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Homework 2

Question 2

Part a-b)

The continuous and discrete signals $x(t) = cos(\Omega t)$ and $x[n] = cos(\omega n)$ for $\Omega, \omega = 0, \pi/2, \pi$, $3\pi/2, 2\pi$ are shown in the Figures 1,2,3,4,5.

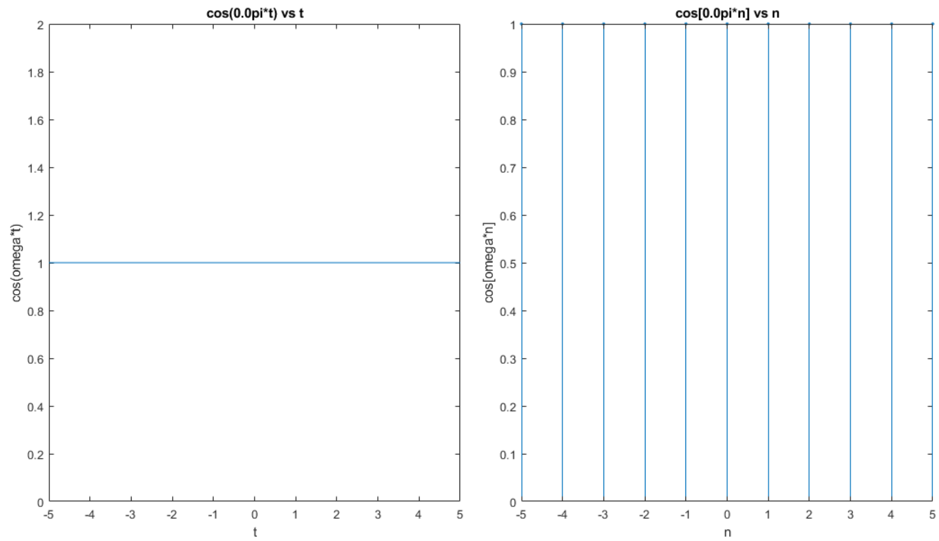

Figure 1: The continuous and discrete cosine wave with $\Omega, \omega = 0$

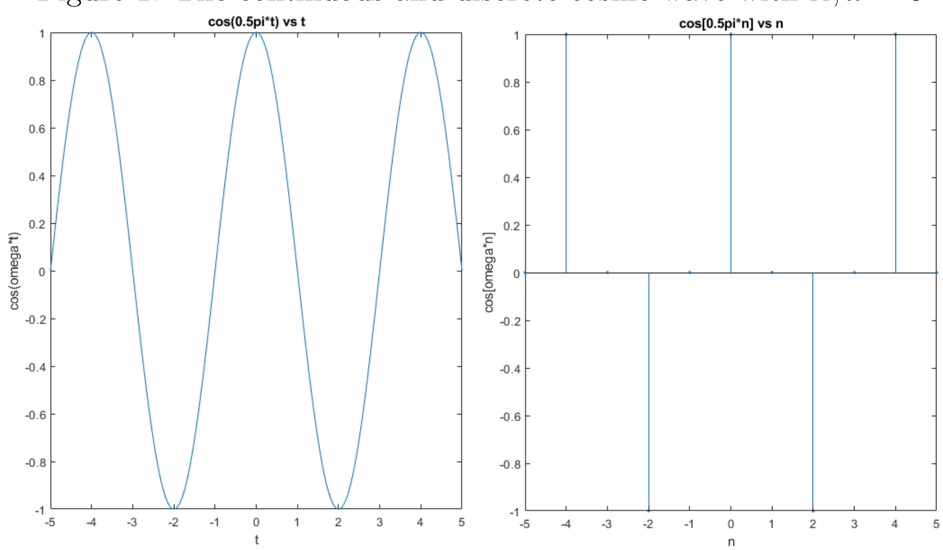

Figure 2: The continuous and discrete cosine wave with $\Omega, \omega = \pi/2$

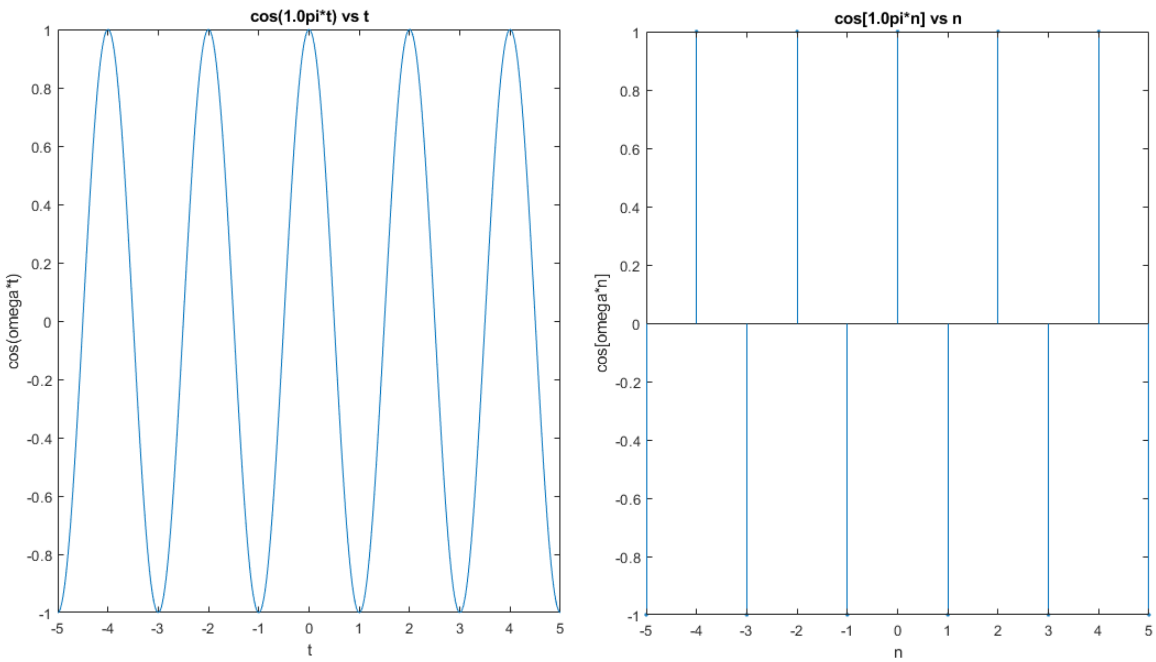

Figure 3: The continuous and discrete cosine wave with $\Omega, \omega = \pi$

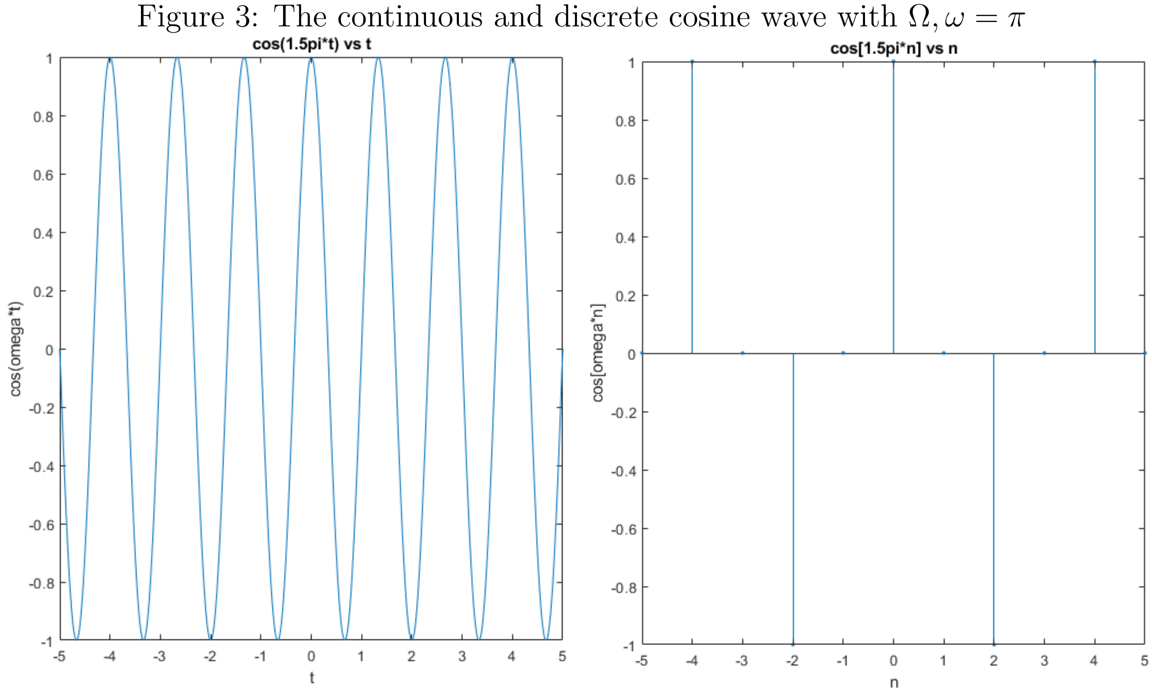

Figure 4: The continuous and discrete cosine wave with $\Omega, \omega = 3\pi/2$

