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Homework 3

***Abstract*—This document contains the solutions, explanations, comments, and related graphs and tables of the questions in the homework 3.**

I. Adaptive Linear Filtering

The filters can be used to remove the noises in the image. In this part, the contaminated version of the circuit board image shown in the figure 1 is used to filter. Firstly, it is contaminated with zero mean Gaussian noise having variance $\sigma_n^2 = 1024$. The version that is obtained by adding the noise is shown in the Figure 2.

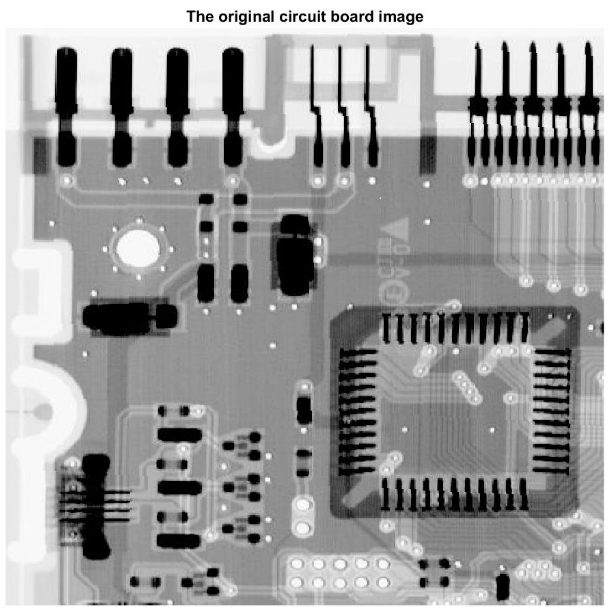

Fig. 1. The original circuit board image

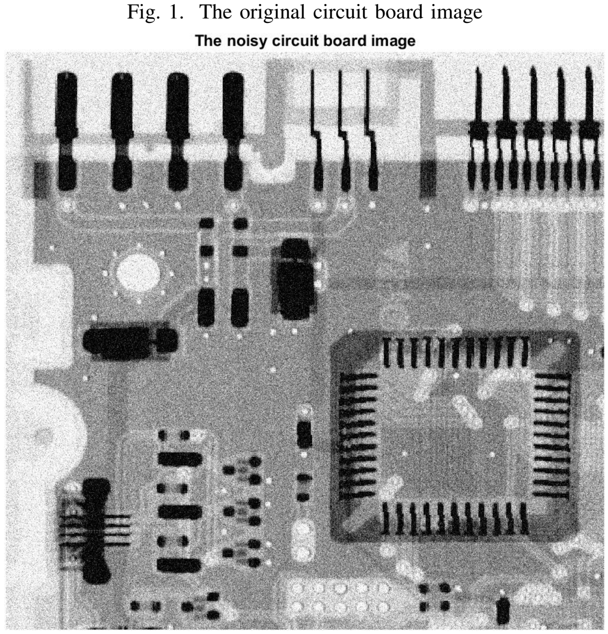

Fig. 2. The noisy circuit board image

In order to attenuate a zero mean Gaussian noise, arithmetic mean filter and geometric mean filter can be used.

A. Arithmetic Mean Filter

An example of a linear filter is arithmetic mean filter. This method uses an n by n matrix (filter window) that moves all over the image, and evaluates the arithmetic mean of its entries, then makes the corresponding pixel of the center entry equal to the mean. In order no to exceed the original image size at the edges, the image is extended with zero padding.
When the filter with the 7×7 filter window is applied to the noisy circuit board, the Figure 3 is obtained.

