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Homework 3

Introduction

Phase and timing estimation is applied to the modulation type QPSK with unit energy and with the symbol constellation of $\phi_i = \{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$ for $i = 1, 2, 3, 4$. The transmitted signal $s(t)$, and the received signal $r(t)$ under perfect synchronization are shown in the following expression 1.

$$s(t) = \sum_{n=1}^{N} g(t - nT)cos(2\pi f_c t + \phi_n),\ r(t) = \sum_{n=1}^{N} g(t - nT)cos(2\pi f_c t + \phi_n) + n(t) \quad (1)$$

$g(t)$ is a raised cosine pulse shaping filter with a roll-off factor of $\alpha = 0.5$, $T = 1\mu s$ is the symboling period, N is the number of transmitted symbols, $f_c = 1GHz$ is the carrier frequency, ϕ_n is the phase of the n^{th} information symbol, and $n(t)$ is the additive white Gaussian noise. For following cases, N pilot symbols are transmitted, namely, the receiver knows the transmitted information before the communication starts. And the symbol $\phi_n = \pi/4$ is chosen for all n without loss of generality.

1 Case 1: Phase Estimation

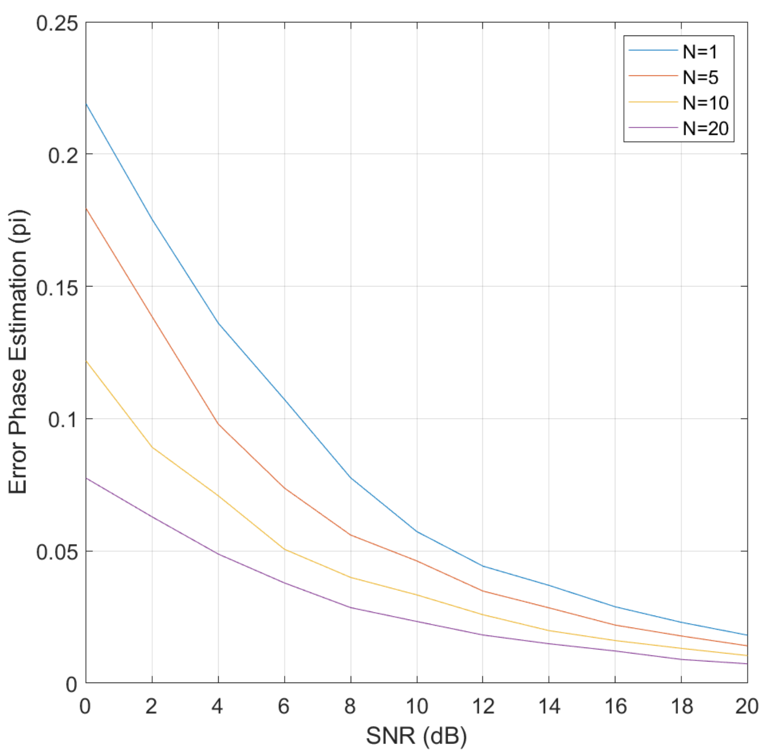

Figure 1: The error in phase estimation vs SNR

There exists only phase shift $\phi = \pi/4$ in the transmitted signals. Since the receiver is not aware of ϕ, the ϕ value should be estimated.

The communication system with the given specification above is simulated with $SNR = \{0, 2, 4, ..., 20\}$ and $N = 1, 5, 10, 20$. For each SNR and N value, the phase estimation $\hat{\phi}$ and the phase error $\phi_e = |\hat{\phi} - \phi|$ at the receiver are obtained. The plot of the phase errors in π as a function of SNR in dB is given in the Figure 1.

The method *the decision directed carrier phase estimate* is used to estimate the phase. In order to apply this method, the signals should be taken to the base band, and they should be handled in the base band. The phase $\hat{\phi}$ is determined in a way that it maximizes the correlation of the received pilot signals and the pilot signals with phase $\hat{\phi}$ that is produced by the receiver. The formula *decision directed* which is derived in the class gives the phase $\hat{\phi}$ that maximizes the correlation.

