Advanced Derivatives: Implied Volatility Construction

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We loaded the data from the excel sheet and extract the implied volatilities for the respective strike values and times to maturity. The underlying asset price is set to $S_0 = 100$ and a fixed interval dK between strikes is defined.

Firstly we have computed the initial option prices based on their intrinsic value if the options were exercised immediately at $t = 0$.

We computed a set of theoretical option prices using the BSM formula and the implied volatilities loaded. This is clearly done only for strikes and time to maturities that have positive implied vol.

We set a finite difference framework with the Dupire Forward Equation to compute call option prices for the next maturity T_{i+1} based on the prices of T_i , this is done by discretizing the equation over strikes and solving for prices and next time step.

We then set an optimization problem to minimize the squared difference between the computed prices form the Dupire approach and the actual observed prices at know maturities and strikes in order to adjust the local volatility parameters.

The next step involves inverting the BSM formula for each price derived before in order to get the implied volatility surface for each strike and time to maturity.

In the second phase, we interpolated the implied volatilities for maturities that are not explicitly given in the market data but fall between known expirations. We assumed that volatility remains constant between two given maturities. Using the Andreasen-Huge method, we constructed a tridiagonal matrix $\tilde{A}$ based on the same discretization approach as before, with a finer time step $\Delta\tilde{T} = \tilde{T} - T_j$. We propagated the option prices for intermediate maturities, and finally used the Black-Scholes model to derive the implied volatilities for these interpolated maturities.

Finally we plot the implied volatility surface

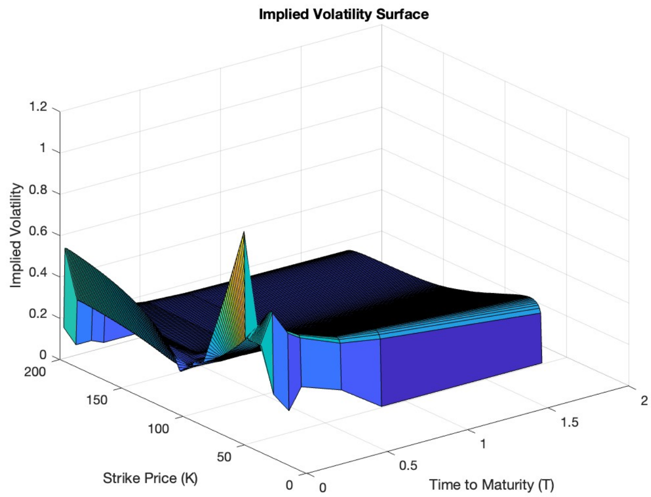

Figure 1: Implied Volatility Surface