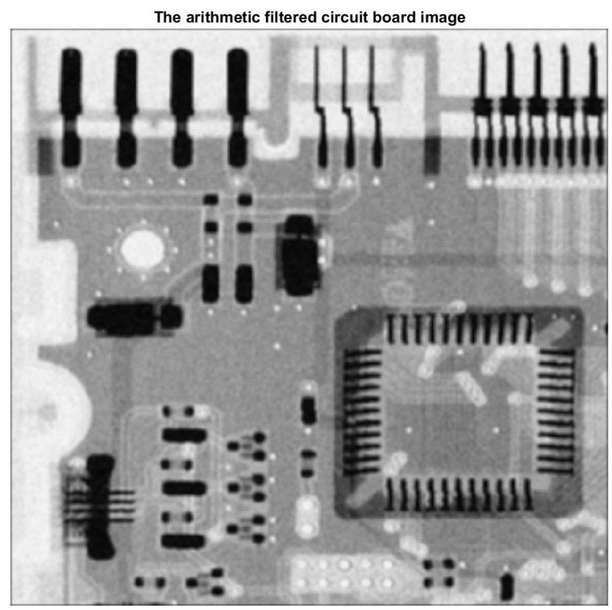

Fig. 3. The arithmetic filtered circuit board image

Since averaging is a low pass operation meaning that it suppresses the instant changes, and sharp edges in the image, the filter removes the noise, but it blurs the image and causes some losses in the detail.
It can be seen from the Figure 3 that the details in the sharp edges such as legs of the circuit components, writings and conducting paths are blurred and lost. However, it removes the noise and provides a much clearer image. There exist lines in the edges since the zero padding is applied and it effects the values of the pixels in the edges.

B. Geometric Mean Filter

Another example of filters is geometric mean filter. It also uses a filter window that moves all over the image, and

evaluates the geometric mean of its entries, then makes the corresponding pixel of the center entry equal to the mean. And zero padding is applied.
When the filter with the 7×7 filter window is applied to the noisy circuit board, the Figure 4 is obtained.

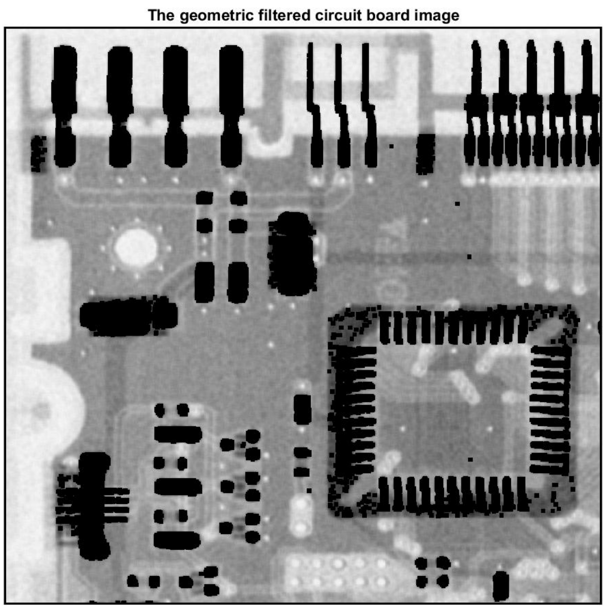

Fig. 4. The geometric filtered circuit board image

It can be seen from the Figure 4 that the filter is able to remove the noise; and also, the black regions are conserved. However, it adds some extra black regions and it disrupts the image in a way that it interprets the filter window as black (having 0 pixel value) whenever it has even one black pixel (having 0 pixel value). So, it causes extra black regions in the image.
There exist black lines in the edges due to the zero padding and it makes the values of the pixels in the edges equal to 0 since the geometric mean is 0.

C. Adaptive Arithmetic Mean Filter

The arithmetic mean filter can be improved in a way that it doesn't cause so much blurring and preserves the sharp edges in the image by taking into account the variances of the filter window. The variance gives information about whether the filter window has a sharp edge meaning that its pixel values vary a lot, or not. Since the noise also causes the fluctuation, the variance of the noise should be considered. Namely, the ratio of the noise variance and the overall variance of the filter window is the deciding factor.

The following expression 1 is used to determine the pixel values of the filtered image.

$$
\begin{aligned}
g(x,y) = f(x,y) - \frac{\sigma_n^2}{\sigma_{xy}^2}\left(f(x,y) - \mu_{xy}\right)\ ,\ \textit{where} \\
\textit{if}\ \frac{\sigma_n^2}{\sigma_{xy}^2} > 1\ \textit{then set it to}\ 1
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
g(x,y) &: \textit{the filtered image} \\
f(x,y) &: \textit{the noisy image} \\
\sigma_n^2 &: \textit{the noise variance} \\
\sigma_{xy}^2 &: \textit{the filter window variance} \\
\mu_{xy} &: \textit{the filter window mean}
\end{aligned}
$$

The expression can be interpreted as

When the filter with the 7×7 filter window is applied to the noisy circuit board, the Figure 5 is obtained.

