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Homework 4

EE 475
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***Abstract*—This document contains the solutions, explanations, comments, and related graphs and tables of the questions in the homework 4.**

I. Color Coordinates

Three valid colors c_1, c_2, and c_3 with coordinates (x_1, y_1), (x_2, y_2), and (x_3, y_3) in the chromaticity diagram can represent any point which lies within the triangle, shown in the Figure 1 whose vertices are at the coordinates of c_1, c_2, and c_3 by mixing the corresponding colors with relative percentages. Namely, these relative percentages of the vertices colors specify any color within the triangular. In order to calculate these percentage, the following 2 steps can be considered; finding the line, that includes c_{23x} in terms of c_2 and c_3, and finding the intersection of the lines (c_1, c_x) and (c_2, c_3). After obtaining the coordinates of c_{23x} in terms of c_1, c_2, c_3, and c_x, the relative percentage values can be found by calculating relative distances.

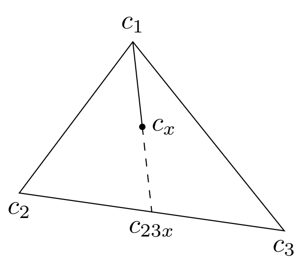

Fig. 1. A triangular in the chromaticity diagram

A. The line representing c_{23x} in terms of c_2 and c_3

The line equation on which the two points are known can be easily written. So, the line equation containing c_{23x} in terms of c_2 and c_3 is expression in following 1.

$$y = \frac{y_3 - y_2}{x_3 - x_2} x + y_3 - \frac{y_3 - y_2}{x_3 - x_2} x_3 \tag{1}$$

B. Coordinates of c_{23x} in terms of c_1, c_2, c_3, and c_x

The coordinates of c_{23x} can be obtained by finding the intersection of the line described above and the line formed by c_1 and c_x that is shown in the following expression 2.

$$y = \frac{y_x - y_1}{x_x - x_1} x + y_1 - \frac{y_x - y_1}{x_x - x_1} x_1 \tag{2}$$

By solving the equation 1 and 2 together to obtain the intersection point c_{23x}, the following expression 3 finds the coordinates (x_{23x}, y_{23x}).

$$\begin{aligned} x_{23x} &= \frac{y_1 - \frac{y_x - y_1}{x_x - x_1} x_1 - y_3 + \frac{y_3 - y_2}{x_3 - x_2} x_3}{\frac{y_3 - y_2}{x_3 - x_2} - \frac{y_x - y_1}{x_x - x_1}} \\ y_{23x} &= \frac{y_3 - y_2}{x_3 - x_2} x_{23x} + y_3 - \frac{y_3 - y_2}{x_3 - x_2} x_3 \\ &= \frac{y_3 - y_2}{x_3 - x_2} \frac{y_1 - \frac{y_x - y_1}{x_x - x_1} x_1 - y_3 + \frac{y_3 - y_2}{x_3 - x_2} x_3}{\frac{y_3 - y_2}{x_3 - x_2} - \frac{y_x - y_1}{x_x - x_1}} \\ &+ y_3 - \frac{y_3 - y_2}{x_3 - x_2} x_3 \end{aligned} \tag{3}$$

C. The relative percentages

Finally, the relative percentages, p_1, p_2, p_3 can be computed by the distance between c_1 and c_x, c_2 and c_{23x}, c_3 and c_{23x}, respectively. The following expression 4 shows the relative percentages.

$$\begin{aligned} p_1 &= \frac{d(c_1, c_{23x}) - d(c_1, c_x)}{d(c_1, c_{23x})} 100 \\ p_2 &= \frac{d(c_2, c_3) - d(c_2, c_{23x})}{d(c_2, c_3)} (100 - p_1) \\ p_3 &= \frac{d(c_2, c_3) - d(c_3, c_{23x})}{d(c_2, c_3)} (100 - p_1) \end{aligned} \tag{4}$$

For a given c_x color, mixing c_1 color with percentage p_1, c_2 color with percentage p_2, and c_3 color with percentage p_3 will generate the color c_x.