It can be seen from the Figure 1 that increasing the number pilot symbols gives better performance. However, increasing them requires more energy, also it reduces the load size of the transmitted signals; namely, instead of sending data, the pilot signals should be sent.

2 Case 2: Timing Estimation

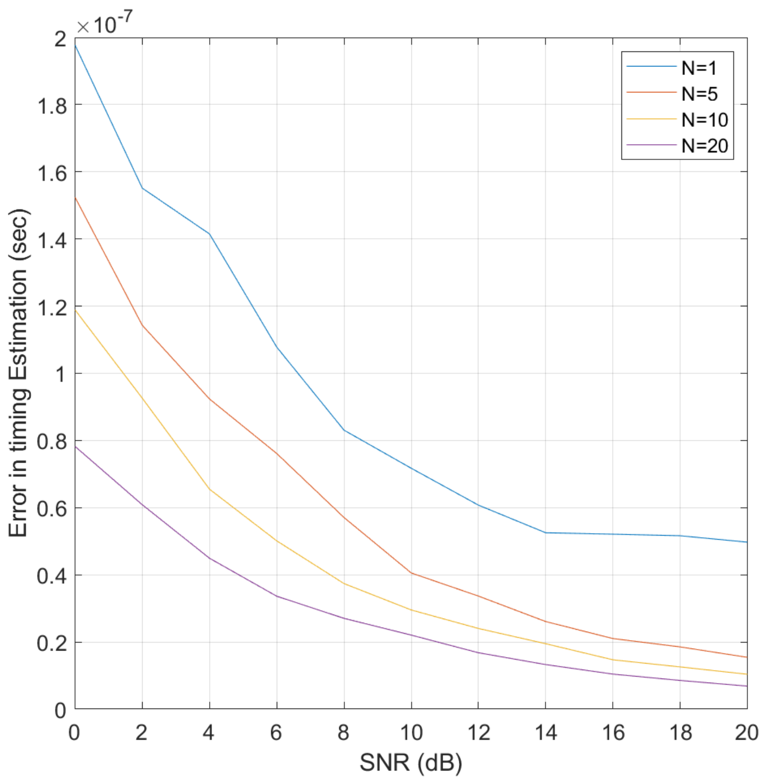

Figure 2: The error in timing estimation vs SNR

There exists only timing error $\tau = T/4$ in the transmitted signals. Since the receiver is not aware of τ, the τ value should be estimated.

The communication system with the given specification above is simulated with $SNR = \{0, 2, 4, ..., 20\}$ and $N = 1, 5, 10, 20$. For each SNR and N value, the timing estimation $\hat{\tau}$ and the timing error $\tau_e = |\hat{\tau} - \tau|$ at the receiver are obtained. The plot of the timing error in *sec* as a function of SNR in *dB* is given in the Figure 2.

The method *decision directed maximum likelihood timing estimation* is used to estimate the timing shift. In order to apply this method, the signals should be taken to the base band, and they should be handled in the base band. The timing shift $\hat{\tau}$ is determined in a way that it maximizes the correlation of the received pilot signals and the pilot signals with timing shift $\hat{\tau}$ that is produced by the receiver. In order to find the timing shift $\hat{\tau}$, the τ values from 0 to T with an increment of $0.01T$ are applied to the pilot signals and the correlation of them with the received signals are calculated.

It can be seen from the Figure 2 that increasing the number pilot symbols gives better performance. With the same reasoning from the previous part, increasing them requires more energy, also it reduces the load size of the transmitted signals.