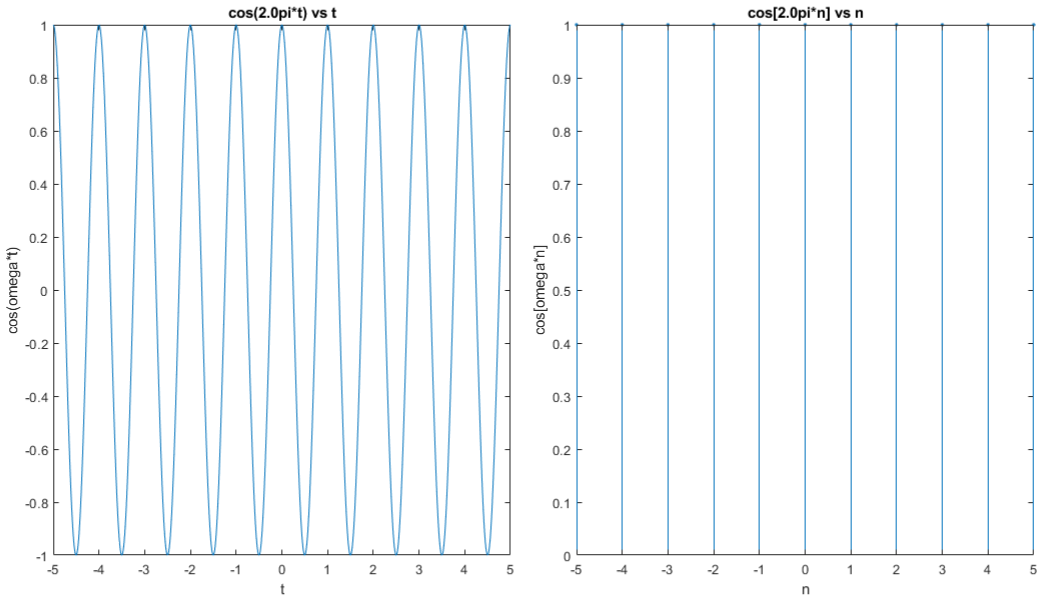

Figure 5: The continuous and discrete cosine wave with $\Omega, \omega = 2\pi$

Part c)

It can be seen from the above figures that the continuous time cosine signals have an inversely proportional relationship between frequency and period such that $T = 1/f$. The frequencies and periods of the continuous time cosine signals can be seen in the following expression 1.

$$\begin{gathered}
\Omega = 0 \rightarrow f = 0, T = \infty \\
\Omega = \pi/2 \rightarrow f = 1/4, T = 4 \\
\Omega = \pi \rightarrow f = 1/2, T = 2 \\
\Omega = 3\pi/2 \rightarrow f = 3/4, T = 4/3 \\
\Omega = 2\pi \rightarrow f = 1, T = 1
\end{gathered} \tag{1}$$

On the other hand, in the discrete time cosine signals, there exists no direct relationship between frequency and period that is valid for all values; due to the fact that the periodicity should be an integer value. The frequencies and periods of the discrete time cosine signals can be seen in the following expression 2.

$$\begin{gathered}
\omega = 0 \rightarrow f = 0, N = 1 \\
\omega = \pi/2 \rightarrow f = 1/4, N = 4 \\
\omega = \pi \rightarrow f = 1/2, N = 2 \\
\omega = 3\pi/2 \rightarrow f = 3/4, N = 4 \\
\omega = 2\pi \rightarrow f = 1, N = 1
\end{gathered} \tag{2}$$

The difference in the periodicity of the continuous and discrete time signals can easily be seen in $\Omega, \omega = 0, 3\pi/2$. However, the definition of the frequency in both discrete and continuous time signals is the same, so their frequencies are equal, respectively.

