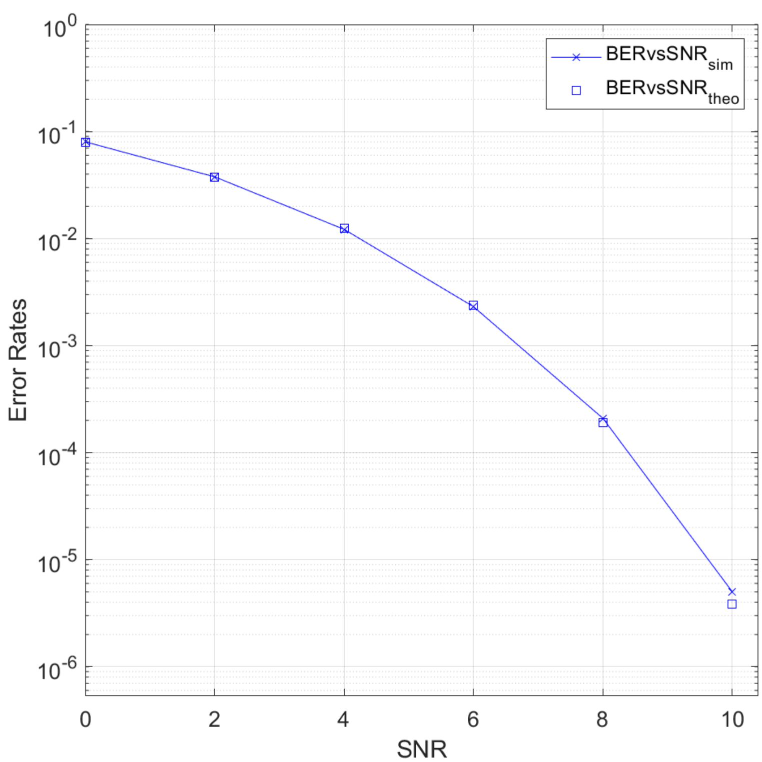

Figure 2: The simulation and theoretical values of BER for BPSK

The simulation is applied for BPSK in order to observe BER. Since in BPSK, a symbol has a single corresponding bit, BER is equal to SER value. The theoretical values of SER for BPSK is given in the following expression 3.

$$P_{SER} = Q(\sqrt{2SNR}) \tag{3}$$

The resulting graphs are plotted BER as a function of SNR. They are shown in the Figure 2. So, it can be seen from the figure that the simulation results almost match with the theoretical ones. And the performance of BPSK is better than the QPSK since the distance of distinct symbols on the constellation diagram is larger in BPSK according to QPSK.

3 8PSK

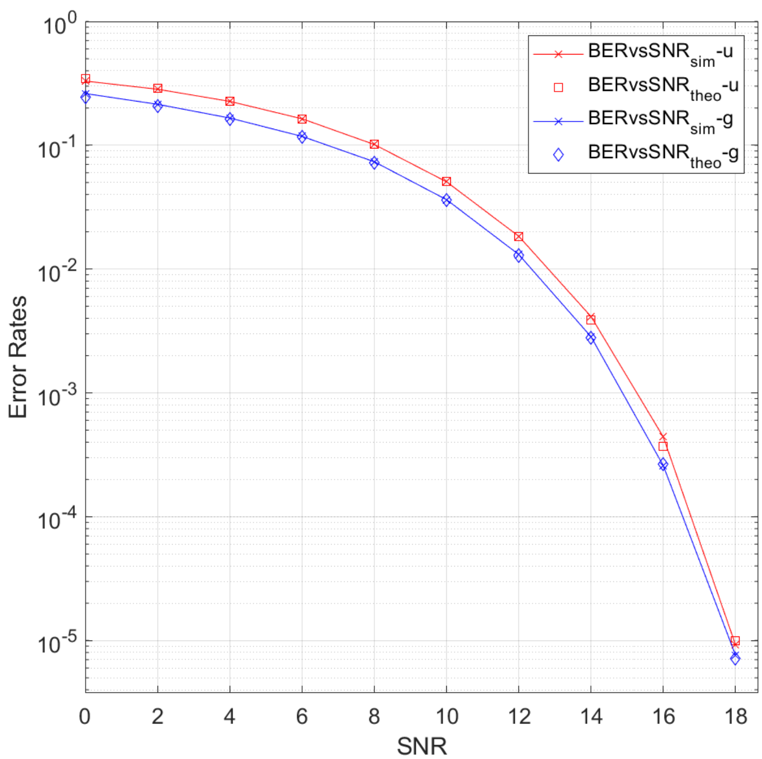

Figure 3: The simulation and theoretical values of BER for 8PSK

The simulation is applied for 8PSK in order to observe BER in both uniform mapped constellation diagram and gray mapped constellation diagram. The BER values vary with the mapping of the constellation diagram since one symbol error in different mapping can cause different numbers of bit errors. The theoretical values of BER can be found from SER values. The SER for 8PSK is given in the following expression 4.