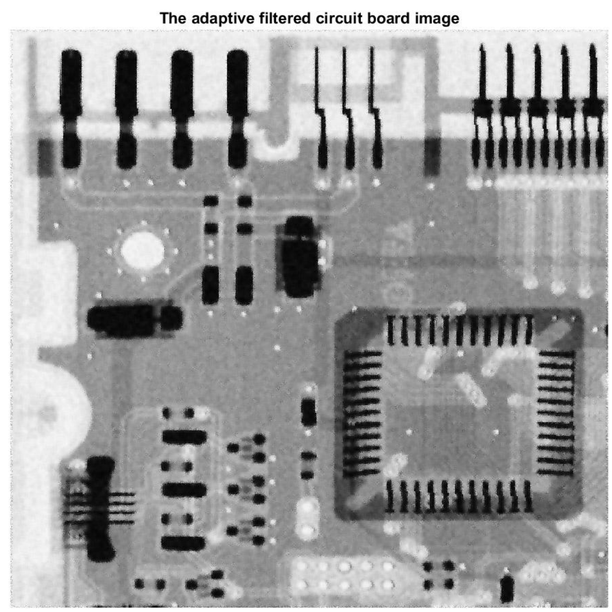

Fig. 5. The adaptive arithmetic filtered circuit board image

The figure 5 shows that the adaptive filter can remove the Gaussian noise while preventing the blurring considerably. Therefore, the details in the sharp edges such as legs of the circuit components, writings and conducting paths are preserved. Therefore, the performance of the adaptive filter is much better than the arithmetic and geometric mean filters since it can keep the sharp edges, considerably.

D. Estimation of the Noise Variance

In general, the value of the noise variance is not known; so it should be estimated in order to use the adaptive arithmetic filter. The following steps[1] can be followed in order to estimate the noise variance.
Firstly, the original image should be suppressed so that its influence is minimized. This procedure can be achieved by a difference operator. The differences of the image by its next-to neighbors horizontally and vertically are taken. This operation doesn't affects the variance of the noise since it is assumed that the noise is independent from the original image.
Secondly, a histogram of a standard deviation is computed. The local variances in the neighbors of the pixels, meaning that a square window is considered, are calculated, and their means are used in the function named as $h(k)$ in the algorithm[1] to plot the histogram.
Finally, the histogram is evaluated in order to find the desired variance. It is obtained by computing the mean square value of the histogram.

II. Adaptive Median Filter

In this part, the contaminated version of the Pentagon image shown in the Figure 6. Firstly, it is contaminated with 25% salt and 25% pepper noise. The version that is obtained by adding the noise is shown in the Figure 7.