Question 2

Part a)

When the discrete-time Fourier transform of the equation (1) in the question is taken, the following expression 3 is obtained:

$$Y(e^{jw}) = X(e^{jw}) + \alpha X(e^{jw})e^{-jwN} \tag{3}$$

And, the impulse response of the echo system is derived by the following expression 4.

$$h[n] = F^{-1}\left(\frac{Y(e^{jw})}{X(e^{jw})}\right) = F^{-1}\left(1 + \alpha e^{-jwN}\right) = \delta[n] + \alpha\delta[n-N] \tag{4}$$

Part b)

The impulse response of the echo system with $N = 1000$ and $\alpha = 0.5$ is derived in MATLAB using $filter$ function with unit impulse and the coefficients of the $x[n]$ and $y[n]$. So, the resulting response in the range $n = 0, ..., 1000$ is given in the Figure 6.

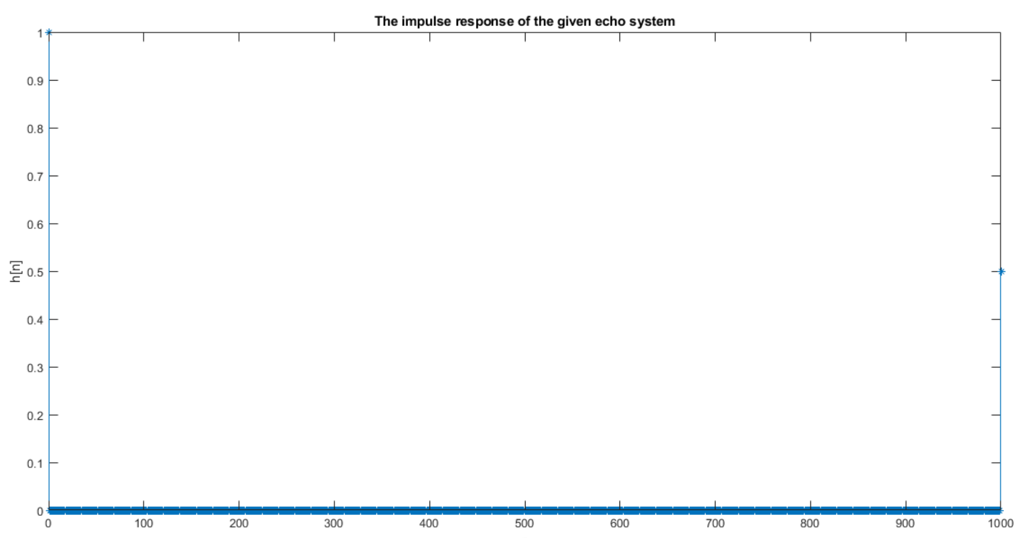

Figure 6: The impulse response of the given echo system in the range $n = 0, ..., 1000$

Part c)

The given echo removal system has an infinite-length impulse response, since the output $z[n]$ is recursive in the given difference equation. Therefore, there should be a feedback mechanism in the implementation of the system.

Part d)

With the echo removal system, the original speech signal can be obtained. The diagram in the Figure 7 shows the overall system.

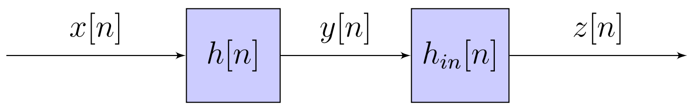

Figure 7: The diagram of the overall system

In the time domain, the overall system can be desribed by the following expression 5.

$$z[n] = y[n] * h_{in}[n] = x[n] * h[n] * h_{in}[n] \tag{5}$$

If the discrete time Fourier transform of the expression 5 is taken, the following 6 will be obtained.

$$Z(e^{jw}) = X(e^{jw})H(e^{jw})H_{in}(e^{jw}),\ where\ H(e^{jw}) = 1 + \alpha e^{-jwN},\ H_{in}(e^{jw}) = \frac{1}{1 + \alpha e^{-jwN}} \tag{6}$$

After cancellations, $Z(e^{jw}) = X(e^{jw})$ is obtained. Then, their inverse discrete time Fourier transform is taken and $z[n] = x[n]$ will be found.
Actually, the equality of the $x[n]$ and $y[n]$ can also be derived from the given difference equations. When the $y[n]$ in the 2^{nd} equation is substituted to the 1^{st} equation, $z[n] = x[n]$ is obtained easily.
However, the equality will be valid when the absolute value of the α is smaller than 1, since the inverse discrete time Fourier transform of the echo removal system exists in this range.

Part e)

The phrase *lineup* with echo is given as an input to the echo removal system that is described by the given difference equation. The response of the system is given in the Figure 8. And the impulse response of the echo removal system is given in the Figure 9.