3 Case 3: Phase and Timing Estimation together

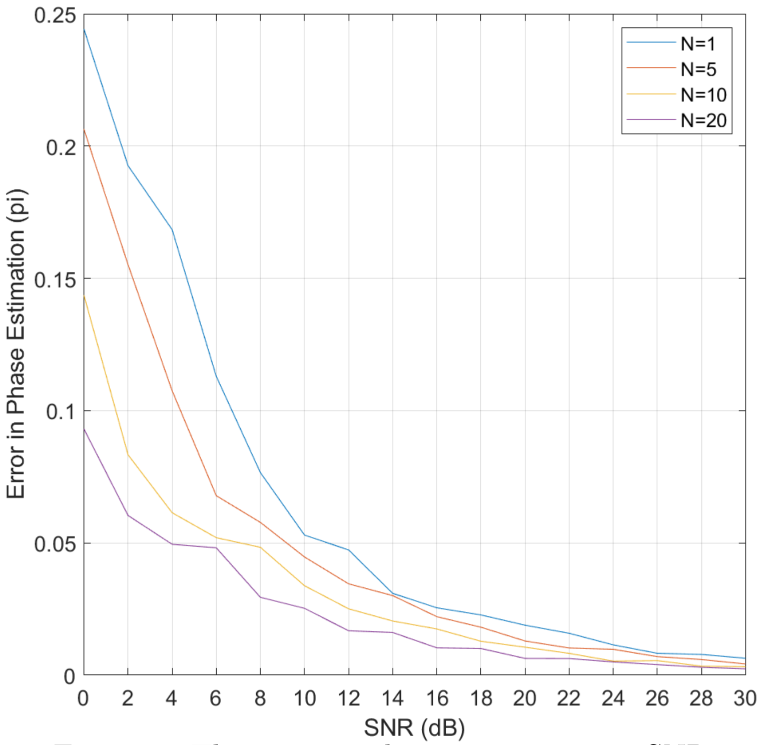

Figure 3: The error in phase estimation vs SNR

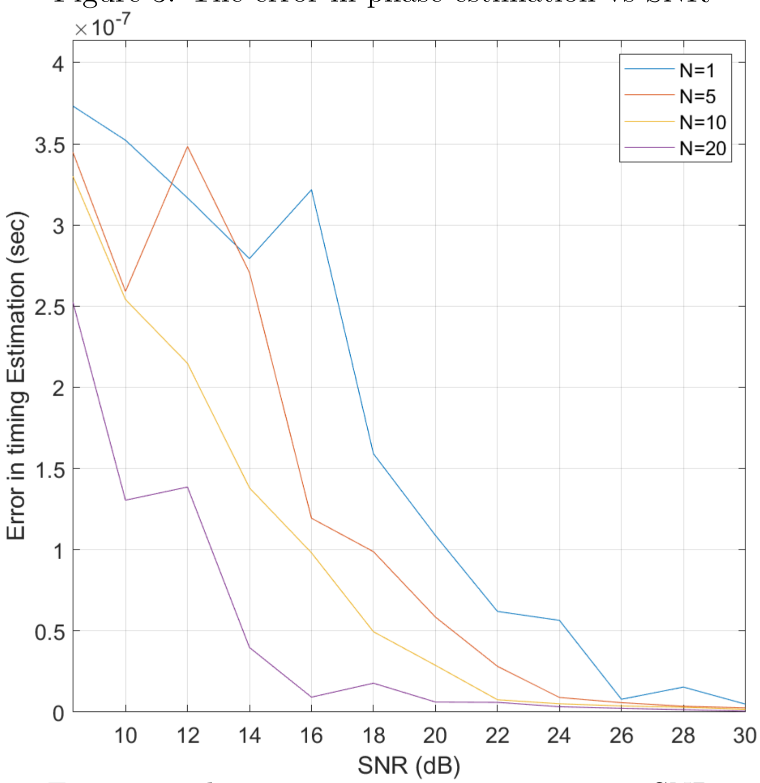

Figure 4: The error in timing estimation vs SNR

There exist both timing error $\tau = T/30$ and phase shift $\phi = 2\pi/3$ in the transmitted signals. Since the receiver is not aware of τ, and ϕ, their values should be estimated.

