EE 473 HW 3 (Fall 2019)

1) Homework is due **November 7, Thursday!**
2) Do NOT exceed 3 pages when you turn in your solution to **3**. Additional pages will not be graded, or even looked at.
3) Be neat and well-organized with your submission and coding. Sloppy homeworks, including hand-written ones, will be rewarded with a 25-point deduction from now on.

1: *Reading.* We have completed Section 4 of Oppenheim and Schafer/2e in class except for Sections 4.8 and 4.9, which are about quantization, and hence, truly digital signals. You are already familiar with some of these concepts but do read Section 4.8. That is, you will be responsible for 4.8.

2: *Homework 1.* There are several versions of "the phase" depending on the specified range of the function. We will cover these briefly in class but you are referred to Section 5.3, pp. 255 (last paragraph)-257 for detail. Do not blindly trust the results you get from Matlab, or any other software, without proper interpretation and verification.

3: *Frequency Sampling-Based Filter Design.* The frequency response of any stable discrete-time LTI system is given by the discrete-time Fourier transform (DTFT) of its impulse response. In this problem, you will use DTFT to design an FIR filter based on a set of samples from the desired frequency response. This method is aptly named frequency sampling- or DTFT-based filter design.

The goal is to obtain an FIR approximation to the ideal low-pass filter

$$H_{\text{ideal}}\left(e^{j\omega}\right) = \begin{cases} 1, & |\omega| < \pi/2, \\ 0, & \pi/2 < |\omega| < \pi. \end{cases}$$

(a) Draw the desired frequency response magnitude over the range $0 < |\omega| < 2\pi$.

(b) Construct the vector that consists of $N = 9$ equally-spaced frequencies in $[0, 2\pi)$: $\mathtt{w} = 2 * \pi * \mathtt{k}/\mathtt{N}$ for $\mathtt{k} = [0 : \mathtt{N} - 1], \mathtt{N} = 9$. Store in vector `Hm` the desired magnitudes $|H_{\text{ideal}}\left(e^{j\omega}\right)|$ that correspond to the frequencies in `w`. Plot the desired frequency response magnitude against `w`. Does this look like a low-pass filter?

(c) With the desired magnitude response out of the way, the next step is to specify the phase. If the impulse response $h[n]$ were symmetric around $n = 0$, the frequency response, $H\left(e^{j\omega}\right)$, could be made purely real, in which case $H\left(e^{j\omega}\right)$ would have zero phase. However, we restrict the design to causal systems, and the impulse response will be right-sided. What is the phase of the causal $h[n]$ that is related to the aforementioned zero-phase filter by a delay of $(N-1)/2$ samples? Store in vector `Hp` the values of this phase at the frequency samples in `w`.

(d) The vectors `Hm` and `Hp` you just created contain nine pieces of information about the 9-point $h[n]$. Each of the samples of the DTFT of $h[n]$ can be stated in the form of a linear equation that involves $h[n]$. By simultaneously solving these nine equations for the nine unknowns, $h[0], h[1], h[2], \ldots, h[8]$, you can recover the impulse response of the low-pass filter. In matrix form, these equations are

$$\underbrace{\begin{bmatrix} 1 & e^{j\omega_0} & e^{j2\omega_0} & \ldots & e^{j8\omega_0} \\ 1 & e^{j\omega_1} & e^{j2\omega_1} & \ldots & e^{j8\omega_1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & e^{j\omega_8} & e^{j2\omega_8} & \ldots & e^{j8\omega_8} \end{bmatrix}}_{\mathbf{F}} \begin{bmatrix} h[0] \\ h[1] \\ \vdots \\ h[8] \end{bmatrix} = \underbrace{\begin{bmatrix} H\left(e^{j\omega_0}\right) \\ H\left(e^{j\omega_1}\right) \\ \vdots \\ H\left(e^{j\omega_8}\right) \end{bmatrix}}_{\mathbf{H}}$$

where $\omega_k = 2\pi k/9$. The matrix $\mathbf{F}$ on the left-hand side is known as the discrete Fourier transform (DFT) matrix. We will see more about DFT later on, but for now, beware that DFT and DTFT are not the same. The desired frequency response vector `H` is constructed by combining `Hm` and `Hp`. Solve the set of linear equations and plot the resulting impulse response.

(e) Verify that the impulse response has the desired DTFT magnitude at the specified frequency samples by plotting the magnitude of the DTFT of $h[n]$ and the magnitude of the desired DTFT samples on the same plot. Be sure to plot at least 1000 evenly-spaced points of the DTFT magnitude to see what happens between the specified samples.