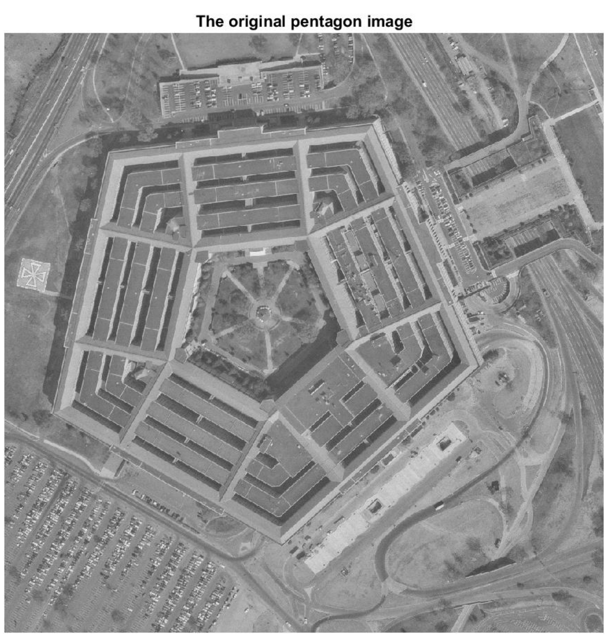

Fig. 6. The original Pentagon image

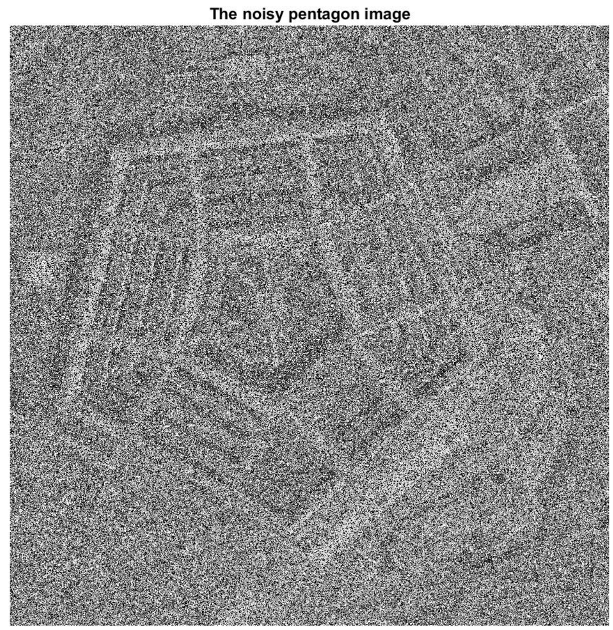

Fig. 7. The original Pentagon image

The median filters are effective in attenuating the impulsive noises such as salt and pepper noise, However, it can lead to poor results for the noise density above 20% and also for very large or very small window sizes. Therefore, an adaptive median filter is obtained by modifying the standard median filter in order to overcome these issues.

A. Median Filter

An example of a nonlinear filter is median filter. This method uses a filter window that moves all over the image, and evaluates the median of its entries, then makes the corresponding pixel of the center entry equal to the median. In order no to exceed the original image size at the edges, the image is extended with zero padding.

When the filter with the 7×7 filter window is applied to the noisy Pentagon image, the Figure 8 is obtained.
It can be seen from the Figure 8 that the median filter is successful in the attenuation of the impulsive noise; however, it can cause some distortion in the image. So, some details in the image such as cars in the parking slots, the helipad, the roofs of the buildings, etc. are lost in the filtered image.

There exist lines in the edges since the zero padding is applied and it effects the values of the pixels in the edges.

Fig. 8. The median filtered Pentagon image

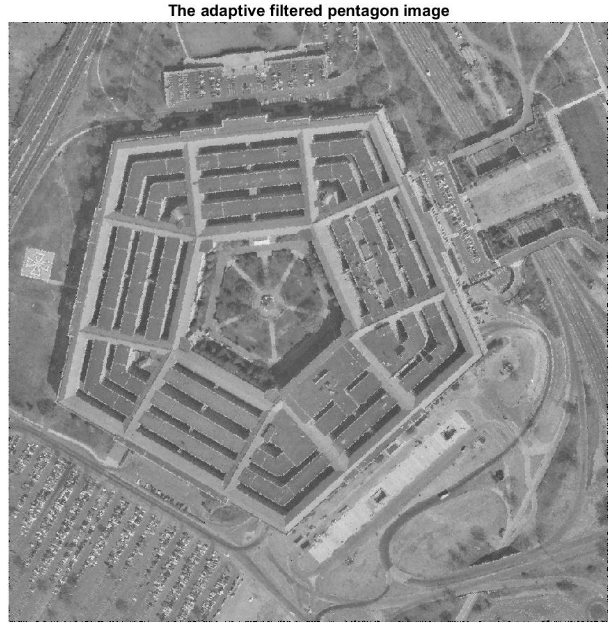

Fig. 9. The adaptive median filtered Pentagon image

B. Adaptive Median Filter

The median filter can be modified in order to preserve the details in the image that has impulsive noises by an adaptive approach. The objectives of the adaptive median filter are removal of the salt and pepper noise, smoothing the non-impulse noises, and reduction of the image distortion and excessive thinning of image details.