II. Understanding the Color Cube

A. The front face of color cube

An image colors can be decomposed into 3 parts, red-green-blue (RGB) components. These components represent any color in the color cubic that is shown in the Figure 2. For example, on the front face of the cube,

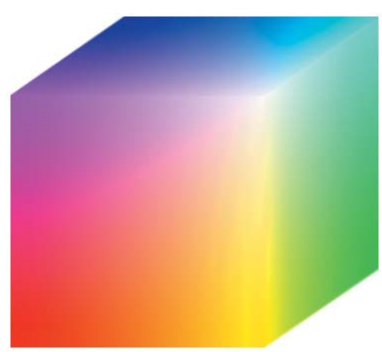

Fig. 2. The RGB color cube[1]

the gray level of the red component is all 255, the green component's is the columns of 0's up to 255's, and the blue component's is the rows of 255's up to 0's.[1]

B. CMY colors

The corresponding color in the vertices of the RGB color cubic is given in the following table I. There exists another representation of RGB component with their secondary colors cyan-magenta-yellow (CMY). This representation has a subtractive characteristic, for example, cyan subtracts red light from the white light and reflects green and blue lights; whereas the RGB representation has a additive characteristic, for example, the red and blue light add up to the light with magenta color. The transformation from RGB to CMY is given in the following expression 5.

TABLE I
THE COLOR OF VERTICES IN RBG REPRESENTATION

000	001	010	011	100	101	110	111
black	blue	green	cyan	red	magenta	yellow	white

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix} \tag{5}$$

The corresponding color in the vertices of the CMY color cubic is given in the following table II.

TABLE II
THE COLOR OF VERTICES IN CMY REPRESENTATION

000	001	010	011	100	101	110	111
white	yellow	magenta	red	cyan	green	blue	black

C. Saturation values of edges of color cube

The saturation component is given by the following expression 6.

$$S = 1 - \frac{3}{R+G+B} min(R,G,B) \tag{6}$$

Since on the edge of the RGB color cube, at least one of the RBG component is 0, except the ones that contain white, the saturation value is 1, meaning that they are fully saturated; except the ones that contain black point since it causes 0 division. Therefore, the edges that don't contain black or white point are fully saturated; and the saturation of the edges that contain white increases from white vertices to the other vertices, and similarly, the saturation of the edges that contain black increases from black vertices to the other vertices.

[1]It is assumed that the origin is taken as left-top, and the image is 8-bit.

III. Solid Color Squares

A color image represented in RGB color space can be converted into hue-saturation-intensity (HSI) color space at which the color image is decomposed into intensity components, descriptor of monochromatic images, hue-saturation component, color carrying information. The color image represented in RGB color space shown in the Figure 3 is examined by converting into HSI color space.

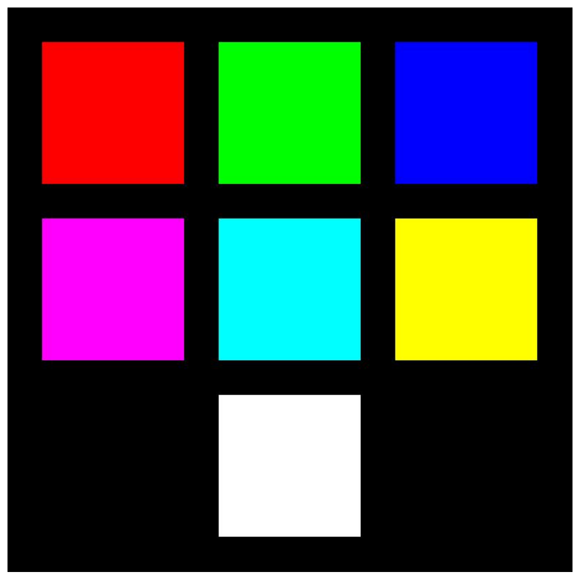

Fig. 3. An image represented in RGB color space

A. The hue image

The hue component of an image can be calculated by the following expression 7.

$$\begin{aligned} H &= \begin{cases} \theta & B \leq G \\ 360 - \theta & B > G \end{cases}, \text{ where} \\ \theta &= cos^{-1}\left(\frac{\frac{1}{2}[(R-G)+(R-B)]}{[(R-G)^2+(R-B)(G-B)]^{1/2}}\right) \end{aligned} \tag{7}$$

From the above formula, the Figure 4 shows the hue values of the solid colors, red, green, blue, yellow, cyan, and magenta.

