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Homework 7

Question 2

The given utterance is recorded using Wavesurfer at 16-kHz sampling rate and 16-bit PCM encoding. And white Gaussian noise with the energy of $1/20^{th}$ of that of signal is added using *awgn* function in MATLAB. The waveforms of the original and noisy sound are given in the Figure 3, and 4. And the frequency domain representation of them is shown in the Figure 1,and 2.

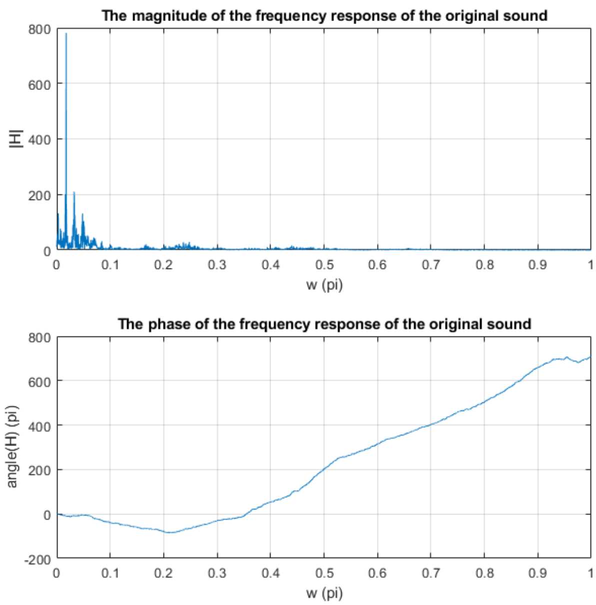

Fig. 1. The frequency domain representation of the original sound

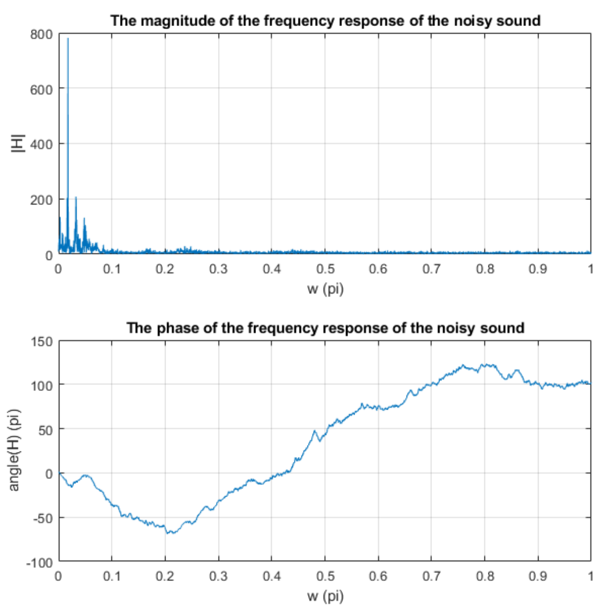

Fig. 2. The frequency domain representation of the noisy sound

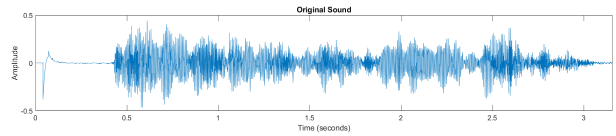

Fig. 3. The waveform of the original sound

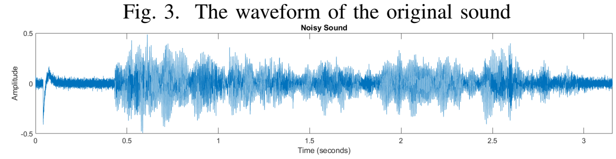

Fig. 4. The waveform of the noisy sound

In order to filter out the noise, a low pass filter should be used. So, the following approaches are used to design a low pass filter.