The adaptive median filter has two check points. Firstly, it searches for the median of the filter window which is distinct from the maximum and minimum of the filter window. If there exists no such median value, then the algorithm increases the size of the filter window up to the given maximum size. Then, it checks whether the center pixel of the filter window is the maximum or minimum of the filter window, or not. If the center is not the maximum or minimum, then it picks the center pixel value as the filtered image pixel value; If not, it picks the median value from the first check point as the filtered image pixel value.

When the adaptive filter with starting from the 3×3 filter window up to size of 7×7 is applied to the noisy Pentagon, the Figure 9 is obtained.
It can be seen from the Figure 9 that the adaptive median filter gives much better results. It is able to remove the noise while preserving the details of the image, considerably.

C. Comparison of Median and Adaptive Median Filter

By examining the Figures 8 and 9, it can be said that the adaptive median filter gives more accurate result than the median filter, since the adaptive one takes into account not only the median of the filter window but also the original pixel value. For example, the helipad, the cars in the parking slots, and the roof of the buildings are much clearer in the adaptive filtered image since the adaptive one assigned the original pixel values in these regions rather than the median of their filter window.

III. Notch Filtering

A. The Spectrum of the Astronaut and the Car Images

Images can be also represented in the frequency domain by taking 2-Dimensional Fourier transform. The spectrum in the frequency domain can show the periodic interference and the amount of DC parts of the image, which have low frequency values. This representation is used to attenuate the patterned interference.

The Fourier transforms of the astronaut and the car images which are given in the Figures 10, 11 are calculated and the spectrum of them is shown in the Figures 12, 13; respectively.

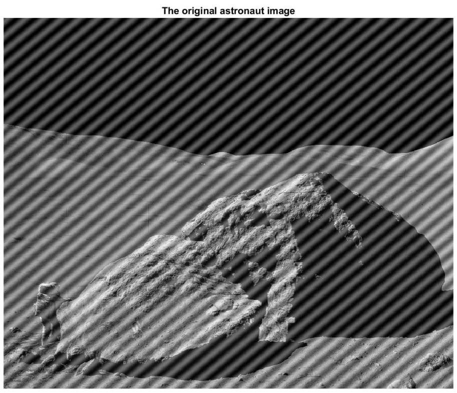

Fig. 10. The original astronaut image

Fig. 11. The original car image

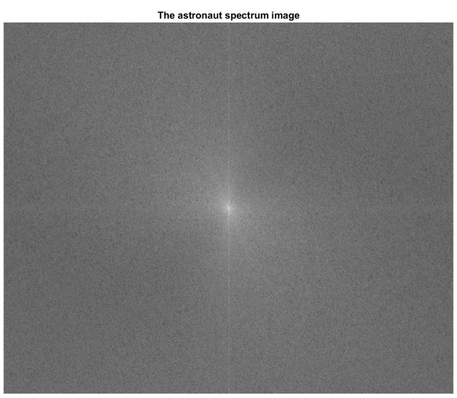

Fig. 12. The spectrum of the astronaut image

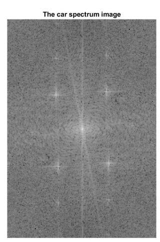

Fig. 13. The spectrum of the car image

The shifted spectrums are shown in the Figures 12, 13 in order to observe the low frequencies at the center as the common understanding of spectrum of the low and high frequencies. Also, the logarithm values of the spectrum is shown since the low frequencies (DC values) are too dominant to be able to distinguish the high frequencies.

B. Setting Threshold to the Spectrums

A threshold is considered in order to pick the peaks in the spectrum. For the astronaut image, it is 92% of its maximum spectrum values; for the car image, it is 77% of its maximum spectrum values.

The peaks of the spectrums[1] are shown in the Figures 14, 15. The peaks outside of the center are due to the interference.

