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Homework 6

Question 2: Kaiser

In order to design the Kaiser filter with the given specifications, the shape parameter, β and the filter order M should be determined. They are computed by the following expression 1:

$$\beta = \begin{cases} 0.1102(A-8.7) & A > 50 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21) & 21 \leq A \leq 50 \\ 0 & A < 21 \end{cases} \tag{1}$$

$$M = \frac{A-8}{2.285\Delta w} \quad \text{, where} \qquad A = -20log(\delta) \quad \Delta w = w_s - w_p$$

$\delta = min(\delta_s, \delta_p) = min(0.01, 0.05) = 0.01 \quad \Rightarrow \quad A = -20log(0.01) = 40$
$\beta = 0.5842(A-21)^{0.4} + 0.07886(A-21)|_{A=40} = 3.395$
$M = (A-8)/(2.285\Delta w)|_{A=40,\Delta w=0.2\pi} = 22.29 \quad \Rightarrow \quad M = 23$

By using *kaiser* function with the parameter β and M, the window for the low pass filter is found. Then, by using *fir1* function with the parameter $M-1$, $Wc = (w_s + w_p)/2$, $'low'$, and the found window above, the desired low pass filter with type 1 FIR is designed.

The frequency response of the designed Kaiser filter is given in the Figure 1.

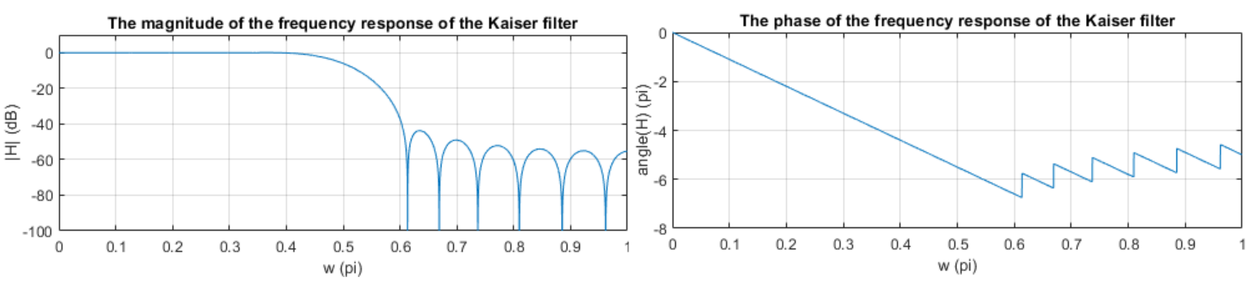

Figure 1: The frequency response of the Kaiser low pass filter

Question 3: Parks-McClellan

In order to design the Parks-McClellan filter with the given specifications, the algorithm *fircheb* that is given *Remez* documentation is used. The filter order is determined as same as the above filter, namely it is 23.

First, the initial $L+2$[1] extreme frequencies are assigned as equally spaced over the passband and stopband. Then, the algorithm solves the equation**(11)** in the *Remez* documentation to find the type 1 FIR filter coefficients and δ value. After computing the coefficients of the filter, the amplitude response of the filter is found by using the following expression 2[2].

$$H(e^{jw}) = e^{-jwL}A(e^{jw}) \quad \Rightarrow \quad A(e^{jw}) = e^{jwL}H(e^{jw}) \tag{2}$$

[1] $L = (M-1)/2$, only the positive side is considered since it is symmetric with respect to y axis.
[2] This formula is valid for type 1&2 FIR

After finding the amplitude response, the weighted error is computed by taking the difference between the found amplitude response and desired amplitude response, and multiplying with the given weights[3]. Then, the extreme frequencies are updated[4] with the extremum points of the found weighted error. After updating the extreme frequencies, the algorithm repeats these procedure until the updates are smaller than the given stopping criteria, $SN = 10^{-8}$.

The frequency response of the designed Kaiser filter is given in the Figure 2.