Question 3

Impulse invariance approach:

Discrete time filter specifications are given in the following expression 1.

$$\begin{aligned} 0.95 \le |H(e^{j\omega})| \le 1 \qquad & 0 \le |\omega| \le 0.1\pi \\ |H(e^{j\omega})| \le 0.05 \qquad & 0.2\pi \le |\omega| \le \pi \end{aligned} \tag{1}$$

By using $\omega = \Omega T_d$, where T_d can be set to 1, the specifications are converted to the continuous time shown in the expression 2:

$$\begin{aligned} 0.95 \le |H(j\Omega)| \le 1 \qquad & 0 \le |\Omega| \le 0.1\pi \\ |H(j\Omega)| \le 0.05 \qquad & 0.2\pi \le |\Omega| \le \pi \end{aligned} \tag{2}$$

Using Butterworth filter design, the specification can be satisfied. So, the parameter N, and Ω_c should be found in order to design the filter. The procedure to find these parameter is shown in the equation 3.

$$\begin{aligned} &|H_c(j\Omega)|^2 = \frac{1}{1 + (j\Omega/j\Omega_c)^{2N}} \\ &|H_c(j0.1\pi)| \ge 0.95 \qquad |H_c(\pi)| \le 0.05 \\ &\rightarrow 1 + \left(\frac{0.1\pi}{\Omega_c}\right)^{2N} = \left(\frac{1}{0.95}\right)^2 \qquad 1 + \left(\frac{0.2\pi}{\Omega_c}\right)^{2N} = \left(\frac{1}{0.05}\right)^2 \\ &N = \frac{log\left[\left(\left(\frac{1}{0.95}\right)^2 - 1\right) \Big/ \left(\left(\frac{1}{0.05}\right)^2 - 1\right)\right]}{2log(0.1\pi/0.2\pi)} = 5.9253 \approx 6 \end{aligned} \tag{3}$$

If the function *butter* in MATLAB is called Ω_c with $'s'$, it designs an analog filter with the given parameters. Therefore, the function *butter* with the parameter $N = 6$ and $\Omega_c = 2*\pi*1000$ is used to design the analog lowpass filter. Then, *impinvar* is used to design the digital one with the impulse invariance approach.

So, the frequency response of the filtered noisy sound are shown in the Figure 5.

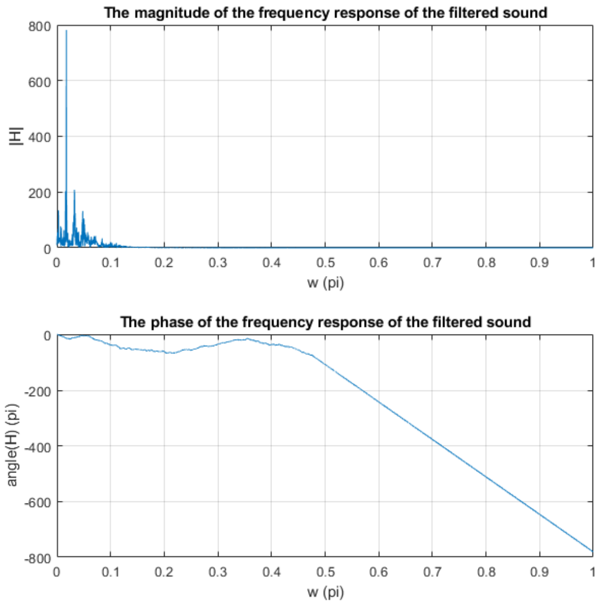

Fig. 5. The frequency response of the filtered noisy sound

Bilinear transformation approach:

The same specification shown in the equation 1 are also valid for the bilinear transformation. In this transformation, the relationship between Ω and ω is different from the previous one. It is $\Omega = \frac{2}{T_d}tan(\frac{\omega}{2})$, where T_d can be set to 1.

Also, in this approach, Butterworth filter design is used. The above calculation can be used to determine N. N is found by the following equation 4.

$$N = \frac{log\left[\left(\left(\frac{1}{0.95}\right)^2 - 1\right) \Big/ \left(\left(\frac{1}{0.05}\right)^2 - 1\right)\right]}{2log[tan(0.05\pi)/tan(0.1\pi)]} = 5.72 \approx 6 \tag{4}$$

