Sefa Kayraklık
2015401066

Homework 4

Introduction

In this homework, the channel is no longer ideal, so it has an impulse response different from an impulse at $n = 0$. Therefore, the intersymbol interference occurs through the channel and also there exists an AWGN noise ($n_k = N(0, \frac{N_0}{2})$). So the received signal y_k is given in the equation 1.

$$y_k = \sum_{n=-L}^{L} f_l x_{k-l} + n_k \tag{1}$$

f_l is the ISI channel impulse response, x_k is the binary data symbol at time k ($x_k \in \{-1, 1\}$), and n_k is the noise.

The three methods, which deal with the ISI issue, zero forcing equalizers (ZFE), minimum mean square equalizer (MMSE), and decision feedback equalizer (DFE) are covered in the class.

ZFE uses the following formula 2 which is derived in the class to estimate the equalizer impulse response:

$$\vec{w}_{opt} = \big((HH^H)^{-1}H\vec{e}\big)^H \tag{2}$$

H is the matrix[1] which contains the channel impulse response and it is padded with zeros. $\vec{e}$ is the desired output of the equalizer so its elements are zero except the center is 1.

MMSE uses the following formula 3 which is derived in the class to estimate the equalizer impulse response:

$$\vec{w}_{opt} = \left(\big(HH^H + \frac{1}{SNR}I\big)^{-1}H\vec{e}\right)^H \tag{3}$$

H is the matrix which contains the channel impulse response and it is padded with zeros. $\vec{e}$ is the desired output of the equalizer so its elements are zero except the center is 1.

DFE uses two equalizer filters, feedforward and feedback. The equation 3 with only the nonpositive indices of the channel coefficients is used to estimate the feedforward equalizer impulse response. This equalizer is used to removes the effects of current send signal at the past signals. the feedback filter impulse response is found by the convolution of the positive indices of the channel coefficients with the feedforward filter impulse response. The detected signal is given to this equalizer to remove the future effects of the current signals. Namely, the feedback equalizer mimics the cascaded system of the channel and the feedforward equalizer for estimation of the effect of the current detected to the future signals, time advance effects.

The *ZFE* with 9-tap and 15-tap; the *MMSE* with 9-tap and 15-tap; and the *DFE* with 4-taps for feedback, 5-taps for feedforward, and 7-taps for feedback, 8-taps for feedforward are implemented to remove the ISI in the following channels.

[1] It is generated as defined in the class.

1 Channel 1:

The first channel is $f_l|_{l=-4}^{4} = [0.04 \quad -0.06 \quad 0.07 \quad -0.21 \quad -0.5 \quad 0.72 \quad 0.36 \quad 0.21 \quad 0.06]$.

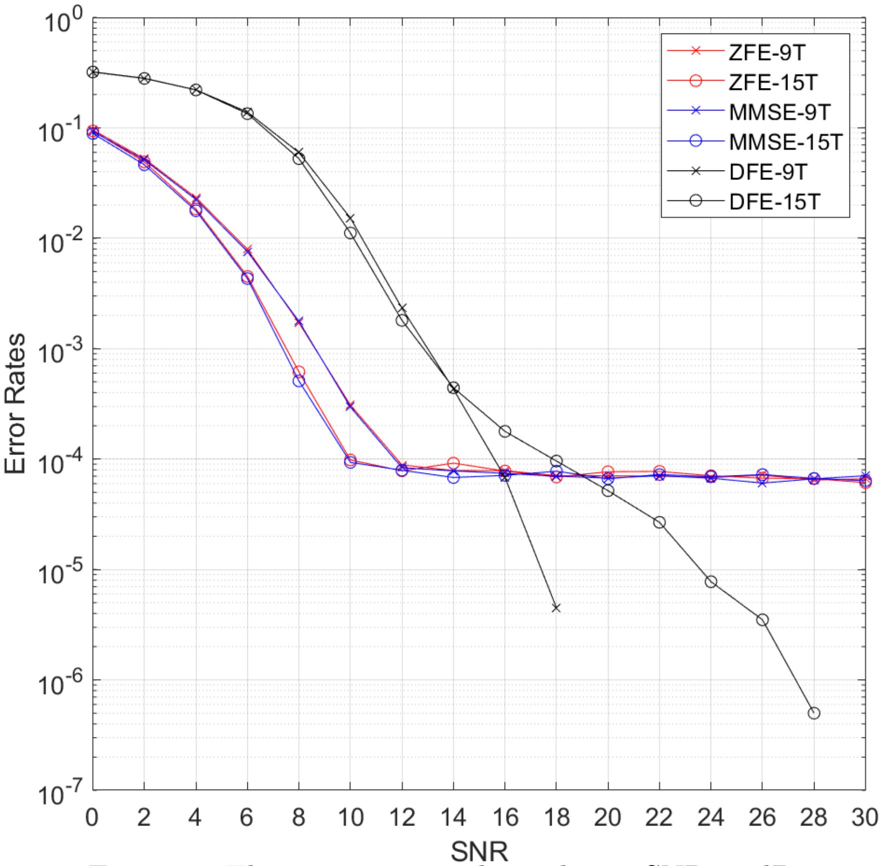

Figure 1: The error rate in channel 1 vs SNR in dB

The Figure 1 shows the plot of the bit error rate of the equalizers as a function of signal-to-noise ratio.

