



## Problem D

### Parity Game

Parity Game is played with  $N$  stones (numbered from 1 to  $N$ ) that are arranged sequentially on a straight line from left to right. The leftmost stone is numbered 1 while the rightmost stone is numbered  $N$ . Each stone has a positive integer  $W_i$  written on it which serves as the stone's value. Note that a stone's number and a stone's value are two different things.

Two players play alternatingly starting from the first player. In their turn, the player should take one or more consecutive stones which include the stone with the smallest or largest **number** that are still available. The game ends when there is no stone left on the game. For example, consider a game with  $N = 4$  stones numbered from 1 to 4, i.e. (1, 2, 3, 4). There are 7 valid moves for the first player on their first turn.

- [1, 1] – take stone number 1, and the game becomes (2, 3, 4).
- [1, 2] – take stone number 1 and 2, and the game becomes (3, 4).
- [1, 3] – take stone number 1, 2, and 3, and the game becomes (4).
- [1, 4] – take stone number 1, 2, 3, and 4, and end the game.
- [2, 4] – take stone number 2, 3, and 4, and the game becomes (1).
- [3, 4] – take stone number 3 and 4, and the game becomes (1, 2).
- [4, 4] – take stone number 4, and the game becomes (1, 2, 3).

If the game is (4, 5, 6) which looks like a middle game, then there are 5 valid moves.

- [4, 4] – take stone number 4, and the game becomes (5, 6).
- [4, 5] – take stone number 4 and 5, and the game becomes (6).
- [4, 6] – take stone number 4, 5, and 6, and end the game.
- [5, 6] – take stone number 5 and 6, and the game becomes (4).
- [6, 6] – take stone number 6, and the game becomes (4, 5).



In the last example,  $[5, 5]$  or  $[2, 4]$  are not valid moves. The former does not include the stone with the smallest or largest number (4 or 6), while the latter includes stones that are not available in the game (i.e. stones numbered 2 and 3 are already taken).

Each player then sums up the **values** on all the stones they collected. The first player wins if and only if he collects a total of odd value **and** the second player collects a total of even value; **otherwise**, the second player wins.

You are to assume both players play optimally, that is, if there exists a valid move that guarantees their win, then they will certainly play that move.

Your task in this problem is to determine how many valid **first moves** for the first player such that he will win the game if he plays that move.

### Input

Input begins with an integer  $T$  ( $1 \leq T \leq 10$ ) representing the number of cases.

Each case begins with an integer  $N$  ( $1 \leq N \leq 2000$ ) representing the number of stones in the game. The next line contains  $N$  integers  $W_i$  ( $1 \leq W_i \leq 10^9$ ) representing the value of the  $i^{th}$  stone, respectively.

### Output

For each case, output in a line "Case #X: Y" (without quotes) where X is the case number (starts from 1) and Y is the output for the respective case.

### Sample Input #1

```
3
3
5 11 7
5
8 103 59 74 61
4
100 101 102 103
```

### Sample Output #1

```
Case #1: 1
Case #2: 3
Case #3: 0
```

### *Explanation for the sample input/output #1*

For the 1<sup>st</sup> case, the only valid move that guarantees a win for the first player is [1, 3], i.e. take all the stones at once with a total value of  $5 + 11 + 7 = 23$  (an odd value). The second player is left with 0 (an even value), thus, the first player wins.

For the 2<sup>nd</sup> case, there are 3 valid first moves for the first player to win the game.

- [1, 5] – take all the stones and end the game; the total value is  $8 + 103 + 59 + 74 + 61 = 305$  (an odd value), while the second player's total value is 0 (an even value); thus, the first player wins.
- [2, 5] – take the last 4 stones; after this, the second player's only valid move is [1, 1]; the first player collects a total value of  $103 + 59 + 74 + 61 = 297$  (an odd value), while the second player's total value is 8 (an even value); thus, the first player wins.
- [5, 5] – take the last stone; after this, there are 7 valid moves for the second player, but none can cause the second player to win the game (the first player always has a counter move to make him win); thus, the first player wins.

Some example games:

- $P1[5, 5] \rightarrow P2[1, 1] \rightarrow P1[2, 4]$  – the first player's total value is  $61 + 103 + 59 + 74 = 297$  (an odd value) while the second player's total value is 8 (an even value).
- $P1[5, 5] \rightarrow P2[1, 2] \rightarrow P1[4, 4] \rightarrow P2[3, 3]$  – the first player's total value is  $61 + 74 = 135$  (an odd value) while the second player's total value is  $8 + 103 + 59 = 170$  (an even value).
- $P1[5, 5] \rightarrow P2[2, 4] \rightarrow P1[1, 1]$  – the first player's total value is  $61 + 8 = 69$  (an odd value) while the second player's total value is  $103 + 59 + 74 = 236$  (an even value).
- ...

In all those games, there exists a strategy for the first player to win the game.

For the 3<sup>rd</sup> case, it is not possible for the first player to win the game.