$$P_{SER} \approx 2Q(\sqrt{2SNR}sin(\pi/8)) \tag{4}$$

The BER values for 8PSK that has an uniform mapped constellation diagram can be derived by calculating the average bit errors per symbol error as it is done in the expression 2. The following expression 5 gives the BER value for 8PSK.

$$\begin{aligned} e_{\text{average bit error}} &\approx \frac{1}{8}\left(2+\frac{3}{2}+\frac{3}{2}+2+2+\frac{3}{2}+\frac{3}{2}+2\right)=\frac{7}{4} \\ P_{BER} &\approx \frac{7}{12}P_{SER} \end{aligned} \tag{5}$$

In a gray mapped constellation diagram, one symbol error corresponds to one bit error in according to the nearest neighbors. So, $P_{BER} \approx (1/3)P_{SER}$. However, this approximation gives a little bit shift to down, so the coefficient is replaced with 5/12.

The resulting graphs are plotted BER as a function of SNR. They are shown in the Figure 3. So, it can be seen from the figure that the gray mapped one gives a better performance as expected, since uniform mapping causes 7/4 many bit errors per symbol error whereas gray mapping causes 1 bit errors per symbol error. (As mentioned above, this value is replace with 5/4.)

4 4PAM

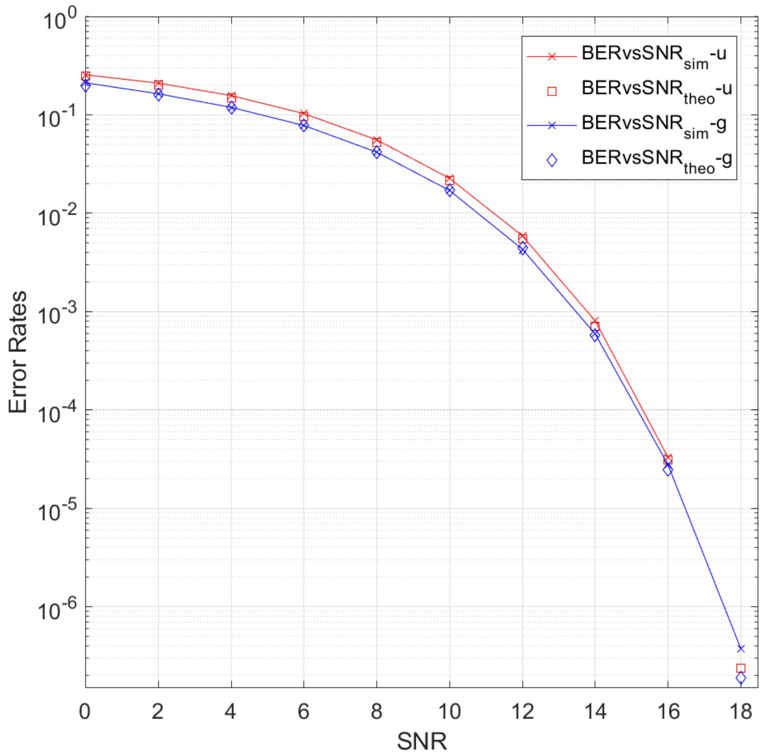

Figure 4: The simulation and theoretical values of BER for 4PAM

The simulation is applied for 4PAM in order to observe BER in both uniform mapped constellation diagram and gray mapped constellation diagram. The BER values vary with the mapping of the constellation diagram since one symbol error in different mapping can cause different numbers of bit errors. The theoretical values of BER can be found from SER values. The SER for 4PAM is given in the following expression 6.

$$P_{SER} \approx \frac{3}{2} Q\left(\sqrt{\frac{2}{5} SNR}\right) \tag{6}$$

The BER values for 4PAM that has an uniform mapped constellation diagram can be derived by calculating the average bit errors per symbol error as it is done in the expression 2. The following expression 7 gives the BER value for 8PSK.

$$\begin{aligned} e_{\text{average bit error}} &\approx \frac{1}{4}\left(1+\frac{3}{2}+1+\frac{3}{2}\right) = \frac{5}{4} \\ P_{BER} &\approx \frac{5}{8} P_{SER} \end{aligned} \tag{7}$$

In a gray mapped constellation diagram, one symbol error corresponds to one bit error in according to the nearest neighbors. So, $P_{BER} \approx (1/2)P_{SER}$.