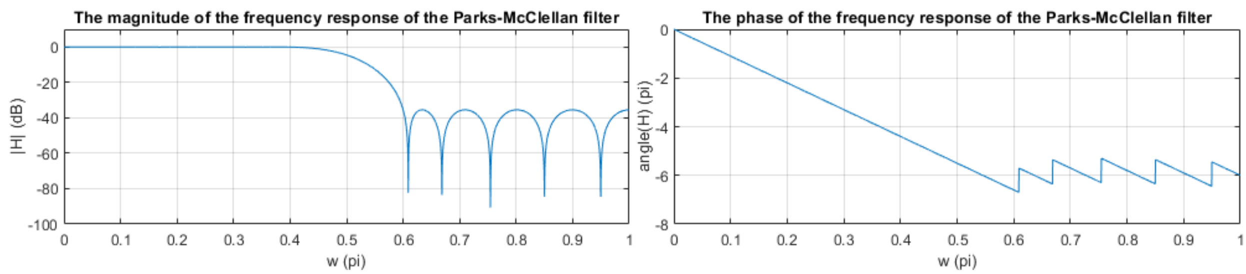

Figure 2: The frequency response of the Parks-McClellan low pass filter

Question 4: Comparison

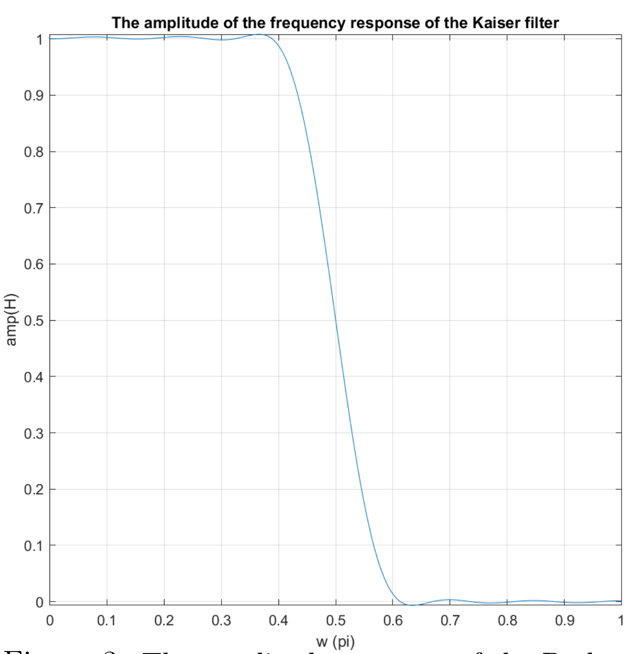

Figure 3: The amplitude response of the Parks-McClellan low pass filter

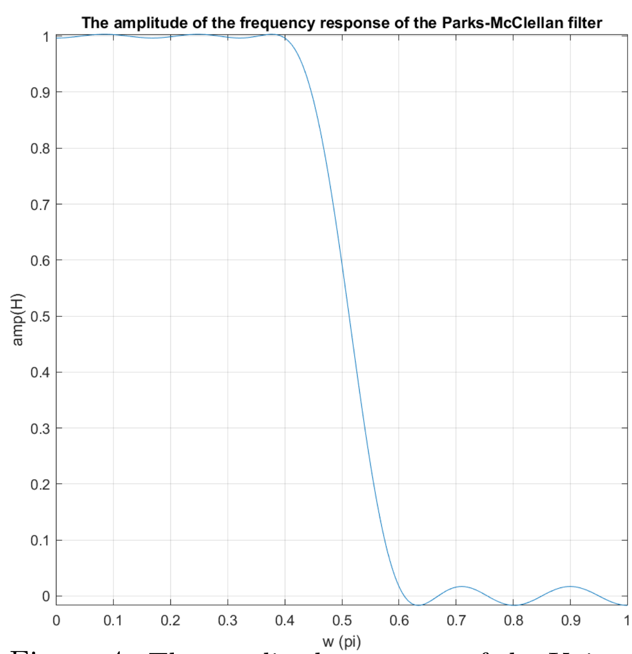

Figure 4: The amplitude response of the Kaiser low pass filter

[3] In our case, it is given as $K = \delta_p/\delta_s = 5$

[4] The 0 and π are added since they are starting and ending point.

