Milestone 1

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Abstract. We study the solar continuum at visible and near-infrared wavelengths..

1. Introduction

2. Method

Here we present the methods we have used to obtain the results in Section 3.

2.1. Theory

The Friedmann-Robertson-Walker metric for flat space is defined as

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)(\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + sin^2\theta\mathrm{d}\phi^2)) \tag{1}$$

$$= a^2(t)(-\mathrm{d}\eta^2 + \mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + sin^2\theta\mathrm{d}\phi^2)), \tag{2}$$

where $a(t)$ is the scale factor, and η is the conformal time defined as

$$\frac{\mathrm{d}\eta}{\mathrm{d}a} = \frac{c}{a^2H} = \frac{c}{a\mathcal{H}}. \tag{3}$$

where c is the speed of light in vacuum, H is the Hubble parameter, and where we have defined the scaled Hubble parameter $\mathcal{H} \equiv aH$. This is the background cosmology for this project.

From the first Friedmann equation, we get an expression for the Hubble parameter,

$$H = H_0\sqrt{(\Omega_{\mathrm{b},0} + \Omega_{\mathrm{m},0})a^{-3} + (\Omega_{\mathrm{r},0} + \Omega_{\nu,0})a^{-4} + \Omega_{\Lambda,0}}, \tag{4}$$

and the scaled Hubble parameter,

$$\mathcal{H} = H_0\sqrt{(\Omega_{\mathrm{b},0} + \Omega_{\mathrm{m},0})a^{-1} + (\Omega_{\mathrm{r},0} + \Omega_{\nu,0})a^{-2} + \Omega_{\Lambda,0}a^2}, \tag{5}$$

where $H_0 = 70$ km s^{-1} Mpc$^-1$ is the Hubble parameter of today, $\Omega_\mathrm{b} = 0.046$, $\Omega_\mathrm{m} = 0.224$, $\Omega_\mathrm{r} = 8.3 \cdot 10^{-5}$, $\Omega_\nu = 0$ and $\Omega_\Lambda = 0.730$ are the density parameters of today of baryonic matter, dark matter, radiation, neutrinos, and dark energy, respectively. We will do our calculations without neutrinos. The subscript 0 means the parameter's value at present day.

These density parameters evolve in time as follows;

$$\Omega_\mathrm{b} = \Omega_{\mathrm{b},0}\left(\frac{H}{H_0}\right)^2 a^{-3}, \tag{6}$$

$$\Omega_\mathrm{m} = \Omega_{\mathrm{m},0}\left(\frac{H}{H_0}\right)^2 a^{-3}, \tag{7}$$

$$\Omega_\mathrm{r} = \Omega_{\mathrm{r},0}\left(\frac{H}{H_0}\right)^2 a^{-4}, \tag{8}$$

$$\Omega_\Lambda = \Omega_{\Lambda,0}\left(\frac{H}{H_0}\right)^2. \tag{9}$$

For convenience, we will not be working with the scale factor a, but its natural logarithm,

$$x = \ln a. \tag{10}$$

We define $a_0 = 1$ to be the scale factor of present day, meaning that $x \leq 0$. Equation 3 can then be rewritten as

$$\frac{\mathrm{d}\eta}{\mathrm{d}x} = \frac{c}{\mathcal{H}} \tag{11}$$

We will solve this ordinary differential equation (ODE) numerically.

We wish to know how the Hubble parameter H evolve with the scale factor x, and the cosmological redshift z. The cosmological redshift is defined as

$$1 + z = \frac{a_0}{a} = e^{-x}. \tag{12}$$

2.2. Implementation & Algorithm

The objective in this project is to find how the density parameters Ω_X evolve with x, and how H evolve with x and z. To do this, we will have to solve the ODE in Equation 11, the first Friedmann equation 4 and 5, and the equations 6, 7, 8, and 9, which we will do numerically.

We set up a gird of x values which we will use for our computations. The initial and final values of x corresponds to $a_\mathrm{initial} = 10^{-10}$ and $a_\mathrm{final} = 1$. The algorithm is

$$x_{i+1} = x_\mathrm{initial} + i \cdot \frac{(x_\mathrm{initial} - x_\mathrm{final})}{n-1}, \tag{13}$$

where we have n gird points. These grid points are denoted `x_eta` in our code.

To find $\eta(x)$ we use a pre-existing ODE solver employing a Runge Kutta method. To use this method, we need the initial value of $\eta(x)$. We can use that in the early days of the universe, around a_init, the cosmos consisted almost entirely of radiation, meaning $\Omega_\mathrm{r} = 1$. If we integrate Equation 3, we get

$$\eta_\mathrm{init} = \int_0^{a_\mathrm{init}} \frac{c}{H_0\sqrt{\Omega_\mathrm{r}}}\mathrm{d}a = \frac{a_\mathrm{init}c}{H_0}. \tag{14}$$

2.2.1. Future projects

We set up a grid of x and a values during ($z = 1630 - 614$) and after ($z = 614 - 0$) recombination. In two separate loops with respectively $n_1 - 1$ and n_2 number of iterations , where n_1 and

n_2 are the number of grid points during and after recombination ,we use the following algorithm:

$$x_{i+1} = x_{\text{start rec}} + i \cdot \frac{(x_{\text{end rec}} - x_{\text{start rec}})}{n_1 - 1}, \tag{15}$$

$$x_{n_1+i} = x_{\text{end rec}} + i \cdot \frac{(x_{\text{end rec}} - x_0)}{n_2 - 1}, \tag{16}$$

where $x_{\text{start rec}}$, $x_{\text{end rec}}$, and x_0 denote the value of x at the three stated redshifts, $z = 1630$, 614 and 0, respectively. Using the relation in Equation 10, we can make a similar grid of a values. These grid points are denoted `x_t` and `a_t`.