Since in 4PAM, the constellation energy is not unity yet it is unity in PSK cases; so the constellation energy should be normalized. The average energy is 5 when [-3 -1 1 3] constellation points are used; then, they are should be diveded by $\sqrt{5}$.

The resulting graphs are plotted BER as a function of SNR. They are shown in the Figure 4. So, it can be seen from the figure that the gray mapped one gives a better performance as expected, since uniform mapping causes 5/4 many bit errors per symbol error whereas gray mapping causes 1 bit errors per symbol error.

5 16QAM

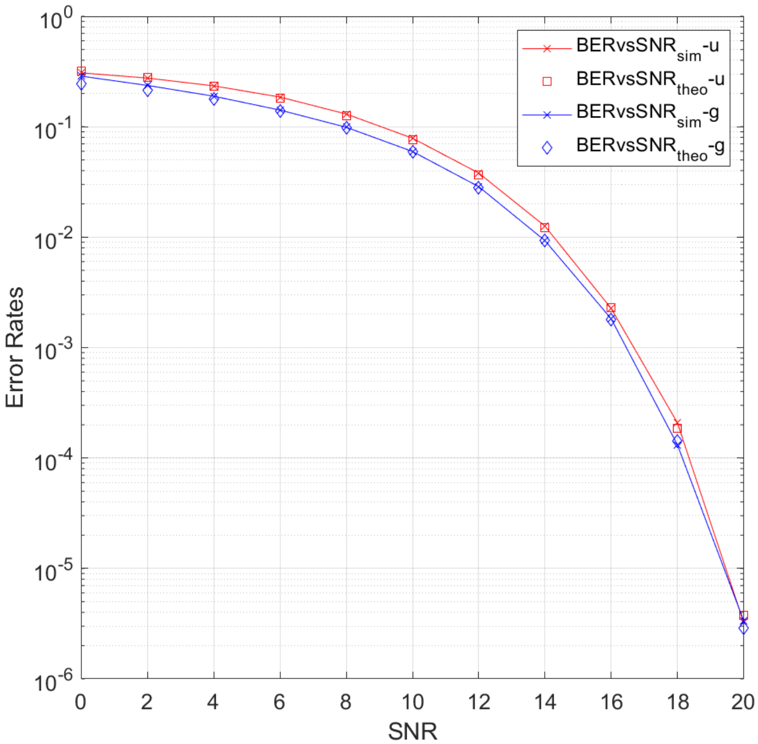

Figure 5: The simulation and theoretical values of BER for 16QAM

The simulation is applied for 16QAM in order to observe BER in both uniform mapped constellation diagram and gray mapped constellation diagram. The BER values vary with the mapping of the constellation diagram since one symbol error in different mapping can cause different numbers of bit errors. The theoretical values of BER can be found from SER values. The SER for 16QAM is given in the following expression 8.

$$P_{SER} \approx 3Q\left(\sqrt{\frac{1}{5}SNR}\right) \tag{8}$$

The BER values for 16QAM that has an uniform mapped constellation diagram can be derived by calculating the average bit errors per symbol error as it is done in the expression 2. The following expression 9 gives the BER value for 8PSK.

$$\begin{aligned} e_{aveb} &\approx \frac{1}{16}\left(1+\frac{4}{3}+\frac{4}{3}+1+\frac{4}{3}+\frac{6}{4}+\frac{6}{4}+\frac{4}{3}+1+\frac{4}{3}+\frac{4}{3}+1+\frac{4}{3}+\frac{6}{4}+\frac{6}{4}+\frac{4}{3}\right)=\frac{31}{24} \\ P_{BER} &\approx \frac{31}{96}P_{SER} \end{aligned} \tag{9}$$

In a gray mapped constellation diagram, one symbol error corresponds to one bit error in according to the nearest neighbors. So, $P_{BER} \approx (1/4)P_{SER}$.

Since in 16QAM, the constellation energy is not unity yet it is unity in PSK cases; so the constellation energy should be normalized. The average energy is 10 when [-3 -1 1 3; -3 -1 1 3] constellation points are used; then, they are should be diveded by $\sqrt{10}$.

The resulting graphs are plotted BER as a function of SNR. They are shown in the Figure 5. So, it can be seen from the figure that the gray mapped one gives a better performance as expected, since uniform mapping causes 31/24 many bit errors per symbol error whereas gray mapping causes 1 bit errors per symbol error.