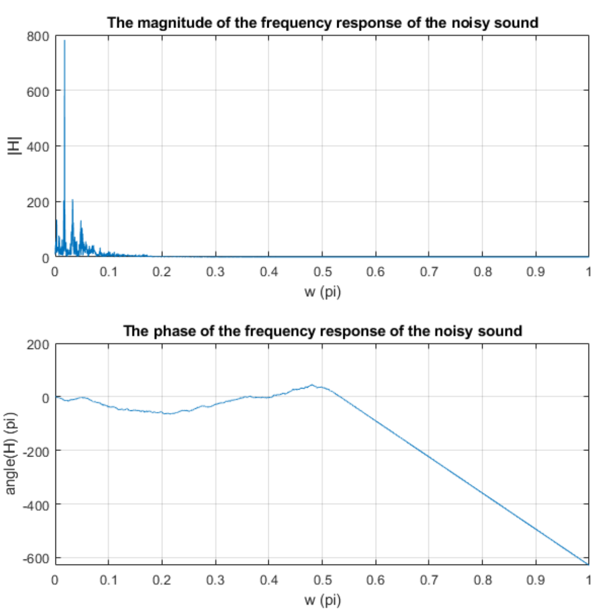

Fig. 6. The frequency response of the filtered noisy sound

If the function *butter* in MATLAB is called ω_c in the range of $[0,1]$, it designs the corresponding digital filter with the given parameters using bilinear transformation. Therefore, the function *butter* with the parameter $N = 6$ and $\omega_c = 0.15$ is used to design the lowpass filter.

So, the frequency response of the filtered noisy sound are shown in the Figure 6.

Parks-McClellan approach:

In order to design the filter with the Parks-McClellan approach, the filter order, n, the cut-off frequencies, w_p, w_s, and the ripple weight, κ, should be specified. In the previous approaches, w_p, w_s, and the ripple weight are selected as 0.1π, 0.2π, and $\kappa = 0.05/0.05 = 1$. The only parameter that will be determined is the order n. It is determined as the smallest one which gives the error, the maximum ripple size, as 0.05. So, $n = 21$ is the one that satisfies the above condition. Therefore, the function *firpm* is called with the parameters, $n = 21$, $w_p = 0.1\pi$, $w_s = 0.2\pi$, and $\kappa = 1$.

So, the frequency response of the filtered noisy sound are shown in the Figure 7.

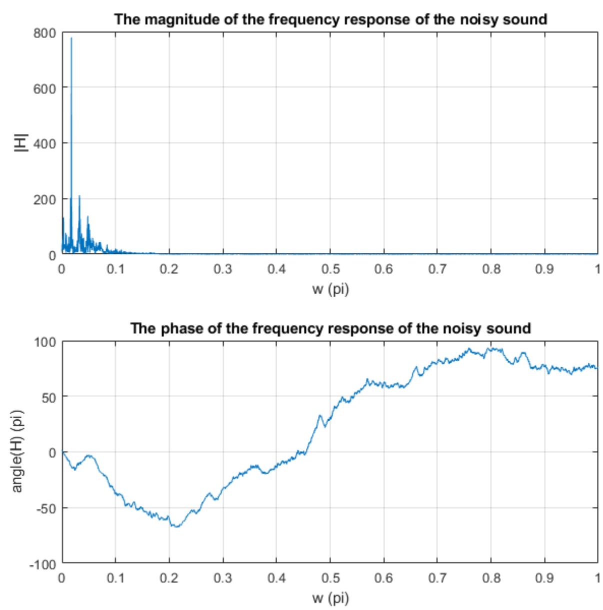

Fig. 7. The frequency response of the filtered noisy sound

Kaiser window approach:

In order to design the filter with the Kaiser window approach, the shape parameter β and the filter order M should be determined. The solution procedure of the HW6 is followed.

$\delta = min(\delta_s, \delta_p) = min(0.05, 0.05) = 0.05$
$\Rightarrow \quad A = -20log(0.05) = 26.02$
$\beta = 0.5842(A-21)^{0.4} + 0.07886(A-21)|_{A=26.02} =$

1.5099

$M = (A - 8)/(2.285\Delta w)|_{A=40,\Delta w=0.1\pi} = 25.1034 \quad \Rightarrow \quad M = 26$

Then, with the found parameters, and the cut-off frequency, $w_c = 0.15\pi$, the low pass filter using Kaiser window is designed.
So, the frequency response of the filtered noisy sound are shown in the Figure 8.