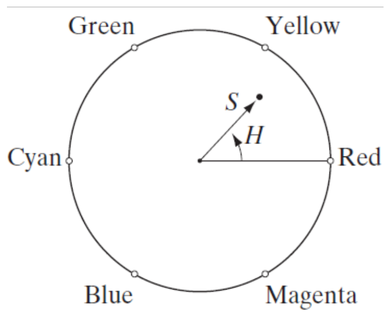

Fig. 4. The hue values of solid colors[2]

So, in 8 quanta representation, the hue value of the red square is 0, the yellow square's is $60\frac{7}{360}$=1, the green square's is $120\frac{7}{360}$=2, the cyan square's is $180\frac{7}{360}$=4, the blue square's is $240\frac{7}{360}$=5, the magenta square's is $300\frac{7}{360}$=6, and the white's is 0 since it doesn't contain any chromaticity. The resulting gray scale image[2] of the hue is given in the Figure 5.

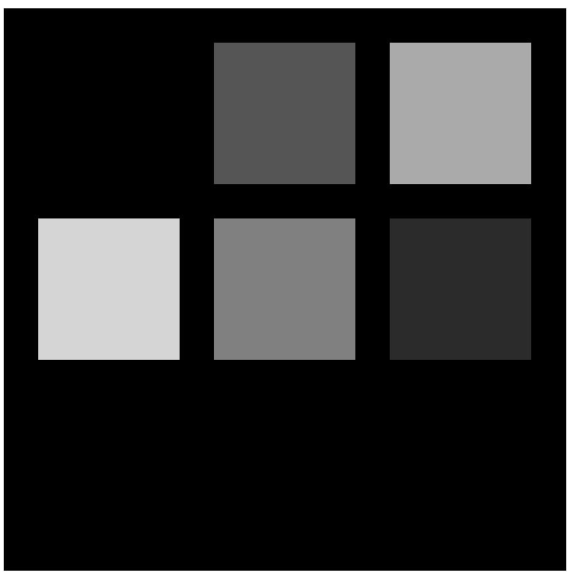

Fig. 5. The hue values of the given image in gray scale

B. The saturation image

The formula for computing the saturation component of an image is given in the equation 6.
As mentioned in the section C of the part II, the solid colors, red, green, blue, yellow, cyan, and magenta are fully saturated since they contain at least one 0 in the RGB components. So, in 8 quanta representation, they are all 7, and the saturation value of the white is 0 since it doesn't contain any chromaticity. The resulting gray scale image of the saturation is given in the Figure 6.

[2]When plotting the resulting images, the 8 quata levels are scaled to the range [0 255].

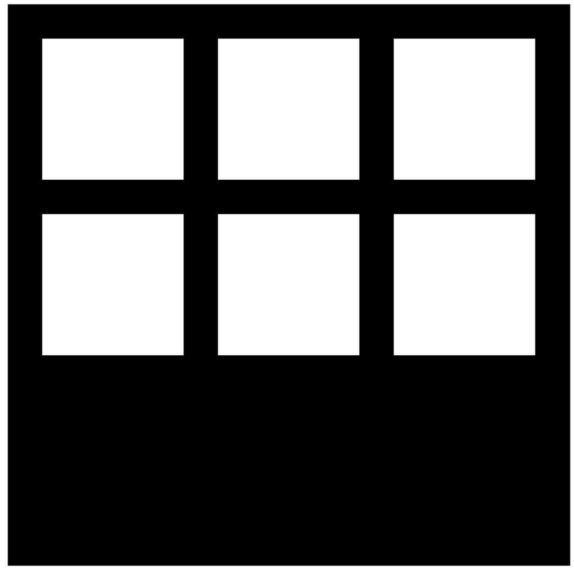

Fig. 6. The saturation values of the given image in gray scale

C. The intensity image

The intensity component of an image can be calculated by the following expression 8.