[1]The figures are zoomed into the center

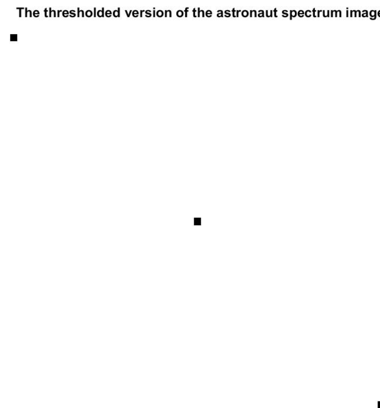

Fig. 14. The peaks of the spectrum of the astronaut image

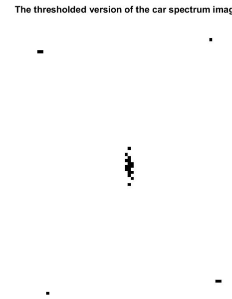

Fig. 15. The peaks of the spectrum of the car image

C. The Gaussian Notch Filter

The Gaussian notch filter can be used to separate the patterned image and the original image from each other. The filter is defined as following expression 2.

$$
\begin{aligned}
&H_{NR}(u,v) = \Pi_{k=1}^{Q} H_k(u,v) H_{-k}(u,v)\\
&H_k(u,v) = 1 - exp(-D_k^2(u,v)/2D_0^2)\\
&D_k(u,v) = \sqrt{(u - \frac{M}{2} - u_k)^2 + (v - \frac{N}{2} - v_k)^2}\\
&\{(u_k, v_k), (-u_k, -v_k)\}_{k=1}^{Q}, \text{ peak pairs}
\end{aligned} \tag{2}
$$

D_0 is the parameter for the bandwidth, and M and N are the size of the image. D_0 is taken 1 for the astronaut image, 11 for the car image.

The corresponding Gaussian notch filters for the astronaut and the car image are given in the Figures 16, 17.

Fig. 16. The Gaussian notch filter for the astronaut image

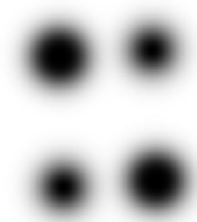

Fig. 17. The Gaussian notch filter for the car image

When the filters shown above are applied to the given astronaut and car images, the following filtered image shown in the Figures 18, 19, and the inference patterns shown in the Figures 20, 21; respectively.

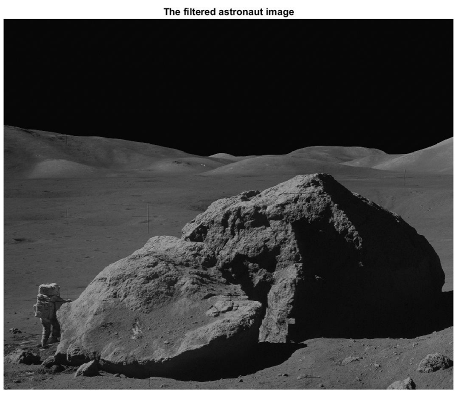

Fig. 18. The filtered astronaut image

Fig. 19. The filtered car image

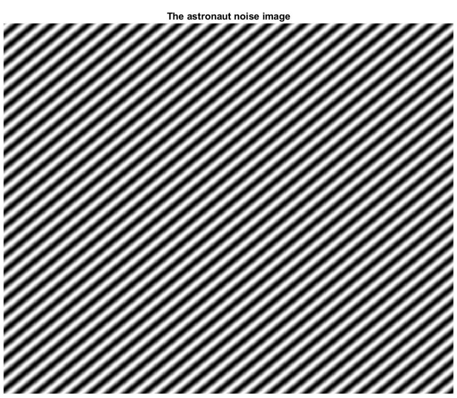

Fig. 20. The inference patterns of the astronaut image

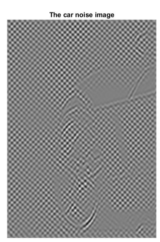

Fig. 21. The inference patterns of the car image

Since in the astronaut image, the spectrum of the pattern interference acts like impulse; the filter works almost perfectly in removing the noise. However, in the car image, the spectrum of the pattern interference spreads, doesn't act like impulse, the filter gives good result but it losses some part of the car image as seen in the inference pattern of the car image in the Figure 21.

IV. Heat Maps

In this part, the behaviour of the median filter is analyzed by applying different amounts of salt and pepper noise density with different sizes of the filter window.