The effect of the number of taps: For *ZFE* and *MMSE*, increasing the number of taps improves the equalizer performance at low SNR values; however, at high SNR values, there exists no improvement in increasing the number of taps since these equalizers are not able to remove ISI in the system, completely. For *DFE*, increasing the number of taps doesn't improve the performance since the energy of the channel 1 is concentrated at its positive indices so its performance is determined by the feedback equalizer. So, the tap number doesn't effect so much since the feedback equalizer's size is determined by the positive indices of the channel impulse response.

The performance of the equalizers: The performances of the *ZFE* and *MMSE* are close to each other since the channel is not symmetric and its energy is mostly at the positive indices; so, the error is mainly caused by ISI. Therefore, the noise consideration in the *MMSE* doesn't have the desired impact on the performance. And their performances stops improving at some point since they are not able to remove it, completely. The performance of the *DFE* is better for the high SNR since the decision device is able to detect more correct bits as increasing SNR; and so the number of correct decisions increases the performance of the feedback equalizer, so it boosts the performance. For the low SNR, its performance is much worse than others since wrong decisions cause more errors through the feedback equalizer since ISI is caused by mostly the positive indices of the channel, so it should be filtered out by the feedback equalizer.

2 Channel 2:

The second channel is $f_l|_{l=-1}^{1} = [0.41 \quad 0.815 \quad 0.41]$.

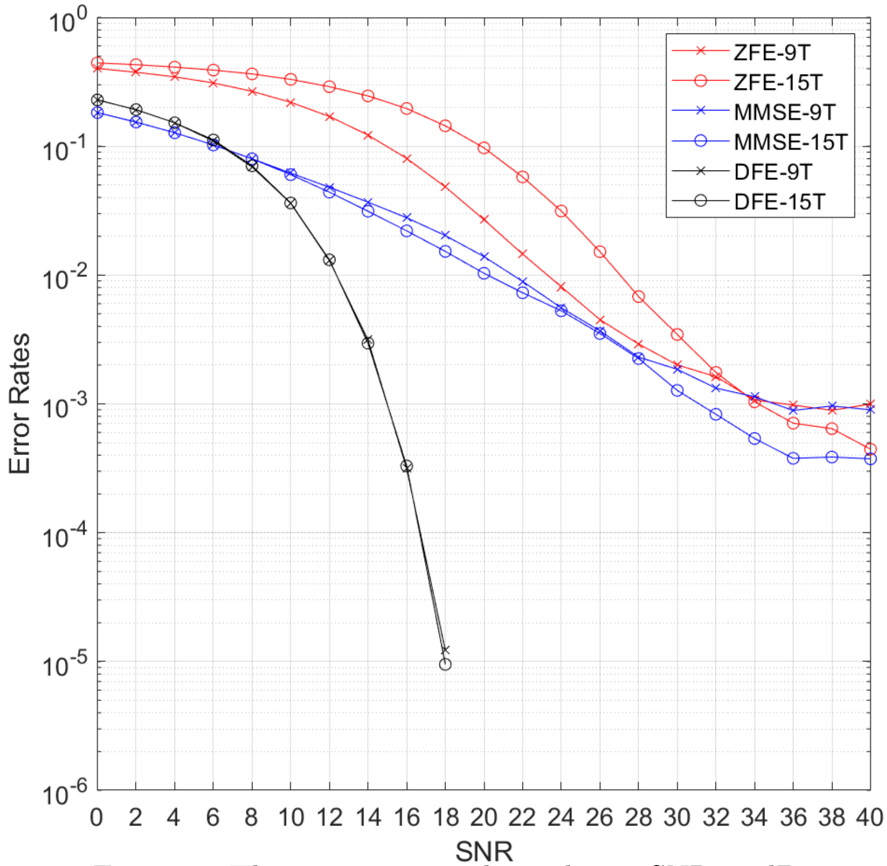

Figure 2: The error rate in channel 2 vs SNR in dB

The Figure 2 shows the plot of the bit error rate of the equalizers as a function of signal-to-noise ratio.