We will not be using these grids in this project, but in future ones

3. Results

The different density parameters are shown in Figure 1 against x. It shows that in the early universe, up until $x = -7.906$, there where radiation domination. This value of x correspond to a redshift of $z \approx 2712$. As the Ω_r decreases toward 0, Ω_m increases, having a maximum value of around $\Omega_\text{m} \approx 0.8$. We can also see that the dominating matter is mostly non-baryonic, dark matter, as Ω_b peaks at just under $\Omega_b = 0.2$ in the same scale frame x. Dark matter dominates from $x = -7.906$ to $x = -0.415$, corresponding with redshift $z \approx 2712 - 0.660$.

Dark matter- dark energy equality occurs at $z = 0.660$, which is when Ω_Λ becomes larger than Ω_m, meaning that in our time at $x = 0$ and $a = 1$, dark energy is dominating the Universe.

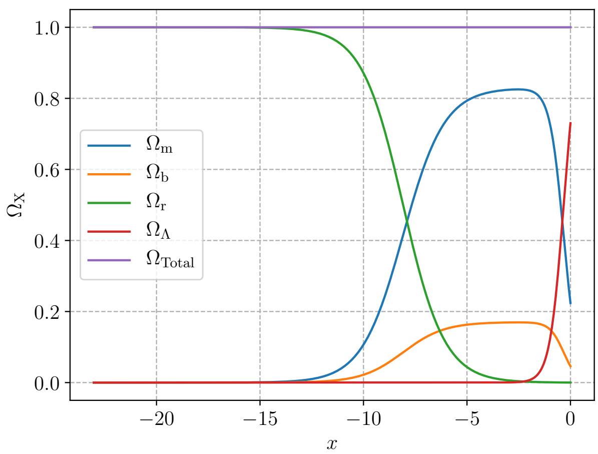

Fig. 1: The different density parameters, Ω, as a function of the natural logarithm of the scale factor, $x = \ln a$. We see that their sum is equal to 1 at all times, which is expected. The graph shows that the Universe was radiation dominated at early times, and that radiation-matter equality occurred at around $x = -7.906$. The radiation density parameter decreases quickly, and is close to zero around $x = -5$. Matter, dark matter, then dominated until matter-dark energy equality at around $x = -0.415$

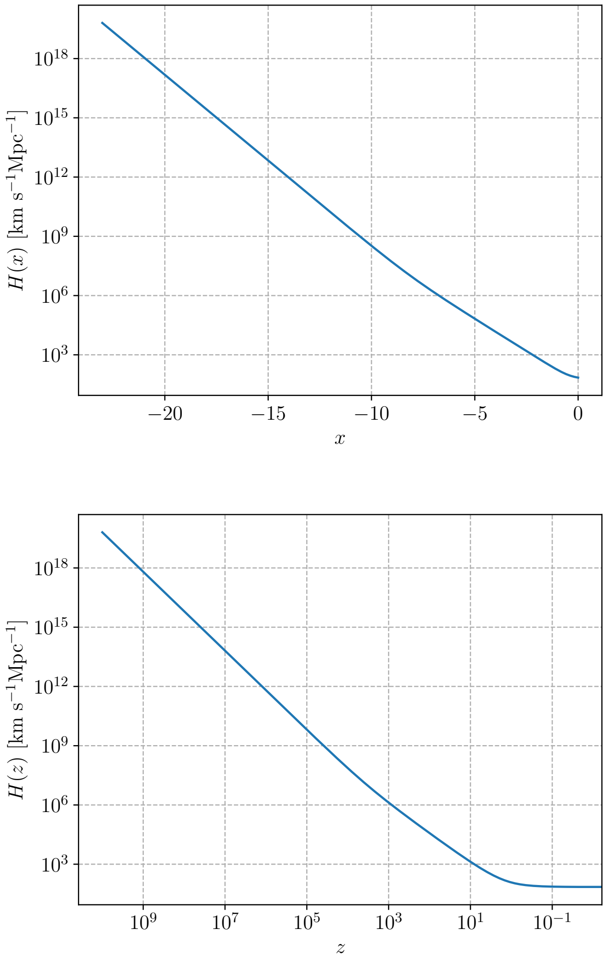

Fig. 2: The Hubble parameter shown against the natural logarithm of the scale factor, $x = \ln a$ (top figure), and against the redshift, z (lower figure). Note the flipped x-axis in the redshift graph, and the log-scale on the y-axis. We see that the Hubble parameter decreases as the scale factor of the universe increases.

4. Discussion

5. Conclusions

Acknowledgements. We are much indebted to Rob Rutten for exemplary instruction.

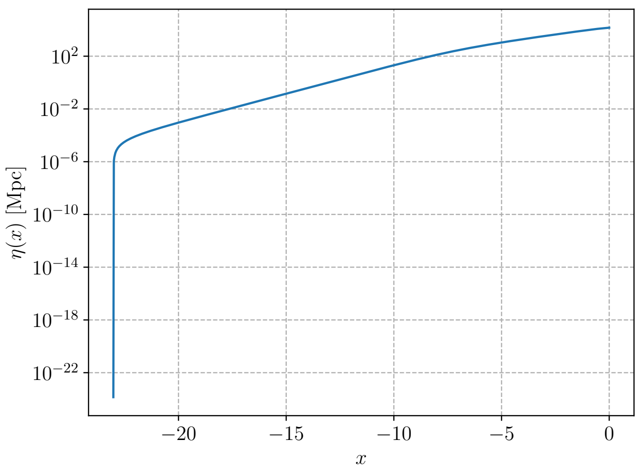

Fig. 3: The conformal time is shown on the y-axis against $x = \ln a$ on the x-axis