The rose image shown in the Figure 22 is used to be contaminated by salt and pepper noises and to be filtered by the median filter.

Fig. 22. The rose image

The range of the noise density is selected as from 3% to 30% with the increments of 3%; and the range of the window size of the filter is selected as 3×3, 5×5, 7×7, 9×9, 11×11, 13×13.

In order to display the improvements, if any, the image quality index $SSIM$[2] is calculated for each combination. Then, the resulting indices from before filtering and after filtering are mapped into a heat map that indicates good performance as blue, and poor performance as red. The heat maps corresponding before and after filtering are given in the Figures 23, 24; respetively.

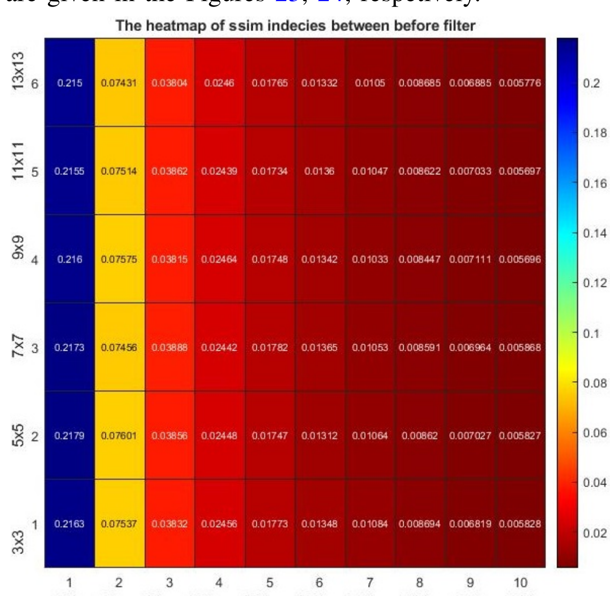

Fig. 23. The heat map before filtering

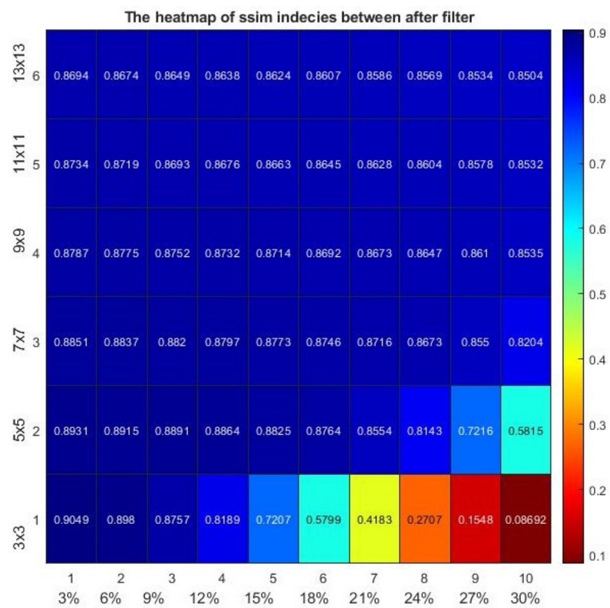

Fig. 24. The heat map before filtering

The Figure 23 shows that the similarity decreases while the noise density is increased, as expected.

It can be seen from the Figure 24 that the worst performance is where the filter window size is the smallest and the the noise density is the largest; and the best performance is where the filter window size is the largest and the the noise density is the smallest.

Also, as the size of the filter window increases, the performance increases; however, at some point (in this case, it is about 7×7) the improvement stops.

REFERENCES

[1] K. Rank, M. Lendl and R. Unbehauen, "Estimation of image noise variance," in IEE Proceedings - Vision, Image and Signal Processing, vol. 146, no. 2, pp. 80-84, April 1999. doi: 10.1049/ip-vis:19990238. Available at: https://ieeexplore.ieee.org/document/788764

[2] En.wikipedia.org. (2019). Structural similarity. [online] Available at: https://en.wikipedia.org/wiki/Structural_similarity [Accessed 3 Nov. 2019].