The effect of the number of taps: For *ZFE* and *MMSE*, increasing the number of taps improves the equalizer performance at high SNR values; however, at low SNR values, the performance is better in 9-tap equalizers since the noise has more impacts on the decision as increase in the equalizer size. For *DFE*, increasing the number of taps doesn't make so much change in the performance, since the performance also depends on the feedback equalizer. So, the tap number doesn't effect so much since the feedback equalizer's size is determined by the positive indices of the channel impulse response.

The performance of the equalizers: The performances of the *ZFE* is worse than *MMSE*'s since The *MMSE* takes into consideration of the noise power, and determines the coefficients of the equalizer accordingly. However, at really high SNR, the performances get closer to each other since the errors are mostly derived from ISI, and their performances stops improving at some point since they are not able to remove it, completely. The performance of the *DFE* is better for the high SNR since the decision device is able to detect more correct bits as increasing SNR; and so the number of correct decisions increases the performance of the feedback equalizer, so it boosts the performance. For the low SNR, its performance is worse than *MMSE* since wrong decisions cause more errors through the feedback equalizer.

3 Channel 3:

The third channel is $f_l|_{l=-2}^{2} = [0.227 \quad 0.460 \quad 0.688 \quad 0.460 \quad 0.227]$.

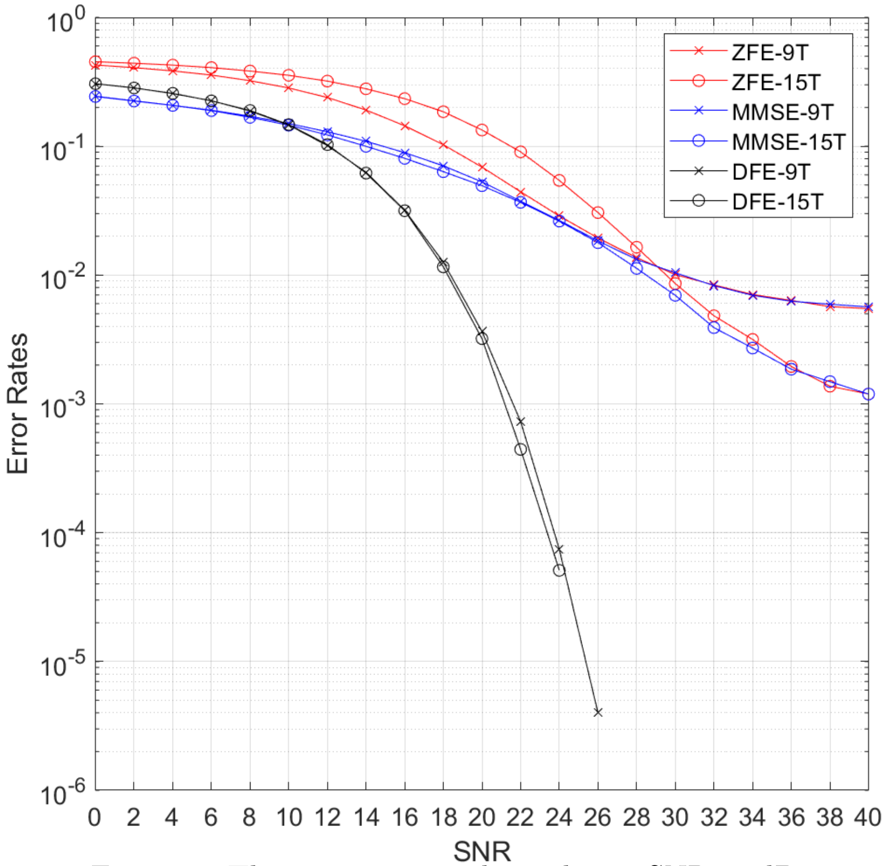

Figure 3: The error rate in channel 3 vs SNR in dB

The Figure 3 shows the plot of the bit error rate of the equalizers as a function of signal-to-noise ratio.

The effect of the number of taps: For *ZFE* and *MMSE*, increasing the number of taps improves the equalizer performance at high SNR values; however, at low SNR values, the performance is better in 9-tap equalizers since the noise has more impacts on the decision as increase in the equalizer size. For *DFE*, increasing the number of taps doesn't make so much change in the performance, since the performance also depends on the feedback equalizer. So, the tap number doesn't effect so much since the feedback equalizer's size is determined by the positive indices of the channel impulse response.

The performance of the equalizers: The performances of the *ZFE* is worse than *MMSE*'s since The *MMSE* takes into consideration of the noise power, and determines the coefficients of the equalizer accordingly. However, at really high SNR, the performances get closer to each other since the errors are mostly derived from ISI, and their performances stops improving at some point since they are not able to remove it, completely. The performance of the *DFE* is better for the high SNR since the decision device is able to detect more correct bits as increasing SNR; and so the number of correct decisions increases the performance of the feedback equalizer, so it boosts the performance. For the low SNR, its performance is worse than *MMSE* since wrong decisions cause more errors through the feedback equalizer.