$$I = \frac{1}{3}(R + G + B) \tag{8}$$

In 8 quanta representation, the intensity value of the primary colors, red, green, and blue is $7/3$=2; the secondary colors, magenta, cyan, and yellow is $(7{+}7)/3$=5; and the white is 7. The resulting gray scale image of the intensity is given in the Figure 7.

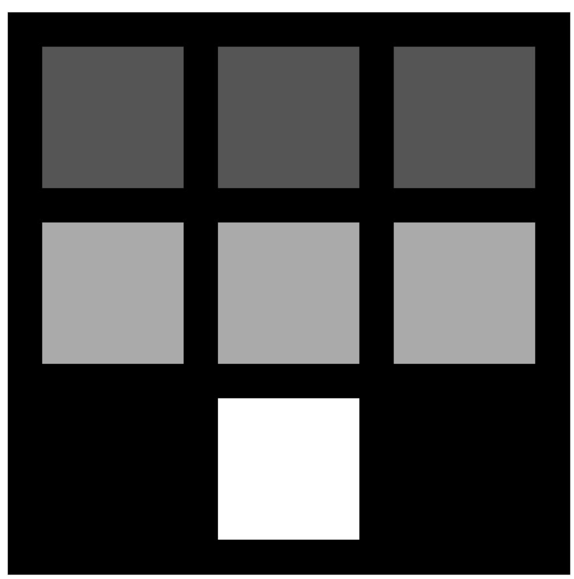

Fig. 7. The intensity values of the given image in gray scale

IV. Display of HIS Components

HSI components of an image can be represented in 8 bit gray scale. Namely, the original values is mapped to the range [0, 255]. The Figure 8 shows 8-bit images which are (left to right) the HSI component images from the Figure 6.16 in the textbook.
So, given the HSI values of some regions on the image, other regions' value can be determined by following:

Fig. 8. The HSI component images the Figure 6.16 in the textbook

A. *The values of the regions in the hue image*

Similarly to the section A of the part III, the values 43, 85, 128 in 8 bit representation correspond to 60, 120, 180 values of the hue respectively. 0 region of the circle might be red, white, or black; but since the combination with the region with 120 hue value gives a color with the hue value of 60, so 0 should correspond to red. By the same reasoning, the unknown color on the left bottom should correspond to blue, 240 hue value, since its combination with the color having 120 hue value gives a color with the hue value of 120. The unknown region which is the combination of 0 region and 240 region should have 300, since it is the mixture of red and blue, namely, magenta. The last region is the combination of all primary color, white, so its hue value is 0.

So, the hue values of the unknown regions are 300, 240, 0, ordered from brighter to darker. And their 8 bit gray levels are 213, 170, and 0, respectively, as calculated similarly in the section A of the part III.

B. *The values of the regions in the saturation image*

255 in 8 bit representation means fully saturated. Since the colors are spectral colors, and their saturation values are 1, the center, the combination of all region, should be white. And the saturation value of the white color is 0 as the value in the 8 bit representation is 0.

C. *The values of the regions in the intensity image*

Since the center is white, it should be the combination of pure red, green, and blue with knowing that the 85 region corresponds to a pure color since $85 * 3$=255. The other regions with the same brightness should be 85 meaning that they are primary colors, red, green, blue. The intersections of the circle correspond to secondary colors, yellow, cyan, magenta; so their gray scale value is $(255 + 255)/3$=170. And since the center region is white, its intensity should be 255.

References

[1] Gonzalez, Rafael and Woods, Richard (2008). Digital image processing. 3rd ed. Harlow: Prentice Hall, p.403.
[2] Gonzalez, Rafael and Woods, Richard (2008). Digital image processing. 3rd ed. Harlow: Prentice Hall, p.409.