The communication system with the given specification above is simulated with $SNR = \{0, 2, 4, ..., 30\}$ (Since quantization also introduces errors, the larger SNR values make more sense, for example, in the algorithm T/30 shift corresponds 3 samples shift out of 100 samples, so it corresponds to 0.3T shift not exactly T/30 shift) and $N = 1, 5, 10, 20$. For each SNR and N value, the timing estimation $\hat{\tau}$, the phase estimation $\hat{\phi}$ and the phase error $\phi_e = |\hat{\phi} - \phi|$ and the timing error $\tau_e = |\hat{\tau} - \tau|$ at the receiver are obtained. The plot of the phase error in π and the timing error in *sec* as a function of SNR in *dB* is given in the Figure 3, 4, respectively. The method *decision directed joint maximum likelihood timing and phase estimation* is used to estimate the timing shift. In order to apply this method, the signals should be taken to the base band, and they should be handled in the base band. It is the combination of the methods that are implemented previous parts. First, the timing shift $\hat{\tau}$ is estimated by trying all the combinations of τ and ϕ values to find which τ gives the maximum correlation of the received pilot signals and the pilot signals with timing shift τ. After finding $\hat{\tau}$, the phase shift $\hat{\phi}$ is estimated by using the formula derived in the class with the y_k's of $\hat{\tau}$, namely the y_k's are calculated at $\tau = \hat{\tau}$.

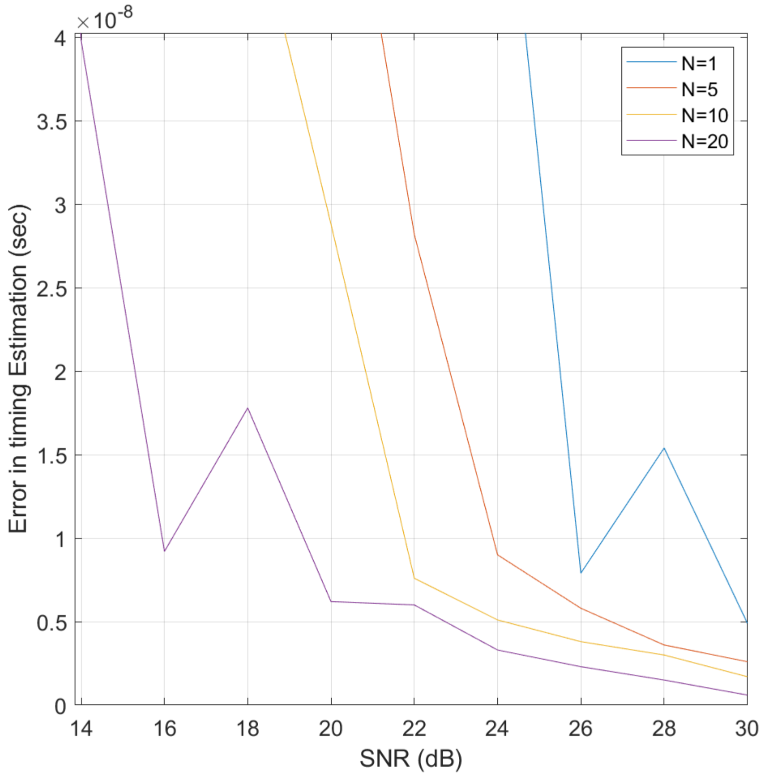

Figure 5: The zoomed version of Figure 4

It can be seen from the Figure 3, 4 that increasing the number pilot symbols gives better performance. With the same reasoning from the previous parts, increasing them requires more energy, also it reduces the load size of the transmitted signals. And at low SNR values, timing estimation is really bad since timing shift is equal to $0.333 * 10^{-7} sec$, and the error falls below after $14dB$ SNR.
The zigzagging occurs due to the insufficient repeat of the algorithm, it is repeated 100 times.

If the graph in the Figure 4 is zoomed after $14dB$ SNR, the Figure 5 is obtained. In order to get better results, SNR can be increased or the quantization interval can be increased. Namely, instead of $0 - 100$ quantization, $0 - 1000$ would give a better result; however, running the code takes already almost thirty minutes, increasing the quantization interval will make is even slower. Also, the workspace of the case 3 is added to the files since running the code takes almost thirty minutes.