6 BFSK

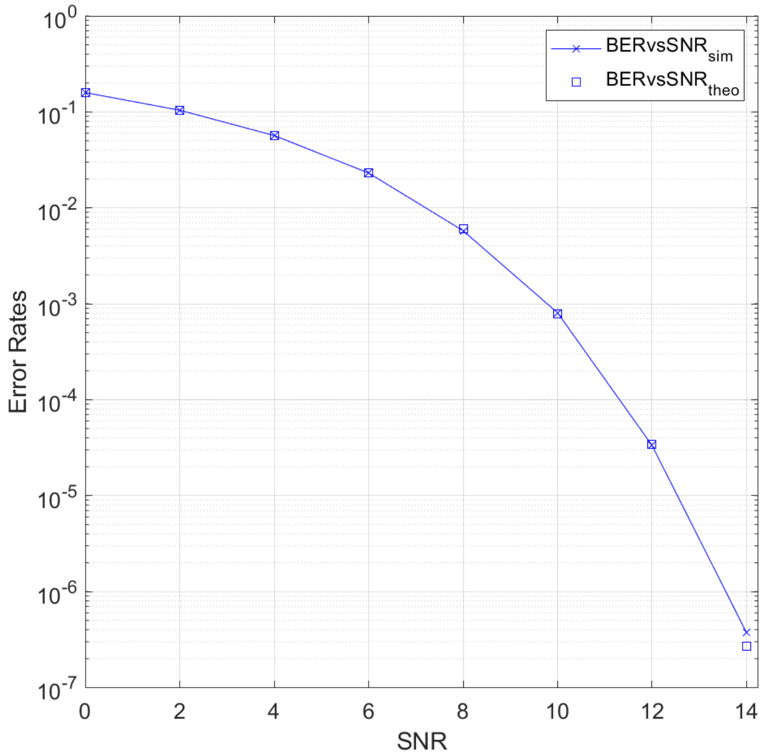

Figure 6: The simulation and theoretical values of BER for BFSK

The simulation is applied for BFSK in order to observe BER. Since in BFSK, a symbol has a single corresponding bit, BER is equal to SER value. The theoretical values of SER for BFSK is given in the following expression 10.

$$P_{SER} = Q(\sqrt{SNR}) \tag{10}$$

The resulting graphs are plotted BER as a function of SNR. They are shown in the Figure 6. So, it can be seen from the figure that the simulation results almost match with the theoretical ones.

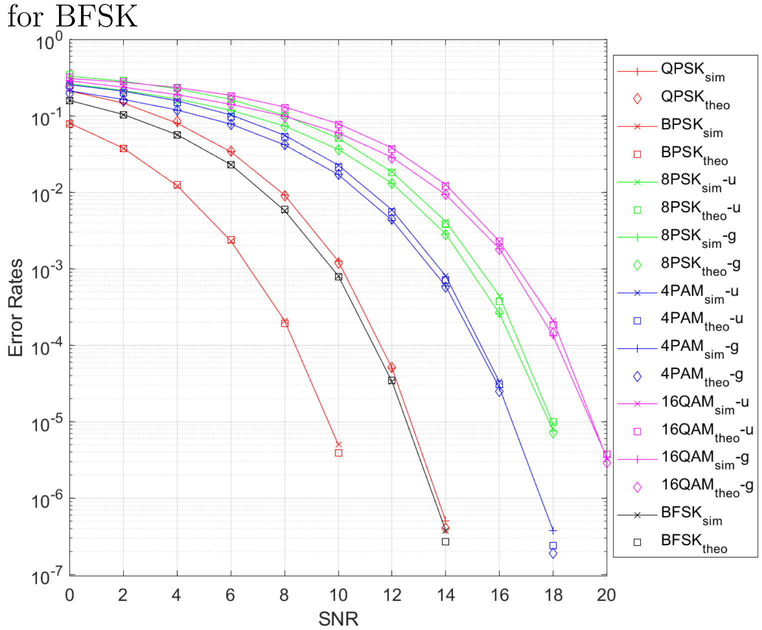

Figure 7: The simulation and theoretical values of BER for all the modulations that are examined

7 Comparison of All

The Figure 7 shows the simulation and theoretical BER values of all the modulations that are examined.

It can be seen that BPSK gives the best result since its points on the constellation diagram are the farthest of the modulations. The closer points on the constellation diagram, the worse performance in the modulation. The ranking from better to worse of all modulations is BPSK, BFSK, QPSK, 4PAM-g, 4PAM-u, 8PSK-g, 8PSK-u, 16QAM-g, 16QAM-u. (At low SNR, 16QAM performs better than 4PAM.)