Question 3

Part a)

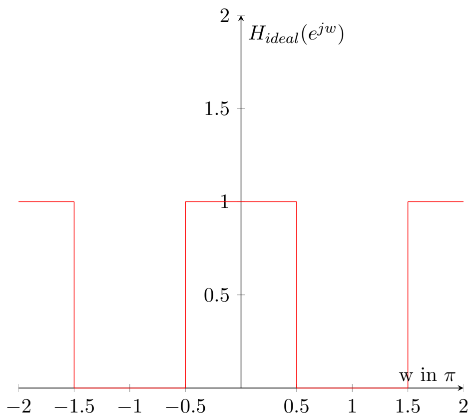

Figure 1: The desired freq. response

The graph of the desired frequency response magnitude over the range $0 < |w| < 2\pi$ is given in the Figure 1.

Since it cannot be implemented exactly, one way to implement approximately is frequency sampling or its another name DTFT-based filter design.

Part b)

The frequency axis is divided into $N = 9$ equally spaced pieces as $w = 2\pi k/N$, where $k = [0 : N - 1]$. And H_m is formed for the desired magnitudes by using the corresponding values of $|H_{ideal}(e^{jw})|$. The plot of H_m against w is given in the Figure 2.

It can be seen from the Figure 2 that it has a low-pass filter characteristics since it passes the low frequencies (approximately $[0, \pi/2]$ and $[3\pi/2, 2\pi]$) and blocks the high frequencies (approximately $[\pi/2, 3\pi/2]$).

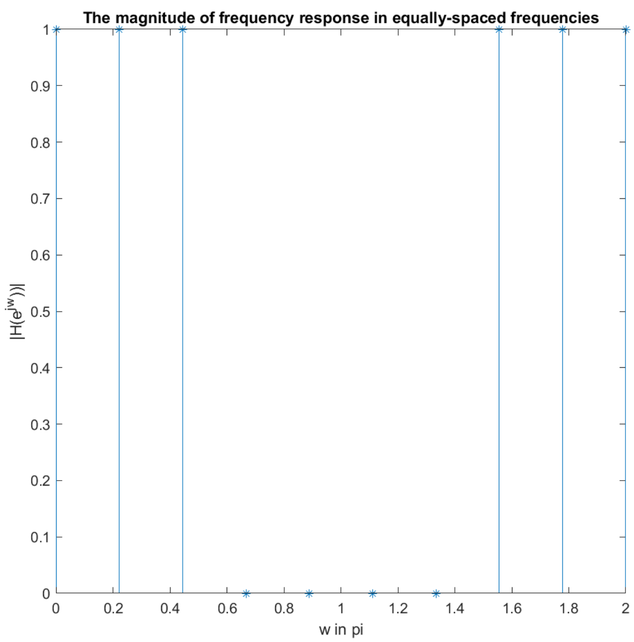

Figure 2: The magnitude of frequency response in equally-spaced frequencies

Since H_m is periodic with 2π, the value of Hm at 2π is equal to $H_m(0)$.

Part c)

In order to have purely real $H(e^{jw})$ meaning that its phase is zero, $h[n]$ should be symmetric around $n = 0$. However, it is desired to have a causal system; therefore, $h[n]$ should be right-sided. Since the delay in the time domain introduces a phase shift in the frequency domain, the phase of the causal $h[n]$ that is related to the ideal zero-phase filter by a delay of $(N-1)/2$, where $N = 9$ samples will be $\angle e^{-jw(N-1)/2} = -4w$.

Part d)

Each of the samples of H that are found in the previous parts can be stated by linear equations in terms of the $h[n]$'s; and $h[n]$'s over $n = [0 : N-1]$ can be recovered. So, the linear equations can be shown as $Fh = H$, where F is the discrete Fourier transform matrix, h is the impulse response sequence, and H is the discrete Fourier transform. From the equations, $h = F^{-1}H$ is derived to find the impulse response; the resulting impulse response[1] is shown in the Figure 3.

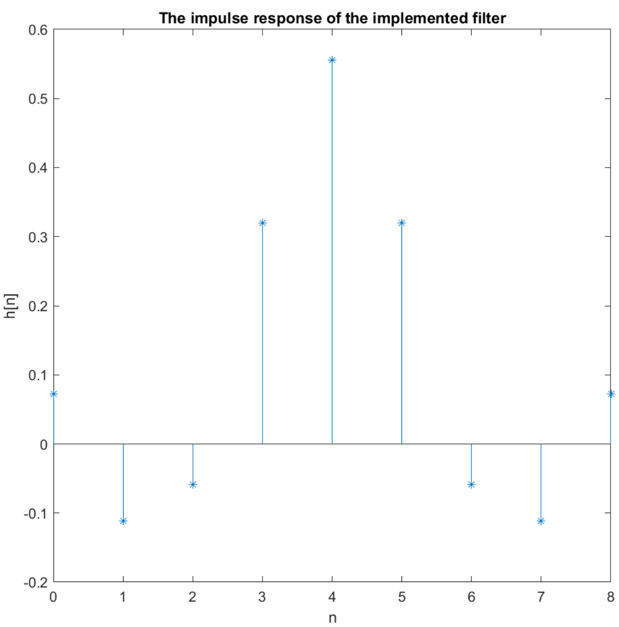

Figure 3: The impulse response of the implemented system

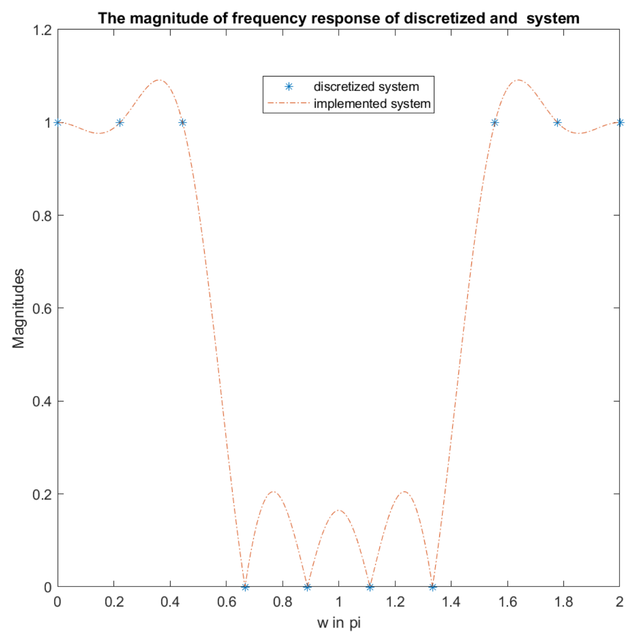

Figure 4: The magnitude of frequency response of discretized and implemented system

Part e)

The discrete time Fourier transform of $h[n]$ is calculated in order to verify the discretized system has the desired DTFT magnitude at the frequency samples specified in the part b. The frequency response of the implemented system is calculated with the 9000 many points in order to be able to see what happens between the specified frequency samples. The magnitude of the frequency response of the discretized and implemented system is shown in the Figure 4. It can be seen that they both give the same values at the specified frequency samples; however, in-between, the implemented system has some oscillations in the flat regions of the discretized system.

[1]The imaginary parts of it are ignored since they are around 10^{-16}