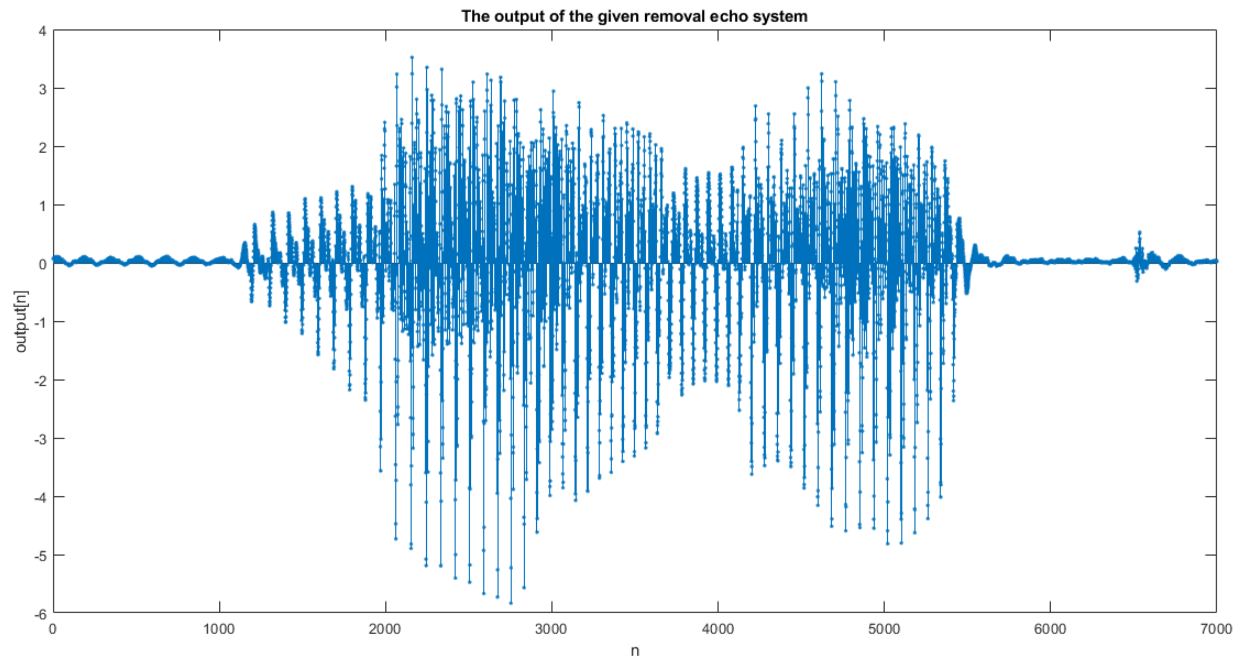

Figure 8: The output of the echo removal system in response to the *lineup*

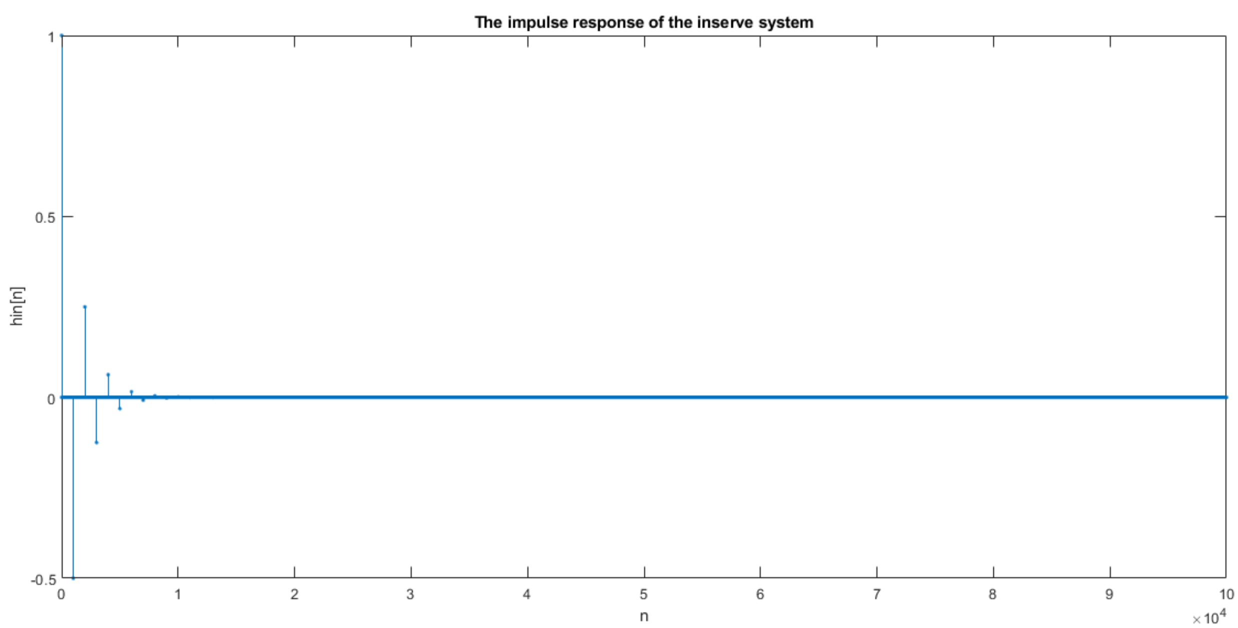

Figure 9: The impulse response of the echo removal system

When the resulting sound is listened; the sound is clearer than the input, the echo is gone, pretty much.

Part f)

The impulse response of the overall system can be achieved by convolving the impulse response of the echo system given in the Figure 6 and the impulse response of the echo removal system given in the Figure 9. The result of the convolution is given in the Figure 10.

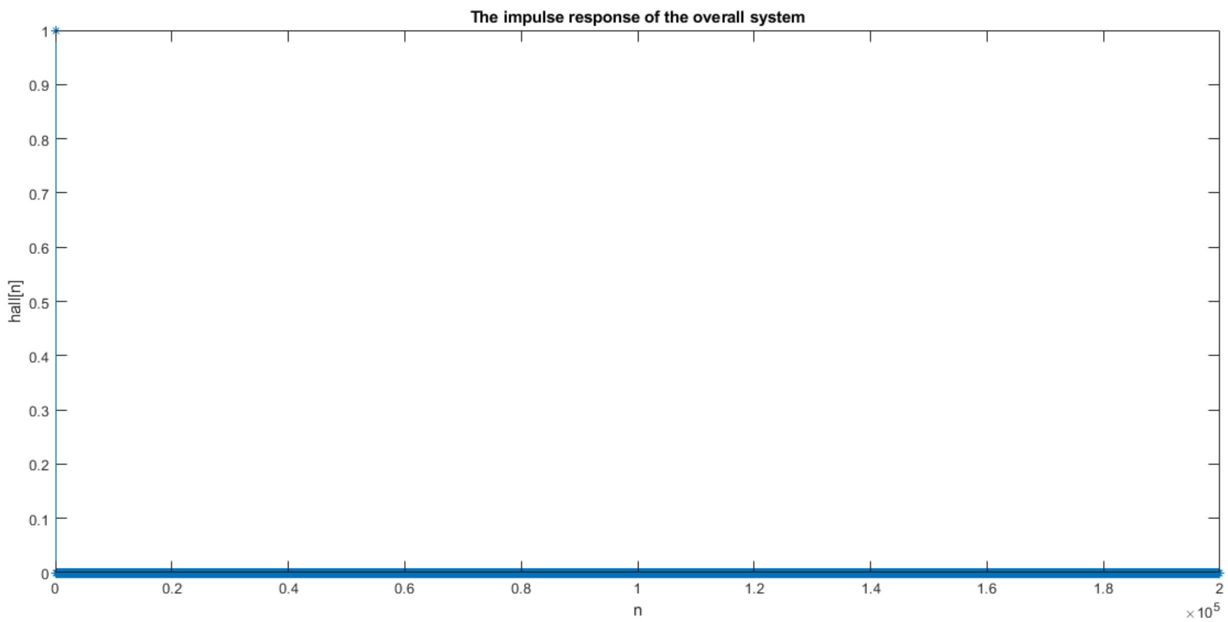

Figure 10: The impulse of the overall system

At first, the unit impulse couldn't be achieved in the overall system, since when in the calculation of the impulse responses of the systems, the size[1] of the unit impulse was too small (the size was $1e3$); therefore, the resulting impulse responses were wrong. After enlarging the size of the unit response (the size is $1e5$, now), the correct impulse responses are obtained. Therefore, in the overall system, the result is obtained as a unit impulse.

[1]It cannot be infinity; since when implementing the unit impulse in the MATLAB, the size should be finite, where the size means the number of elements in the array of unit impulse.