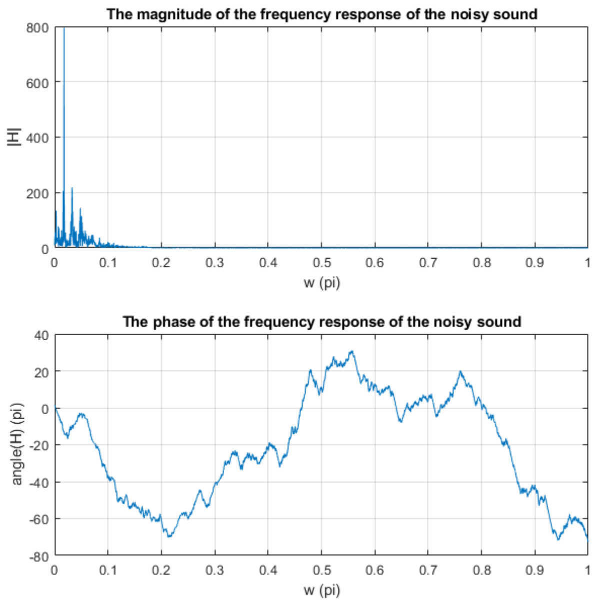

Fig. 8. The frequency response of the filtered noisy sound

Question 4

If the frequency responses of the designed filters are compared, it can be seen from the corresponding figures that the impulse invariance, and bilinear transform approaches disturb the phase response of the sound, whereas the Parks-McClellan and Kaiser window approaches don't disturb the phase response of the sound.

Another observation about the filters can be their magnitude responses and the ripples at the passband and stopband. So the magnitude responses of the filters are shown in the Figure 9, 10, 11, and 12.

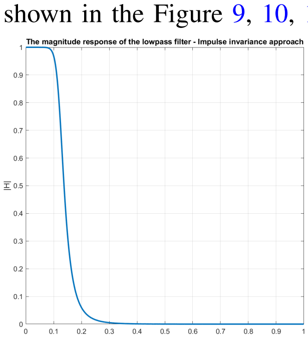

Fig. 9. The impulse invariance approach

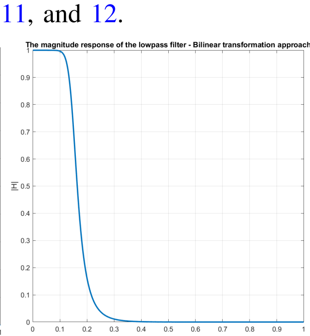

Fig. 10. The bilinear transformation approach

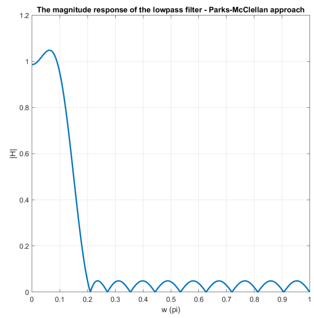

Fig. 11. The Parks-McClellan approach

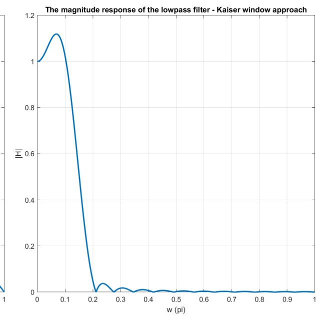

Fig. 12. The Kaiser window approach

The ones which are designed by converting analog filters to digital one gives almost flat response on the passband and stopband whereas the ones which are designed by the FIR filter give some ripples on the passband and stopband.

Therefore, if the phase distortion is not desired, then the Parks-McClellan and Kaiser window approaches will be preferred; whereas if the rippling effects are not desired, the impulse invariance and bilinear transform approaches will be preferred. In fact, both the phase distortion and the rippling effects are not desired, then the Parks-McClellan and Kaiser window approaches with much higher order should be implemented.

If the filter design approaches are ordered in terms of their performance (It is considered according to the discussion above.), and complexity (It can be seen as the filter order.), the following order from best to worst can be obtained: the bilinear transform approach, the impulse invariance approach, and the Kaiser window approach, the Parks-McClellan approach. The FIR filters are listed last since the ripples on the filter response have more impact on the cancellation of the noise. Whereas the phase distortion of the other 2 filters don't affect the sound quality so much. If the performance of the FIR filters is desired to improve, then their order should be increased.

If the filter design approaches are ordered based on the listening the filter outputs the following order from best to worst can be obtained: the Kaiser window approach, the bilinear transform approach, the impulse invariance approach, and the Parks-McClellan approach. The result of the Parks-McClellan approach gives the worst sound since it doesn't cancel the noise completely at the stopband (where the noisy sounds stem from) due to the large ripples. Actually, the performance order doesn't match with above one, since the Kaiser window approach gives the best sound and lower order than Parks-McClellan.