The Figure 3 and 4 show the amplitude response of the filters which is designed using Kaiser and Parks-McClellan, respectively. So, it can be seen from the figures that the Kaiser design achieves the smaller tolerance level from passband and stopband, while the Parks-McClellan design achieves the tolerances with their weights.

The Kaiser low pass filter design tries to achieve the passband and stopband tolerances by considering the smaller one; while the Parks-McClellan low pass filter design tries to achieve them by their predefined weights. So, if it is desired to have difference tolerance levels in the passband and stopband, the Parks-McClellan design can be preferred rather than Kaiser design since Parks-McClellan takes into account different tolerance levels.

In addition, the above explanation can be seen from the magnitude responses of the filters which are shown in the Figure 1 and 2. The Kaiser design has -44 dB relative magnitude of the peak side lope as the Parks-McClellan design has -35.5 dB relative magnitude of the peak side lope. The Kaiser performance is better in the stopband than the Parks-McClellan performance due to the same reasoning mentioned above, it tries to achieve the minimum tolerance whereas the Parks-McClellan design does with their weights.

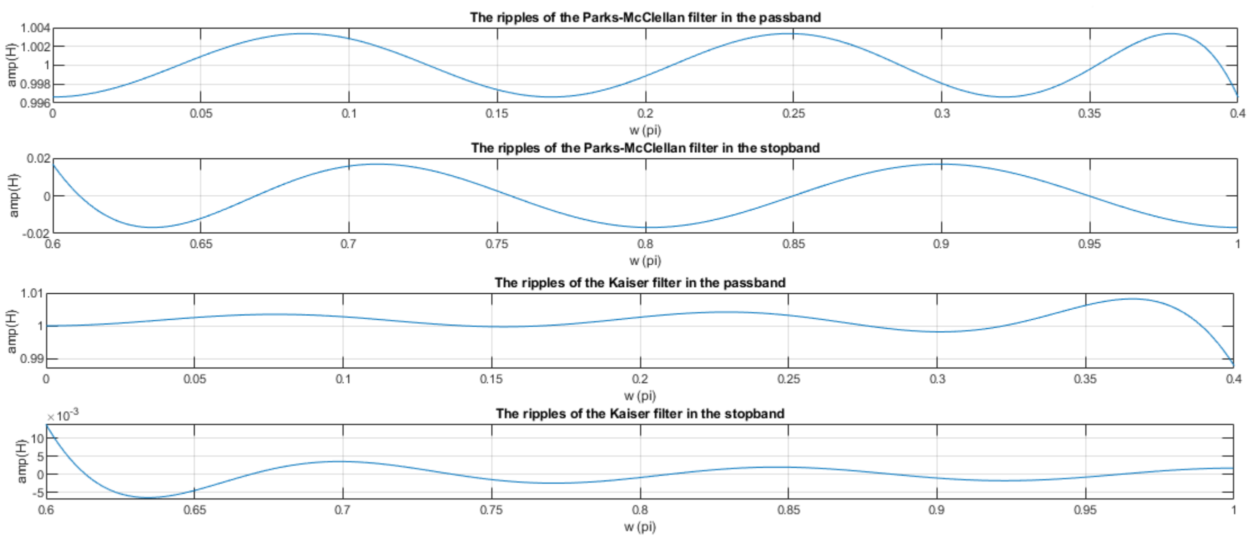

Figure 5: The ripples of the filters

The Figure 5 shows the ripples of the filters in passband and stopband. It can be seen from the figure that Kaiser design can achieve smaller ripples with the same order, yet the ripples are not equiripple; whereas the Parks-McClellan design can give weights to the ripples in different regions and the ripples are equiripple.

Therefore, the Parks-McClellan design is preferred if equiripple regions and different weights are desired; on the other hand, the Kaiser design is preferred if equiripple regions and different weights are not so important, since it can achieve better performance with the same order due to the smaller side lopes.