

Final Round

# Problem B Power of 2 Sum

The number 2 holds a special role in computer science. For example, our conventional computers use a binary system (base-2 number system) as it is much easier for machines to operate on. It can be proven (whilst being obvious as  $2^0=1$ ) that any positive integer can be constructed as a summation of powers of two, e.g.,  $23=2^4+2^2+2^1+2^0=16+4+2+1$ ; or alternatively,  $23=2^0+2^0+\cdots+2^0$ , i.e. 23 terms of  $2^0$ . Of course, we can go further, for example,  $23=2^{-1}+2^{-1}+\cdots+2^{-1}$ , i.e. 46 terms of  $2^{-1}$ ; recall that  $2^{-1}=1/2$ .

In this problem, you are given two positive integers, N and K. Your task is to find a sequence of **exactly** K integers  $(a_1, a_2, \ldots, a_K)$  such that:

$$2^{a_1} + 2^{a_2} + \dots + 2^{a_K} = N,$$

$$-32 < a_i < 32$$
, and

• 
$$a_1 > a_2 > \cdots > a_K$$
.

Among all possible sequences, you should print the lexicographically smallest sequence. A sequence  $b_1, b_2, \ldots, b_K$  is lexicographically smaller than a sequence  $c_1, c_2, \ldots, c_K$  if and only if there exists j  $(1 \le j \le K)$  such that  $b_i < c_j$  while  $b_i = c_i$  for all i < j.

For example, let  ${\cal N}=23$  and  ${\cal K}=5$ . The possible sequences:

$$\bullet \ \ (4,2,1,-1,-1) \rightarrow 2^4 + 2^2 + 2^1 + 2^{-1} + 2^{-1} = 16 + 4 + 2 + 1/2 + 1/2 = 23$$

$$\quad \bullet \quad (4,2,0,0,0) \to 2^4 + 2^2 + 2^0 + 2^0 + 2^0 = 16 + 4 + 1 + 1 + 1 = 23$$

• 
$$(4,1,1,1,0) \rightarrow 2^4 + 2^1 + 2^1 + 2^1 + 2^0 = 16 + 2 + 2 + 2 + 1 = 23$$

• 
$$(3,3,2,1,0) \rightarrow 2^3 + 2^3 + 2^2 + 2^1 + 2^0 = 8 + 8 + 4 + 2 + 1 = 23$$

Among all possible sequences, (3,3,2,1,0) is the lexicographically smallest.

Sometimes it might not be possible to have such a sequence. In such a case, you should output "No" (without quotes).



#### Input

Input begins with an integer T ( $1 \le T \le 20$ ) representing the number of cases.

Each case contains two integers N K ( $1 \le N, K \le 100\,000$ ) representing the positive integer you should construct and the number of terms, respectively.

### Output

For each case, output in a line "Case #X: Y" (without quotes) where X is the case number (starts from 1) and Y is the output for the respective case. If there is a valid sequence for the input, then Y contains exactly K integers ranging from -32 to 32 in nonincreasing order and each separated by a single space. Otherwise, Y is "No" (without quotes).

## Sample Input #1

```
3
23 5
5 1
1 4
```

### Sample Output #1

```
Case #1: 3 3 2 1 0
Case #2: No
Case #3: -2 -2 -2
```

Explanation for the sample input/output #1

For the  $2^{nd}$  case, 5 is not a power of 2, thus, it cannot be represented with a sequence of only a single term.

For the  $3^{rd}$  case, 1 = 1/4 + 1/4 + 1/4 + 1/4.