IE 440 Fall 2019 - Nonlinear Models in Operations Research

Homework # 1

due October 21^{th}*, Monday, 17:00*

Consider the following function:

$$f(x) = x^3 \cos(x)^2 \sin(x) + 3x^2 \cos(x) - 5x$$

You are asked to find a local minimum of this function between values $a = -3$ and $b = 9$ using:

- Bisection method
- Golden section method
- Newton's method
- Secant method

You are expected to write programs finding local minimum of the given function by using the methods above, do the tasks below, and report your results. You are allowed to use C, C++, C#, Python and R. C++ and R are preferred. No hard copies are required. Only submit soft copies by using the link on Moodle.

Caution: No use of packages or libraries are allowed. You MUST make your own implementations of the methods. Homeworks done by using the ready-to-use packages will not be accepted.

Your tasks are as follows.

1. Draw the graph of the function between values $a = -3$ and $b = 9$.
2. Determine at least three different parameter sets ((ϵ, a, b) for Bisection and Golden Section, $(\epsilon, x^{(0)})$ for Newton's and Secant Methods) to check whether your code converges to a local minimum or not.
3. Show the point that your code converges to on the graph.
4. Include in your report the results of your code for all parameter sets.

Your report should consist of a separate section for each method, which contains the following in the given order

- The parameters used
- Graph of the function (with intervals and optimum solutions marked clearly)
- Results of your code in the following output format
- The source code

Output Format:

Solution for Bisection Method:

Iteration	a	b	x	$f(x)$	$\frac{\lvert x^{(k+1)}-x^{(k)}\rvert}{\lvert x^{(k)}-x^{(k-1)}\rvert}$	$-log\lvert x^{(k+1)}-x^{(k)}\rvert + log\lvert x^{(k)}-x^{(k-1)}\rvert$
0	...	...	...	...	...	...
1	...	...	...	...	...	...
...						

$$x^* = ...$$
$$f(x^*) = ...$$

Solution for Golden Section Method:

Iteration	a	b	x	y	$f(x)$	$f(y)$	$\frac{\lvert x^{(k+1)}-x^{(k)}\rvert}{\lvert x^{(k)}-x^{(k-1)}\rvert}$	$-log\lvert x^{(k+1)}-x^{(k)}\rvert + log\lvert x^{(k)}-x^{(k-1)}\rvert$
0	...	...	...	...	...	...	...	...
1	...	...	...	...	...	...	...	...
...								

$$x^* = ...$$
$$f(x^*) = ...$$

Solution for Newton's Method:

Iteration	$x^{(k)}$	$f_k(x^{(k)})$	$f'_k(x^{(k)})$	$f''_k(x^{(k)})$	$\frac{\lvert x^{(k+1)}-x^{(k)}\rvert}{\lvert x^{(k)}-x^{(k-1)}\rvert^2}$
0	...	...	...	...	...
1	...	...	...	...	...
...					

$$x^* = ...$$
$$f(x^*) = ...$$

Solution for Secant Method:

Iteration	$x^{(k)}$	$f_k(x^{(k)})$	$f'_k(x^{(k)})$	$\frac{\lvert x^{(k+1)}-x^{(k)}\rvert}{\lvert x^{(k)}-x^{(k-1)}\rvert^{1.618}}$
0	...	...	...	...
1	...	...	...	...
...				

$$x^* = ...$$
$$f(x^*) = ...$$